FYS 3610

EXERCISES WEEK 38 Thursday

EXERCISE 1
Show that the maximum ion production at a zenith-angle χ is given as

$$q_m = q_{mo} \cos \chi$$

where q_{mo} is the ion production for $\chi = 0$. This equation is a scaling factor for maximum ion production.

EXERCISE 2
Maximum ion production occur at an altitude $z_m^{'}$ where $\tau = \cos \chi$ and we have that $z_m^{'} = \ln \sec \chi$. Show that

$$z_m = z_{mo} + H \ln \sec \chi$$

This is a scale law for how the height of maximum ion production varies with zenith-angle χ.

EXERCISE 3

From the Maxwell's equations we have

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad (2.1)$$

$$\nabla \cdot \vec{B} = 0 \qquad (2.2)$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} \qquad (2.3)$$

and the generalised Ohm's law is given by

$$\vec{j} = \sigma(\vec{E} + \vec{v} \times \vec{B}) \qquad (2.4)$$

a) Show that the time varying magnetic field can be expressed as

$$\frac{\partial \vec{B}}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 \vec{B} + \nabla \times (\vec{v} \times \vec{B}) \qquad (2.5)$$

(Hint: $\nabla \times (\nabla \times \vec{B}) = \nabla(\nabla \cdot \vec{B}) - \nabla^2 \vec{B}$)

b) Infer the magnetic Reynolds number (R_m) by taking the ratio of the second term to the first term on the right hand side of Equation 2.5. Use L as a characteristic scale length for changes of the field and flow.

c) Discuss the usage of the Reynolds number as an indicator of whether the frozen-in-flux concept is valid or not. Why does the frozen-in-flux concept (ideal MHD) break down locally near a reconnection site.

Problem 2 from Mid-term exam 2005

THE EARTH MAGNETIC FIELD AND PARTICLE MOTION

The Lorentz force on a charged particle is given as:

$$\vec{F}_L = q\vec{E} + q\vec{v} \times \vec{B}$$

a) In the absence of an electric field, show that a charged particle's motion can be resolved into two components: one along the magnetic field and one perpendicular to the magnetic field.
b) Show that the gyro gyro radius is given by $r_c = \frac{mv_\perp}{qB}$ and that the gyrofrequency is given by $\omega_c = \frac{qB}{m}$
c) Calculate the gyrofrequencies (in hertz) of proton and an electron in a 100-nT field, a 1000 nT field and a 10.000 nT field. At roughly what distances from Earth centre can these gyrofrequences be found in the equatorial plane (see Table for information about the Earth's magnetic field)?

d) What is the gyroradius of a proton moving with transverse to a 100 nT magnetic field at $2\text{x}10^5 \text{ms}^{-1}$? How does this distance compare with the distance in the equatorial plane over which the Earth's dipole field changes from 100 nT to 200 nT (a factor 2)?