UNIVERSITETET I OSLO

Det matematisk-naturvitenskapelige fakultet

MID-TERM EXAMINATION

Course: Fys 3610 Space Physics

Date: October 8, 2012

Duration: 3 hours

Permitted aid(s): Calculator (with emptied memory/ off-line).

The set of examination consists of 4 Pages and 4 Problems.
Please make sure that you have a complete exam set.

PROBLEM 1 : Earth's atmosphere and the Sun's

a) Draw a sketch of the height variation of temperature in the Earth's atmosphere from sea level to 300 km. Explain in general terms what are the physical mechanisms responsible for this structure. Annotate the different regions of the atmosphere by name.

b) Discuss barosperic density distribution. Derive the aerostatic equation. Derive the barometric law and the define the scale height.

c) Why is the atmosphere well mixed up to ~100km. Describe the diffusion processes below and above the homopause.

d) Name the different regions 1- 6 in Figure 1 on page 2. Give a brief characterization of each region.

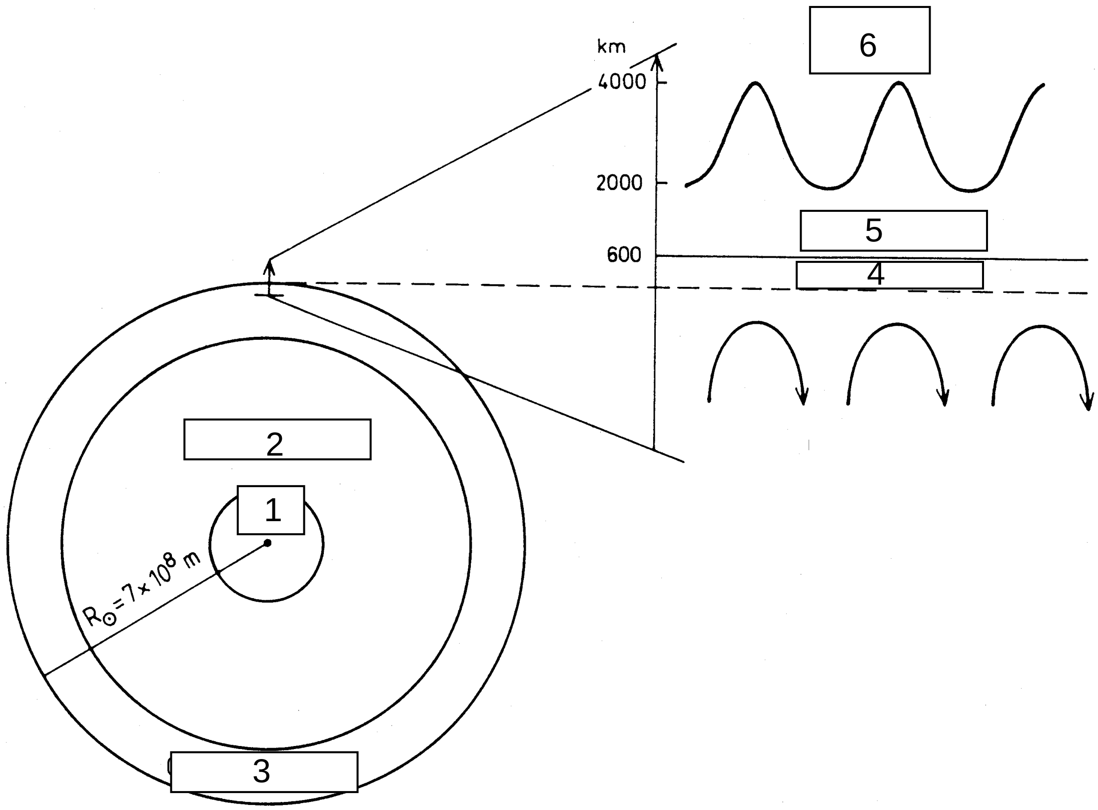


Figure 1

PROBLEM 2 : The earth's magnetic field

a) Draw a sketch of the undisturbed Earth magnetic field using the Earth's rotational axis as a reference. Indicate direction of the magnetic field. What is the strength of the magnetic field near the equator and near the poles?

b) The Earth magnetic field are normally referred to the local coordinate systems (X,Y,Z) or (H, D, Z). Draw a figure that illustrates the Earth magnetic field vector decomposed in the two coordinate systems. Indicate geographic north in your figure.

c) Describe what happens when the solar wind, consisting of electrons and protons, hits the Earth magnetic field on the dayside. Explain the direction of the magnetopause current.

d) What is a typical value for the solar wind stand-off distance? What are the controlling parameters for this distance; i.e. to push it inwards and outwards?

e) What is the magnetic disturbance on ground due to the ring current?

PROBLEM 3 : MHD equations and waves

From Maxwell's equations we have

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \mu_0 \vec{j}$$

And the generalized Ohm's law is given by

$$\vec{j} = \sigma(\vec{E} + \vec{u} \times \vec{B})$$

a) Show that the time varying magnetic field can be expressed as:

$$\frac{\partial \vec{B}}{\partial t} = \frac{1}{\mu_0 \sigma} \nabla^2 \vec{B} + \nabla \times (\vec{u} \times \vec{B})$$

{ **Vector relations**: $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$; $\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) + \nabla^2 \vec{A}$ }

b) Derive the Reynolds number given by $R_m = \mu_0 \sigma \ vL$.

c) Discuss the usage of the Reynolds number as an indicator of whether the use of the frozen-in-flux concept is safe.

d) Start out with the following equations:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \vec{u}_e) = 0$$

$$m_e n_e \left[\frac{\partial \vec{u}_e}{\partial t} + (u \cdot \nabla)\vec{u}_e \right] = -e n_e \vec{E}$$

$$\nabla \cdot \vec{E} = \frac{e(n_0 - n_e)}{\varepsilon_0}$$

Linearize the above equations by introducing perturbations in the particle density, velocity and the electric field:

$$n_e = n_0 + n_1$$

$$\vec{u}_e = \vec{u}_1$$

$$\vec{E} = \vec{E}_1$$

Then introduce

$$\vec{u}_1 = \vec{u}_1 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$
$$n_1 = n_1 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$
$$\vec{E}_1 = \vec{E}_1 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

and solve the equations for ω and show that it becomes $\omega_p = \left(\frac{n_0 e^2}{\varepsilon_0}\right)^{1/2}$

e) Describe the basic properties of Plasma accoustic, Alfvén, and Magnetosonic waves.

PROBLEM 4 : Single particle motion

For each of the following fields, sketch the particle trajectories separately for electrons and protons. Define a coordinate system. Illustrate clearly the direction of the magnetic and electric fields and your coordinate axes. Sketch trajectories in the planes that best illustrate the motions of the charged particles. Describe the form of motion normal to the plane.

a) Assume static uniform magnetic field oriented along the *x*-axis., with no electric field. The particles have an initial velocity $v_x = 0$ and $v_z = v_0$.

b) Assume a static uniform magnetic field oriented along the *z*-axis, with no electric field. Charged particles are initially moving with non-vanishing v_x and v_z.

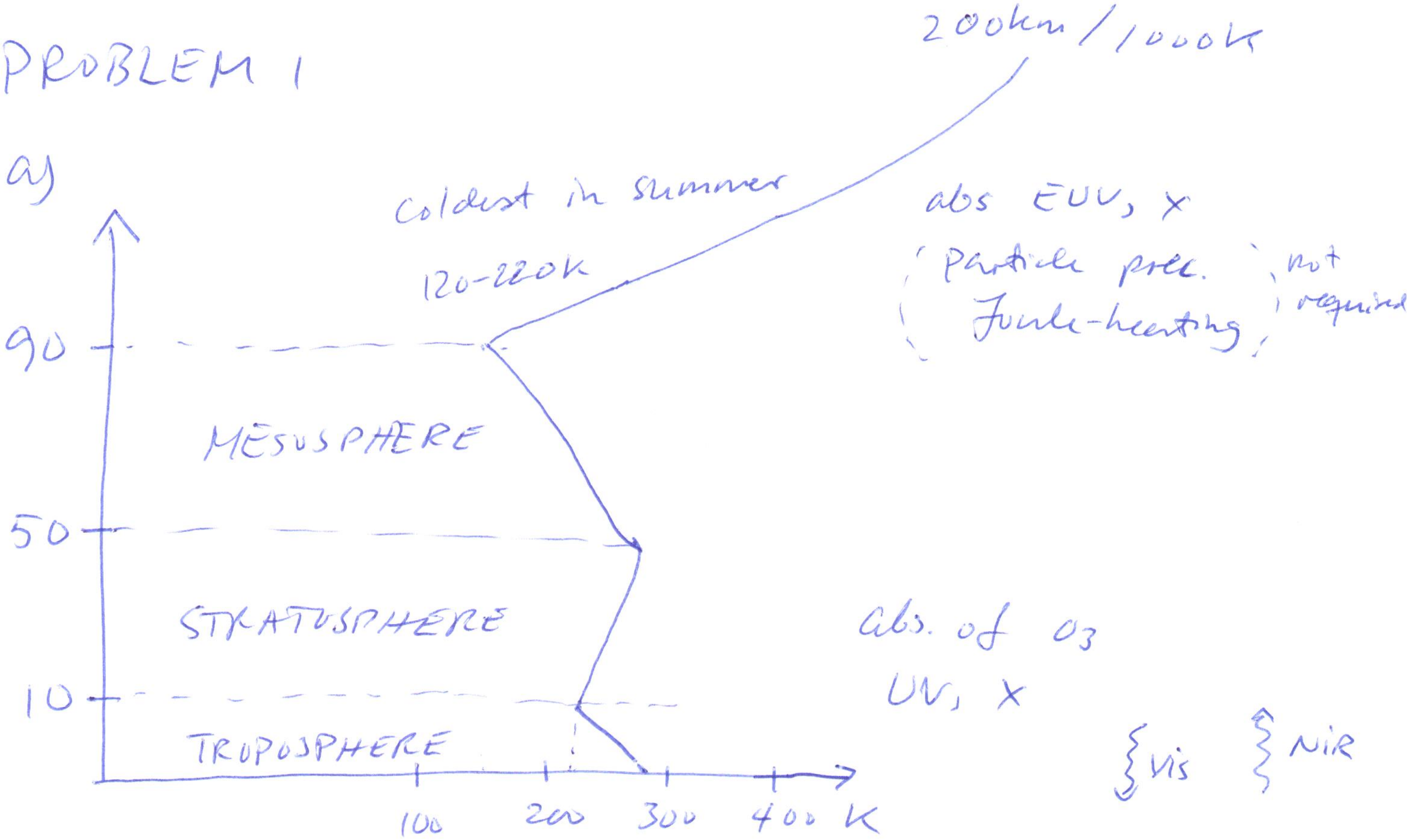
c) Assume a static uniform magnetic field oriented along the *y*-axis, with a static electric field along the *z*-axis. Charged particles are initially at rest.

d) Assume a magnetic field along the *y*-direction increasing in strength with increasing z. Charged particles initially have velocities v_x and v_z, with $v_x << v_z$.

FYS 3610 - MID TERM EXAM 2012

PROBLEM 1

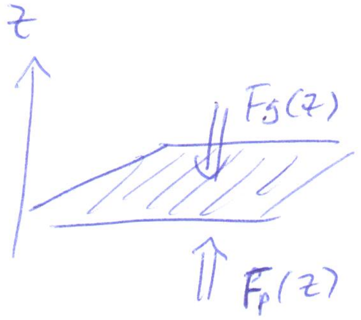
a)



Most important energy sources

SUN : - solar radiation
- solar wind particles

b)



$$F_p(z) = A\, p(z)$$

$$F_g(z) = A \int_z^{\infty} \rho(z')\, g(z')\, dz'$$

$$F_p = F_g$$

$$p(z) = \int_z^{\infty} \rho(z')\, g(z')\, dz'$$

Aerostatic equation: $\dfrac{dp(z)}{dz} = -\rho(z)\, g(z)$ $\quad \boxed{\dfrac{dp}{dz} = -\rho g}$

Barometric law:

$$\rho = \bar{m} n \qquad p = nkT \Rightarrow n = \frac{p}{kT}$$

$$\rho = \bar{m} \frac{p}{kT}$$

$$\frac{dp}{dz} = -\rho g = \frac{\bar{m} g}{kT} p = -\frac{p}{H} \qquad H = \frac{kT}{\bar{m} g}$$

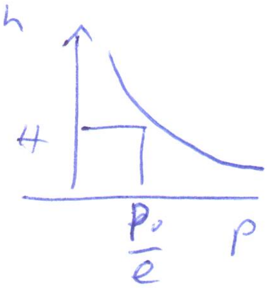
$$\int_{p(h_0)}^{p(h)} \frac{dp}{p} = -\int_{h_0}^{h} \frac{dz}{H(z)}$$

$$\ln \frac{p(h)}{p(h_0)} = -\int_{h_0}^{h} \frac{dz}{H(z)}$$

$$p(h) = p(h_0) \exp\left\{-\int_{h_0}^{h} \frac{dz}{H(z)}\right.$$

assume $H = \text{const} = \frac{kT}{\bar{m} g}$

Barometric law $\boxed{p = p_0 e^{-\frac{h}{H}}}$



$$p(h) = n(h)\, k\, T(h)$$

$$T = \text{konst}$$

$$p(h) = n(h)$$

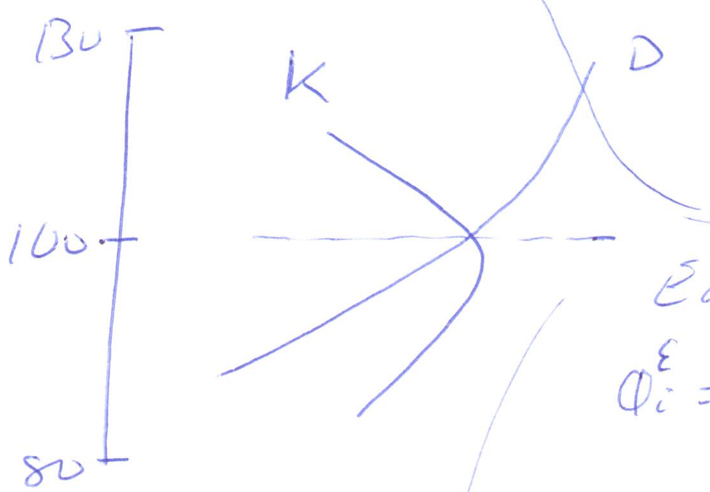
$$n(h) T(h) = n(h_0) T(h_0) \exp\left\{-\int_{h_0}^{h} \frac{dz}{H_z}\right\}$$

$$n(h) = n(h_0) \frac{T(h_0)}{T(h)} \exp\left\{-\int_{h_0}^{h} \frac{dz}{H(z)}\right\}$$

c) Well mixed due to Eddy diffusion, irregular whirling motion. - turbulence.

$\uparrow \dfrac{d(n_i/n)}{dz}$

Altitude profiles depends on



Molecular diffusion

$H = \dfrac{kT}{mg}$ - different scale heights due to different mass

Eddy diffusion

$$\Phi_i^E = -Kn\frac{d(n_i/n)}{dz}$$

gradient in the mixing ratio

$$\Phi^D = n_i u_x = -D\frac{dn_i}{dx}$$

$$D = \frac{kT}{m\nu_{in}}$$

d)

1. Core $T = 1.5 \times 10^7 K$

Fusion reactor

proton - proton chain reactions.

2. Radiative zone

Opaque - scattering processes.

delay the energy transport to the surface by 10 mill. years.

3. Convection zone. Turbulent zone due to rapid temperature decrease.

4. Photosphere. Emits most of the Sun's light.

~~Dark~~ spots - sunspots.

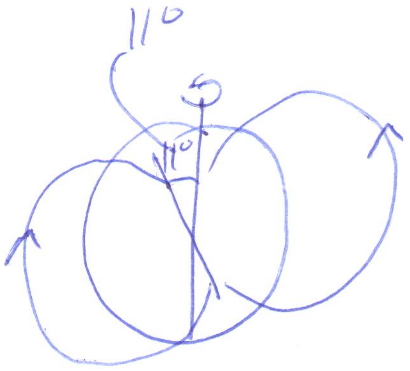
chill down due to strong magnetic field

5. Chromosphere T increases from 4200 - 10000 K

6. Corona $10^6 K$ Dynamic CME, wind streamers etc.

PROBLEM 2

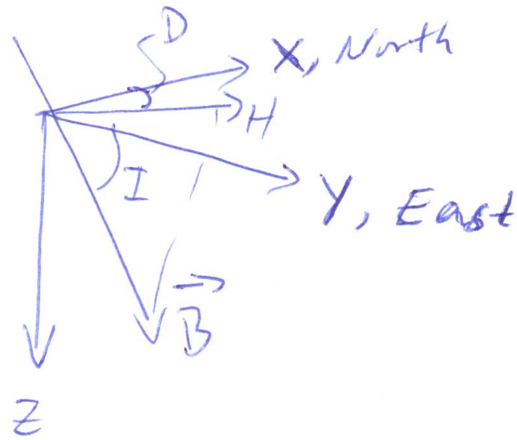
a)



$B_{pol} = 60.000\,nT$

$B_{eq} = 30.000\,nT$

b)



c)

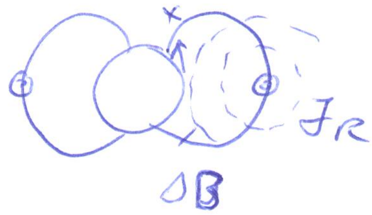
$\vec{F} = q\,\vec{v} \times \vec{B}$

d)

$r = 8 - 10\,R_E$

Depends on density (n) and velocity (v) in the solar wind.

e)



δx - negative

PROBLEM 3

a) $\nabla \times \vec{B} = \mu_0 \vec{j} = \mu_0 \sigma (\vec{E} + \vec{u} \times \vec{B})$

$\nabla \times (\nabla \times \vec{B}) = \mu_0 \sigma (\nabla \times \vec{E} + \nabla \times (\vec{u} \times \vec{B}))$

$\nabla (\overset{=0}{\nabla \cdot \vec{B}}) - \nabla^2 \vec{B} = \mu_0 \sigma (-\frac{\partial \vec{B}}{\partial t} + \nabla \times (\vec{u} \times \vec{B}))$

$$\frac{\partial \vec{B}}{\partial t} = \underbrace{\nabla \times (\vec{u} \times \vec{B})}_{\text{Convection term}} + \frac{1}{\mu_0 \sigma} \underbrace{\nabla^2 \vec{B}}_{\text{Diffusion term}}$$

b) $\nabla \sim \frac{1}{L}$

$$R_m = \frac{|\nabla \times (u \times \vec{B})|}{|\frac{1}{\mu_0 \sigma} \nabla^2 \vec{B}|} = \frac{\frac{1}{L} v B}{\frac{1}{\mu_0 \sigma} \frac{1}{L^2} B} = L \mu_0 \sigma u$$

c) $R_m >> 1$ the diffusion term can be neglected.

d) $$\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot \vec{u}_1 = 0$$

$$m n_0 \frac{\partial \vec{u}_1}{\partial t} = - e n_0 \vec{E}_1$$

$$\nabla \cdot \vec{E}_1 = - \frac{e n_1}{\epsilon_0}$$

(i) $-i\omega n_1 + n_0 i\vec{k} \cdot \vec{u}_1 = 0$

(ii) $-i\omega m n_0 \vec{u}_1 + e n_0 \vec{E}_1 = 0$

iii $i\vec{k} \cdot \vec{E}_1 + e \frac{n_1}{\epsilon_0} = 0$

$\vec{k} \cdot$(ii) $-i\omega m n_0 \vec{k} \cdot \vec{u}_1 + e n_1 \vec{k} \cdot \vec{E}_1 = 0$

$$\omega = \sqrt{\frac{n_0 e^2}{m \epsilon_0}}$$

e) Plasma accoustic:

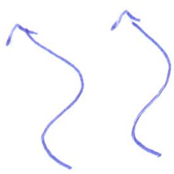
Pressure gradient variations along the magnetic field.

Sound wave along the magnetic field.

$$\left(V_{ps} = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\frac{\gamma k (T_i + T_e)}{m_i}} \right)$$

Alfven wave:

Fluctuations in electromagnetic quantities.



density and ~~pressure~~ variations absent.

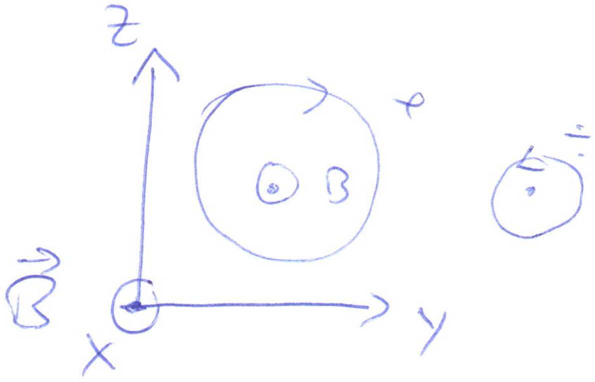
$$\left(V_A = \sqrt{\frac{B^2}{\mu_0 \rho}} \right)$$

Magnetosonic.

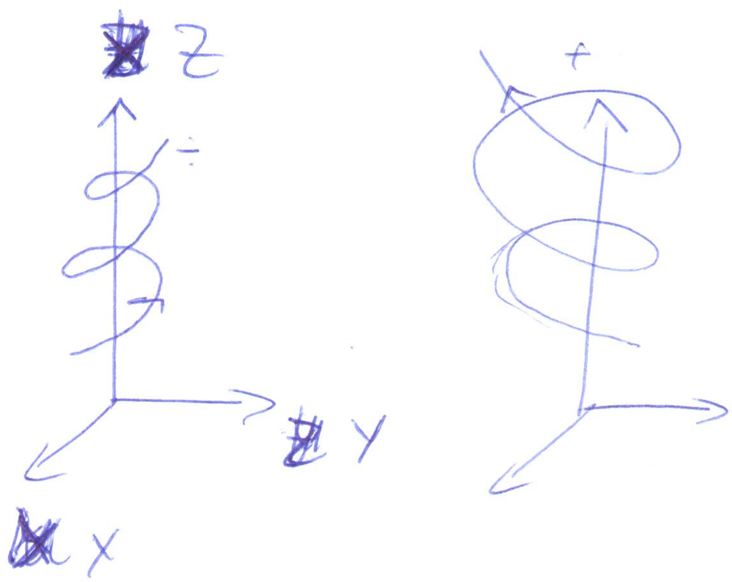
Perturbations in plasma state parameters (ρ, p, $\vec{u}$) as well as electromagnetic (B_z, E_x, j_x)

PROBLEM 4 : Single particle motion

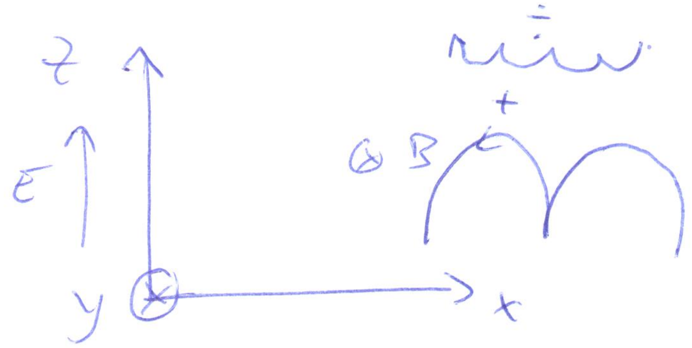
a)



b)

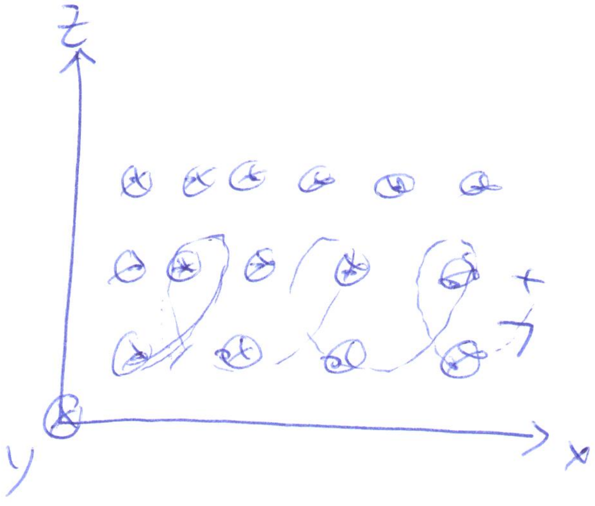


c)



d)

10



$\uparrow \nabla B$