FYS 3610
Exercises week 40

Descriptive questions:

1) For each of the following fields, sketch the particle trajectories separately for electrons and protons. Define a coordinate system. Illustrate clearly the direction of the magnetic and electric fields and your coordinate axes. Sketch trajectories in the planes that best illustrate the motions of the charged particles. Describe the form of motion normal to the plane.

a) Assume static uniform magnetic field oriented along the *x*-axis., with no electric field. The particles have an initial velocity $v_x = 0$ and $v_z = v_0$.

b) Assume a static uniform magnetic field oriented along the *z*-axis, with no electric field. Charged particles are initially moving with non-vanishing v_x and v_z.

c) Assume a static uniform magnetic field oriented along the *y*-axis, with a static electric field along the *z*-axis. Charged particles are initially at rest.

d) Assume a static uniform magnetic field oriented along the negative *y*-axis, with a static electric field along *z*-axis. Charged particles initially are initially moving in the *x*-direction.

e) Assume a magnetic field along the *y*-direction increasing in strength with increasing z. Charged particles initially have velocities v_x and v_z, with $v_x << v_z$.

f) Which of the situations above will a)-e) will give currents in plasmas with equal numbers of positive and negative charges?

2) Describe the radiation belt and ring current (topology and origin).

3) What is the magnetic signature near equator of the ring current.

Exercises 1:

The momentum equations for ions and electrons are given by

$$m_i \frac{d\vec{v}_i}{dt} = e\left(\vec{E} + \vec{v}_i \times \vec{B}\right) \qquad \text{Eq. 1.1}$$

$$m_e \frac{d\vec{v}_e}{dt} = -e\left(\vec{E} + \vec{v}_e \times \vec{B}\right) \qquad \text{Eq. 1.2}$$

a) Assume a static uniform electric field along the y-axis and a static uniform magnetic field along the *z*-axis. Sketch the particle trajectories separately for electrons and ions.

b) Show that the zeroth order drift of the guiding center is given by

$$\vec{v}_{gc} = \vec{u}_E = \frac{\vec{E}_\perp \times \vec{B}}{B^2}, \qquad \text{Eq. 1.3}$$

independent of both the mass and charge.
{ *Hint:* $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$ }

c) Assume a magnetic field along the *z*-direction increasing in strength with increasing *y*, and no electric field. Draw a sketch showing the particle trajectories separately for ions and electrons.

Assume that the magnetic field strength increase linearly with increasing *y*.

d) Show that the magnetic field can be expressed as

$$B_z = B_{0z} + \left(\frac{\partial B_z}{\partial y}\right) r_c \cos\phi \qquad \text{Eq. 1.4}$$

where $r_c = \frac{mv_\perp}{eB_z}$ is the gyro radius, and ϕ is the angle that the position vector $\vec{r} = (x,y)$ makes with *y* in a guiding center reference frame.

e) Show that the average forces over one gyro-period are given by

$$\langle F_x \rangle = \frac{-qv_\perp}{2\pi} \int_0^{2\pi} \left(B_{0z} \sin\phi + r_c \left(\frac{\partial B_z}{\partial y}\right) \sin\phi \cos\phi \right) d\phi \qquad \text{Eq. 1.5}$$

and

$$\langle F_y \rangle = \frac{-qv_\perp}{2\pi} \int_0^{2\pi} \left(B_{0z} \cos\phi + r_c \left(\frac{\partial B_z}{\partial y}\right) \cos^2\phi \right) d\phi \qquad \text{Eq. 1.6}$$

in *x* and *y* directions, respectively.

f) Integrate Eqs. 1.5 and 1.6 and derive the following expression for the gradient drift of the guiding center:

$$\vec{v}_{gc} = -\frac{1}{2}\frac{mv_{\perp}^2}{qB_z^2}\left(\frac{\partial B_z}{\partial y}\right)\hat{x} \qquad \text{Eq. 1.7}$$

$\{$*Hint:* $\int \cos^2 x \, dx = \frac{1}{2}\sin x \cos x + \frac{1}{2} x + C$; $\int \sin x \cos x \, dx = \frac{1}{2}\sin^2 x + C$ $\}$

The general expression for the gradient drift is:

$$\vec{u}_{\nabla B} = \frac{1}{2} m v_{\perp}^2 \frac{\vec{B} \times \nabla \vec{B}}{qB^3} \qquad \text{Eq. 1.8}$$

Exercise 2:

Derive the Dessler-Parker-Sckopke relation.