FYS3520 - Problem set 2

Spring term 2017

Problem 1 – In class

a) What is the nuclear radius? Give several different ways to measure it and compare important differences.

b) All known heavy nuclei exhibit an excess neutrons. Looking eg. at ^{240}Pu, it has more then 40 % more neutrons then protons. Naively one should expect a larger radius for the neutrons then for the protons ($\rightarrow$ How do we usually call these radii), however, they are observed to agree within about 0.1fm. How can this be explained?

c) When analyze the scattering of α-particles on medium and heavy nuclei we can learn about the nuclear radius. How?

d) What is the angular dependence of scattered α-particles? Sketch the cross-section of backscattered nuclei (scattering angle $\theta \approx \pi$) as a function of the incident energy. Take this to estimate the nuclear radius of ^{64}Cu, where the critical energy E_c is about 13.7 MeV.
Hint: What is the relation of kinetic and potential energy in backscattering?

Problem 2

a) The mass of ^{27}Al and ^{235}U has been measured to be 26.9815 u and 235.0439 u respectively. Use this to estimate the binding energy of the two.

b) Find the binding energy of the two nuclei using the semi empirical mass formula. How well does it agree with the result in a)/what are possible reasons for the deviations?
(Use $a_v = 15.5$ MeV, $a_s = 16.8$ MeV, $a_c = 0.72$ MeV and $a_{\text{sym}} = 23$ MeV).

Problem 3

The emiprical mass formula is (neglecting the odd-even effect)

$$M(A, Z) = Z(m_p + m_e) + (A - Z)m_n - a_v A + a_s A^{2/3} + a_c \frac{Z(Z-1)}{A^{1/3}} + a_{\text{sym}} \frac{(A-2Z)^2}{A} \tag{1}$$

Where a_v, a_s, a_c and a_{sym} are constants. Explain the dependence on Z and A for the different terms.

a) Two isobar nuclei 1 and 2 are called mirror nuclei if they result in each other by the exchange of the neutron an proton number: $A_1 = A_2, Z_1 = N_2, N_1 = Z_2$. They have a analogous structure and therefore also the same quantum numbers for the total angular momentrum J and parity π. As an example we look at ^{27}Al and ^{27}Si, both having (J=5/2,π=+1) in the ground state. ^{27}Si decays by β^+-decay into ^{27}Al ($^{27}\text{Si} \rightarrow ^{27}\text{Al} + e^+ + \nu$), where the sum of the kinetic energies of the positron and neutrino has been measured to be at max $E_0 = 3.8$ MeV. Deduce the radius parameter r_0 from E_0.

Hint: If we assume that the nuclear forces are charge independent, the energy released in the decay of mirror nuclei is given by the mass difference $(m_n - m_p)^2 = 1.29$ MeV and the difference in the coulomb energy. The charge distribution can be assumes as a homogeneously charged sphere (total charge Ze and radius R) with the coulomb energy W_c

$$W_c = \frac{3}{5} \frac{Q^2}{4\pi\epsilon_0} \frac{1}{R} \tag{2}$$

b) Use (1) to show that for a constant A the Z value that corresponds to the most stable nucleus is

$$Z_{\text{min}} = \frac{m_n c^2 - (m_p + m_e)c^2 + a_c A^{-1/3} + 4a_{\text{sym}}}{2a_c A^{-1/3} + 8a_{\text{sym}} A^{-1}}$$

c) Determine the most stable isobar with mass number $A = 87$. (Again, use $a_v = 15.5$ MeV, $a_s = 16.8$ MeV, $a_c = 0.72$ MeV and $a_{\text{sym}} = 23$ MeV).

Problem 4

Find the Q-value of the reaction:

$$^{86}\text{Kr} + \text{d} \rightarrow ^{87}\text{Kr} + \text{p}$$

Rember: d = ^{2}H.

Problem 5

Explain how the mass of a nucleus can be calculated from the plot in figure below. Explain briefly some of the main features of the plot and estimate the mass of ^{130}Xe. What is the relation to fission and fusion? How did we get the heavy elements?

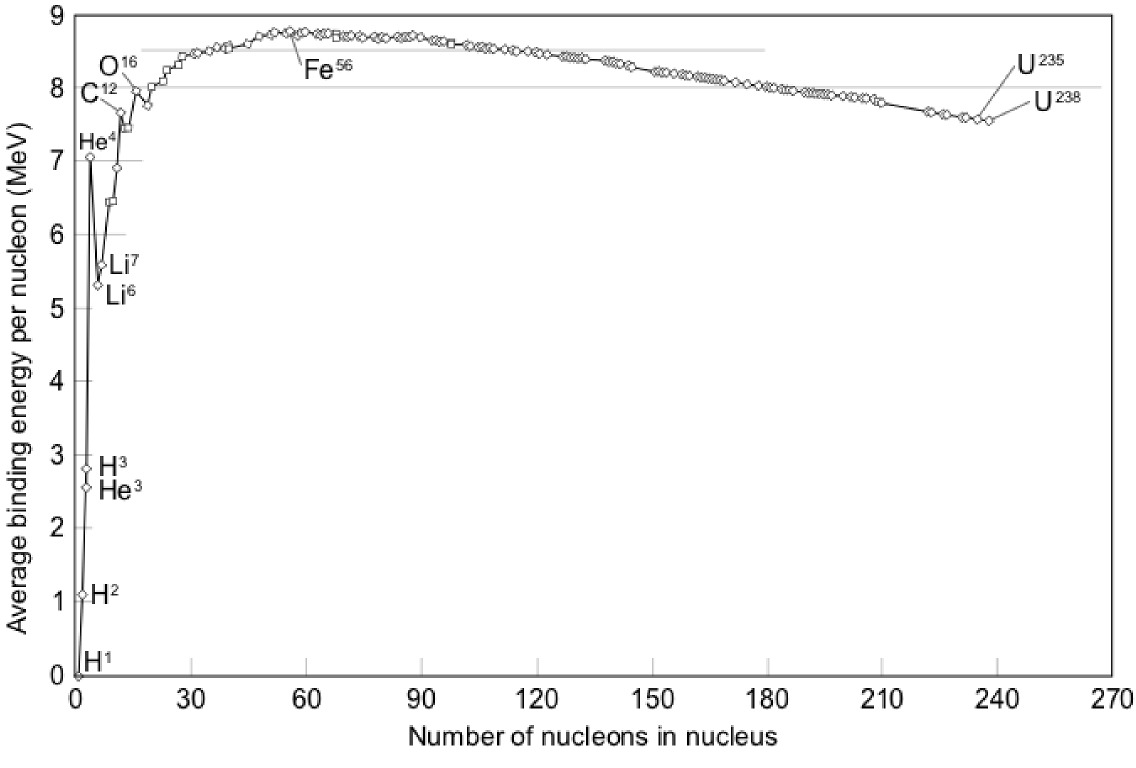

Figure 1: Average binding energy per nucleon for stable nuclei as a function of the mass number.

Problem 6 – not as difficult as it may look like!

We want to refresh our understanding of multipolar radiation. Pen and paper can facilitate the process. There is various approaches and we will only go one first step.

Recall from electrodynamics that the electric field **E** and the magnetic field **B** can be deduced from the scalar potential ′ and the vector potential **A**. In electrostatics we find

$$\boldsymbol{E}(\boldsymbol{r}) = -\boldsymbol{\nabla}\varphi(\boldsymbol{r}), \quad \boldsymbol{B}(\boldsymbol{r}) = \boldsymbol{\nabla} \times \boldsymbol{A}. \tag{3}$$

For the scalar potential $\boldsymbol{\varphi}$ of a spatially localized source with density $\boldsymbol{\rho}(\boldsymbol{r}^p rime) \neq 0$ that can be enclosed in a sphere with finite radius R we find

$$\varphi(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0} \int d^3r' \frac{\boldsymbol{\rho}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}. \tag{4}$$

The evaluation of such a volume integral is not always easy. However, we can often give the asymptotic behavior of $\boldsymbol{\varphi}$ and $\boldsymbol{E}$, so far away from $\boldsymbol{\rho}(\boldsymbol{r}') \neq 0$.

a) Show that if the scalar potential is developed in spherical harmonics,

$$\varphi(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{4\pi}{2l+1} q_{lm} \frac{Y_{lm}(\boldsymbol{\theta}, \boldsymbol{\phi})}{r^{l+1}}, \tag{5}$$

the expansion coefficients q_{lm} are given by $\int Y^*_{lm}(\theta', \phi') r'^l \rho(\boldsymbol{x}') d^3x'$. These coefficients are commonly called the multipole moments.
Hint: You may use the expansion in spherical harmonics

$$\frac{1}{|\boldsymbol{r} - \boldsymbol{r}'|} = 4\pi \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{1}{2l+1} \frac{r_<^l}{r_>^{l+1}} Y^*_{lm}(\theta', \phi') Y_{lm}(\theta, \phi), \tag{6}$$

where $r_<$ is the smaller, and $r_>$ is the (length of the) larger of $\boldsymbol{r}$ and $\boldsymbol{r}'$.

b) Derive the expressions for the monopole q and dipole moment $\boldsymbol{p}$.

c) Compare the results with the expression from rectangular coordinates where

$$\varphi(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0} \left[\frac{q}{r} + \frac{\boldsymbol{p} \cdot \boldsymbol{r}}{r^3} + \ldots \right] \tag{7}$$

Hint: Use the addition theorem $\sum_{m=-l}^{l} |Y_{lm}(\theta, \phi)|^2 = \frac{2l+1}{4\pi}$.

Analog derivations can be obtained for the vector potential $\boldsymbol{A}$ and thus also for the magnetic field $\boldsymbol{B}$.