FYS3520 - Problem set 6

Spring term 2017

corrections: v1: Clarification of Problem 2. (argue for configurations that make sense; there is several options) v2: Corrected β

Problem 1 – in class

a) What is the difference between the Nilsson and the Shell model? For which nuclei are they usually used? Discuss the splitting of the $f_{7/2}$-level of the shell model.

b) What types of nuclear decay are there? Characterize the different types of decay.

c) What is the decay law and how do we derive it?

d) What is the relation between the width Γ of a state and its lifetime? How does Γ effect the notion of discrete states?

e) Explain what the branching ratios is and how this we may measure them.

f) ^{211}At offers many potential advantages for targeted α-particle therapy. Name possibilities how it could be produced. What it the daughter nucleus?

g) Assume you want to measure the half-live against γ-decay in an experiment? What set-up could you choose? What types of uncertainties will be associated to the result?

Problem 2 Shell Model

The ground-state spin of ^{17}F is $J^\pi = 5/2^+$, and of the first excited state it is $J^\pi = 1/2^+$. The second excited state is $J^\pi = 1/2^-$. Give the configurations for protons and neutron of in the ground-state and first excited state and name at least two possible configurations for the second excited state. With the information provided in the lecture it is not possible to calculate the exact energy of these configuration, so you should argue only for two configurations that make sense.

Problem 3 Deformations

a) Two different level schemes are displayed in Figure 1. Which one belongs to a rotational, which one belongs to a vibrational nucleus?

b) In "real life", we usually measure the γ-ray spectra in order to deduce the level scheme. Figure 2 shows a particularly "clean" γ-ray spectrum of ^{152}Dy. Based on this figure, would you conclude that ^{152}Dy is a vibrator or rotator?

c) Once we have deduced the level schemes (Figure 1), we can try to obtain more information about the nucleus. Estimate the frequency for the quadrupole phonon of the vibrational nucleus.
Hint: The N-phonon state of quadrupole mode ($\lambda = 2$) can be analyzed as a 5-dimensional oscillator (since $\mu = -\lambda, -\lambda + 1, \ldots, \lambda$).

Hint: What is the energy difference between two levels for a vibrational (or for a rotational) nucleus?

d) Estimate the moment of inertia for the rotational nucleus.

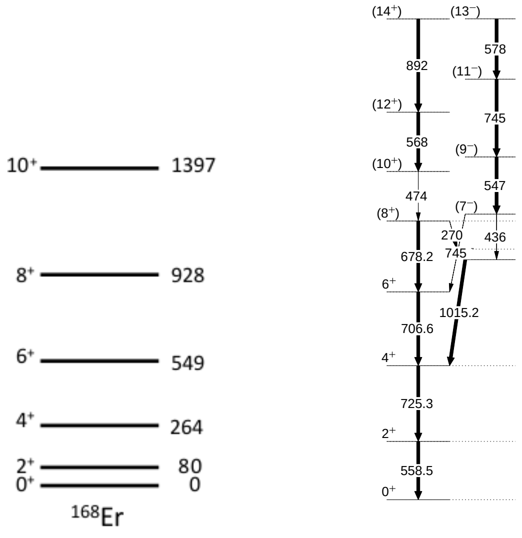
e) The two extremes of calculating the moment of inertia of a ellipsoidal with mass M is to assume that it is either a rigid body or a fluid within a vessel. For a rigid body the moment of inertia becomes:

$$\mathcal{J}_{\text{rigid}} = \frac{2}{5} M R_{\text{avr}}^2 (1 + 0.31\beta)$$

While for a vessel containing a fluid it is:

$$\mathcal{J}_{\text{fluid}} = \frac{9}{8\pi} M R_{\text{avr}}^2 \beta$$

Which of the two models of the inertia fits best with the estimated moment of inertia for the rotational nucleus? Discuss why the experimental result will lie between these two values. The quadrupole deformation parameter for given rotational nucleus is $\beta = 0.340$.



(a) ^{168}Er (b) ^{114}Cd

Figure 1: Parts of the level scheme of ^{168}Er and ^{114}Cd. Which is rotational, which is vibrational? Arrows between levels for ^{114}Cd correspond to γ-ray transitions. Energies are given in keV. Source: ENDSF

Problem 4 Radioactive Sources

Three radioactive sources each have activities of 1 μCi at t=0. Their half-lives are, respectively, 1.0 s, 1.0 h, and 1.0 d.

a) How many radioactive nuclei are present at $t = 0$ in each source?

b) How many nuclei of each source decay between $t = 0$ and $t = 1$ s

c) How many decay between $t = 0$ and $t = 1$ h?

Problem 5 Nuclear Archeology and nuclear physics in archeology

^{14}C is used to determine the age of fossils and other organic materials. The idea is that as long an organism is alive, it constantly is exchanging carbon with its environment (eating and excreting) and so the isotopic composition of the organism matches that of the atmosphere. Once the organism dies, this exchange stops, and the ^{14}C trapped in the system start ^{14}C/^{12}C ratio was the same in the past as it is today (which is almost true, but hang on for a surprise . . .), then if we see less ^{14}C it must be because this isotope has decayed (^{12}C is stable).

a) What determines the time-scales that we can use this method on?

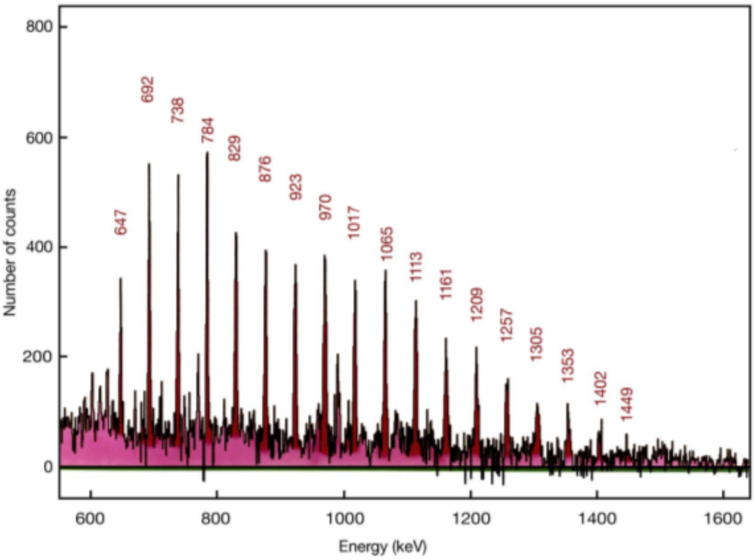


Figure 2: γ-ray spectrum for the high-spin superdeformed bands ^{152}Dy. Does this arise from a rotational or vibrational scheme? Adopted from: PRL 57 (1986) 881.

b) What gave a sharp rise to the fraction of ^{14}C in the atmosphere from about the 1960's.

If we want to look at events that take much longer than 5000 years, it's useful to look for radioactive decays that have much longer half lives. If you poke around the periodic table, you find that heavy elements often have radioactive isotopes with half lives measured in billions of years. Let's focus on Naturally occurring uranium is a mixture of the 238 U (99.28 %) and 235 U (0.72 %) isotopes.

c) How old must the material of the solar system be if one assumes that at its creation both isotopes were present in equal quantities? How do you interpret this result?

d) How much of the ^{238}U has decayed since the formation of the Earth's crust 2.5×10^9 y ago?

Problem 6 Radon

An employee at a Pennsylvania nuclear power plant arrived at work in 1984. He surprised himself and others by triggering the plant's alarm. It was assumed that he had taken radioactivity home for the night, but it was soon determined that he went home without a trace of radioactivity. Surprisingly, what happened was that he carried radon daughters from home to work.

a) The principal health risk from radon arises not from ^{222}Rn, which does not adhere to lungs, but rather from its four radioactive daughters, which chemically attach to aerosols that are trapped in lungs. Give the daughter nuclei.

b) Assume that after the physics auditorium has not been aired for several days, a specific activity A from ^{222}Rn of 100 Bq/m^3 is measured. Assume that the walls, floor and ceiling are made of concrete ($10 \times 10 \times 4$m^3). Calculate the activity of ^{222}Rn as a function of the lifetimes of the parent and daughter nuclei.

c) How high is the concentration of ^{238}U in the concrete if the effective thickness from which the ^{222}Rn decay product can diffuse is 1.5 cm?

Problem 7 Bonus

Part 1

Among the radioactive products emitted in the 1986 Chernobyl readtor accident were ^{131}I ($t_{1/2} = 8.0$ d) and ^{137}Cs ($t_{1/2} = 30$ y). There are about fice times as many ^{137}Cs atoms as ^{131}I atoms produced in fission.

a) Which isotope contributes the greater activity to the radiation cloud? Assume the reactor has been operating continuously for several days before the radiation was released. (Why would you assume so?)

b) How long after the original incident does for the two activities to become equal?

c) About 1% of fission events produce ^{131}I, and each fission event releases an energy of about 200 MeV. Given a reactor of Chernobyl size (1000 MW), caclulate the activity in curies of ^{131}I after 24 h of operation.

Part 2

How many days should you wait until you can drink milk from the region around Chernobyl? Assuming you were member of the scientific advisory committee after a nuclear accident, would you advice to produce cheese for sales out of the milk? What would guide your decision?

After a brainstorming session on the last aspect, I found several research papers that may give you further insight. An (inconclusive) list:

- J.A. Morris, After effects of the Chernobyl accident, British Veterinary Journal, Volume 144, Issue 2, 1988, Pages 179-186, ISSN 0007-1935, `http://dx.doi.org/10.1016/0007-1935(88)90051-6`.
- N.A. Beresford, et al., Thirty years after the Chernobyl accident: What lessons have we learnt?, Journal of Environmental Radioactivity, Volume 157, June 2016, Pages 77-89, `http://dx.doi.org/10.1016/j.jenvrad.2016.02.003`
- Patel, A. et al.. (1993). Decontamination of radioactive milk—a review. International journal of radiation biology, 63(3), 405-412. `http://www.tandfonline.com/doi/pdf/10.1080/09553009314550531`
- P. Froidevaux, et al. 90Sr, 238U, 234U, 137Cs, 40K and 239/240Pu in Emmental type cheese produced in different regions of Western Europe, Journal of Environmental Radioactivity, Volume 72, Issue 3, 2004, Pages 287-298, `http://dx.doi.org/10.1016/S0265-931X(03)00179-6`