FYS3520 - Problem set 5

Spring term 2017

corrections:

v1:

- Problem 2: harmonized node counting of n_r between energy formula and spectroscopic notation (n_r, l), such that the first level is $1s$ and has $n_r = 1, l = 0 \rightarrow N = 0$ such that $E = \hbar\omega(0 + 3/2)$.

- corrected element name in Problem 3 c); + added one more isotope.

Problem 1 – in class

a) What are experimental evidences for the nuclear shell model?

b) Many aspects of the nuclear shell model were "borrowed" from atomic physics. However, there is a difference in the spectroscopic notation in these fields. What is it and why does it maybe make sense to use a different notation?

c) What are the magic numbers?

d) What is the ground state spin ("total angular momentum") and parity I^π of even-even ("gg") nuclei?

e) How can we determine the ground state spin of uneven-even ("ug") nuclei? Give the gs spin and parity for ^{15}O, ^{16}O and ^{17}O.

f) What is the additional challenge for uneven-uneven ("uu") nuclei? What other effects may we see that are not covered by the extremely independent single particle model?

g) If we have time: Motivate how the short range attractive nucleon-nucleon interaction bind nucleons into pairs moving in time reversed orbits.
Hint: Assume in a simplified that analysis that the range interaction is a contact potential $(-V_0\delta(\boldsymbol{r}_1 - \boldsymbol{r}_2))$ and motivate the results for different l orbitals (neglect ls coupling at first).

Problem 2   Generating the shell model, part 1

On problem-set 1 we solved the harmonic oscillator (HO) in 1 dimension. A convenient way to solve the Schödinger equation in three dimensions for an isotropic potential is the splitting in radial and angular part. The radial term reduces to the 1D equation.

a) What are the solutions to the radial and angular functions?

b) The energies of the a simple isotropic 3D HO are given by $E_N = \hbar\omega(N + 3/2)$, where $N = 2(n_r - 1) + l$, and $n_r = 1, 2, 3, \dots$ is the number of nodes in the of the radial solution $R_l(r)$.[1]
What are the possible values for l?
Hint: If you have problems solving this but want to continue, you may find the solution on problem set 3. But take care of the conventions used there.

c) Use this produce a list the levels, their energy and degeneracy up to quantum number $N = 4$. An example is given for $N = 1$

d) If we stick to this model, we reproduce the first magic numbers, but we have a degeneracy that does not look very realistic. How to improve our results? We may chose to a better nuclear potential. We have seen three potentials so far:

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[1]Note that there is different conventions for counting the nodes. Almost always the node at $r = 0$ is neglected, however and we count the node at $r \rightarrow \infty$. However, you will find several place in literature that don't count that either, then we would use $n_r \rightarrow n_r + 1$.

N	$E_N[\hbar\omega]$	(n_r, l)	degeneracy	total cumulative
0				
1	5/2	1p	6	8
2				
3				
4				

- square well potential (rough approximation, not very realistic)
- SHO (problem above, but easy to calculate)
- Woods-Saxon potential: "something in-between"

The Woods-Saxon potential is able to lift the L degeneracy, however it is not analytically solvable. We can understand it's effect by adding another term to the SHO Hamiltonian H_0,

$$H_1 = a_1(L^2 - \langle L^2 \rangle_N)\hbar\omega, \quad a_1 \in \mathbb{R}$$

where $\langle L^2 \rangle_N$ is the average of L^2 taken over each N-shell. It can be shown that it takes following value

$$\langle L^2 \rangle_N = \frac{N(N+3)}{2}.$$

Calculate the expectation value of H_1 for the up to $N = 3$. What sign must a_1 have to reproduce the observed splitting of L values? (What is energetically more favorable, higher or lower L values?)

Hint: The level scheme of the modified harmonic oscillator (MHO) is given in Figure 1

Problem 3 Generating the shell model, part 2

The MHO can still not reproduce the magic numbers correctly. The problem was solved by Maria Goeppert Mayer and J. Hans D. Jensen, who received the Nobel Prize in Physics 1963 "for their discoveries concerning nuclear shell structure". The major step was the introduction of a strong spin-orbit coupling. You have seen the Hamiltonian on Problem set 3,

$$H_{\text{SO}} = -V_{\text{SO}}\hat{\vec{L}} \cdot \hat{\vec{S}}, \quad V_{\text{SO}} \in \mathbb{R}^{>0}.$$

a) Sketch the level scheme for the first 8 levels, giving the spectroscopic notation, degeneracy and cumulative number of nucleons per level.

b) Compare this to the magic numbers.

c) Give the gs spin and parity for $^{27}_{13}\text{Al}$, $^{28}_{14}\text{Si}$, $^{29}_{14}\text{Si}$, $^{31}_{15}\text{P}$, $^{32}_{16}\text{S}$, $^{33}_{16}\text{S}$, $^{141}_{59}\text{Pr}$, $^{45}_{22}\text{Ti}$.

d) Do the same calculations for $^{23}_{11}\text{Na}$ and $^{203}_{81}\text{Tl}$ and look up the experimentally measured value. What do we learn?

e) What is the expectation value of the full shell-model Hamiltonian (assuming MHO as the "basic" potential)?
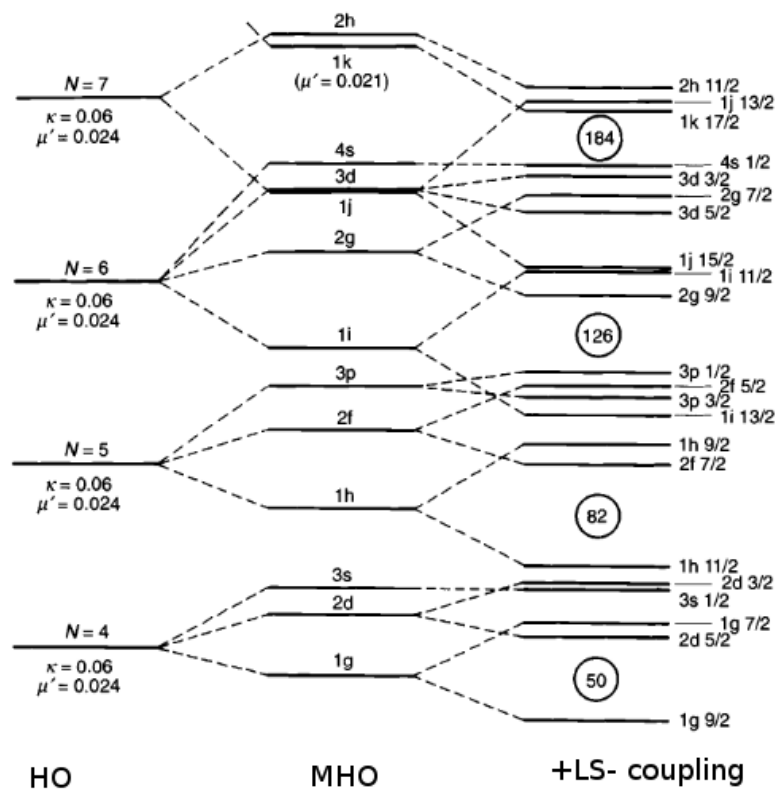
Figure 1: Level scheme with of HO, MHO and eventually LS-coupling for $N \geq 4$. Source: Nilsson, *Shapes and Shells in Nuclear Structure* (1995)