FYS 3610

Exercise Week 35 due 06. September 2013

Questions as they might appear in the mid-term and/or oral exam

Derive the Vlasow equation from first principles, i.e., assuming that the phase space density is a conserved quantity.

Describe Debye shielding. What is the Debye length? What are typical values for Debye lengths in space plasmas.

Derive the induction equation and discuss the magnetic Reynolds number.

What is the frozen-in theorem and what consequences follow from it?

Exercises

For a gas in thermal equilibrium the most probable distribution of velocities can be calculated using statistical mechanics to be the Maxwellian distribution:

$$f(v) = A \int \exp\left(-\frac{m\frac{v^2}{2}}{k_B T}\right) dv$$

a) Determine the constant A.

Hint 1: Remember that the zeroth moment of a distribution function is the particle density.

Hint 2: $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$

b) Determine the bulk flow velocity.

Hint: $\int_0^{\infty} x e^{-x^2} dx = \frac{1}{2}$

Combining Faraday's Law with Ampere's Law (ignoring the displacement currents) and Ohm's Law (assuming ideal MHD) one can derive the induction equation. Use the resistivity $\eta = {}^{1}/_{\mu_0 \sigma}$.

a) Write the induction equation in one dimension, assuming $B = B_z(x,t)$ and $v = v_x(x,t)$, i.e., assuming that all other components of the magnetic and the flow field are zero.

b) Assume that the flow is zero for all x and t. For the following initial magnetic field profile

$$B_z(x, t = 0) = f(x) = A_0 \exp\left({}^{-x^2}/_{L^2} \right)$$

calculate the time-dependent $B_z(x,t)$ from the formal solution of the diffusion equation given by

$$B_z(x,t) = \frac{1}{2\eta\sqrt{\pi t}} \int_{-\infty}^{\infty} f(x-\lambda) \exp\left(-\frac{\lambda^2}{4\eta^2 t}\right) d\lambda$$

by substitution and remembering that $\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{{}^{\pi}/_{\alpha}}$.

c) Using words, describe with the help of the solution to the diffusion equation what happens to the initial magnetic field profile as $t \to \infty$. What happens if the conductivity of the medium is very large?

d) What form does the velocity field $v = v_x(x,t)$ need to have such that the magnetic field profile derived in b) is constant in time? Sketch the magnetic field and flow profile in this situation.