FYS 3610

Exercise Week 34 due 28. August 2015

Questions as they might appear in the mid-term and/or oral exam

For each of the following fields, sketch the particle trajectories separately for electrons and protons. For each case clearly illustrate the direction of the magnetic and electric field as well as the coordinate axes. Sketch the trajectories in a plane that best illustrates the motion of the charged particles. Describe the motion normal to that plane.

a) Assume a static uniform magnetic field oriented along the x-axis, with no electric field. The particles have an initial velocity of $v_{x,y}=0$ and $v_z=v_0$.
b) Assume a static uniform magnetic field oriented along the z-axis, with no electric field. Charged particles are initially moving with non-zero v_x and v_z.
c) Assume a static uniform magnetic field oriented along the y-axis, with a static electric field along the z-axis. Charged particles are initially at rest.
d) Assume a static uniform magnetic field oriented along the negative y-axis, with a static electric field along z-axis. Charged particles are initially moving in the x-axis.
e) Assume a magnetic field along the y-axis increasing in strength with increasing z. Charged particles initially have non-zero velocity v_x and v_z, with $v_x << v_z$.
f) Which of the situations above give rise to currents in plasmas with equal numbers of positive and negative charged particles?

Exercises

The equation of motion for ions and electrons is given by

$$m_e \frac{d\vec{v}_e}{dt} = -e\left(\vec{E} + \vec{v_e} \times \vec{B}\right)$$

$$m_i \frac{d\vec{v}_i}{dt} = +e\left(\vec{E} + \vec{v_i} \times \vec{B}\right)$$

a) Show by substitution that the zeroth order drift of the guiding center is given by:

$$\overrightarrow{v_{gc}} = \frac{\vec{E} \times \vec{B}}{B^2}$$

b) Assume a magnetic field along the z-axis, increasing linearly in strength with increasing y, and no electric field. Show that the magnetic field along the trajectory of a positively charged ion can be expressed as

$$B_z = B_{0z} - \frac{\partial B_z}{\partial y} r_c \sin\varphi$$

where $r = \frac{mv_\perp}{eB_z}$ is the gyro radius, and φ is the angle the instantaneous position vector $\vec{r}(x, y)$ makes with the guiding center reference frame (the gyro-phase), positive anti-clockwise when looking along the magnetic field.

c) Show that the average force over one gyro-period is given by

$$\langle F_x \rangle = -\frac{ev_\perp}{2\pi} \int_0^{2\pi} \left(B_{0z} \cos\varphi - \frac{\partial B_z}{\partial y} r_c \sin\varphi \cos\varphi \right) d\varphi$$

$$\langle F_y \rangle = \frac{ev_\perp}{2\pi} \int_0^{2\pi} \left(B_{0z} \sin\varphi - \frac{\partial B_z}{\partial y} r_c \sin^2\varphi \right) d\varphi$$

d) Execute the integration of the above two equations in order to derive the following expression for the gradient drift of the guiding center:

$$\vec{v}_{gc} = -\frac{1}{2} \frac{mv_\perp^2}{eB_z^2} \frac{\partial B_z}{\partial y} \vec{e}_x$$