FYS 3610

Exercise Week 35 due 04. September 2015

Questions as they might appear in the mid-term and/or oral exam

Derive the Vlasov equation from first principles, i.e., assuming that the phase space density is a conserved quantity.

Derive the induction equation and discuss the magnetic Reynolds number.

What is the frozen-in theorem and what consequences follow from it?

Exercises

For a gas in thermal equilibrium the most probable distribution of velocities can be calculated using statistical mechanics to be the Maxwellian distribution:

$$f(v_x, v_y, v_z)dv_x dv_y dv_z = A\exp\left(-\frac{m\frac{v_x^2}{2}}{k_B T}\right)\exp\left(-\frac{m\frac{v_y^2}{2}}{k_B T}\right)\exp\left(-\frac{m\frac{v_z^2}{2}}{k_B T}\right)dv_x dv_y dv_z$$

a) Determine the constant A.
   Hint 1: Remember that the zeroth moment of a distribution function is the particle density.
   Hint 2: $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$

b) Determine the bulk flow velocity.

   Hint: $\int_0^{\infty} x e^{-x^2} dx = \frac{1}{2}$

Combining Faraday's Law with Ampere's Law (ignoring the displacement currents) and Ohm's Law (assuming ideal MHD) one can derive the induction equation. Use the resistivity $\eta = {}^{1}/_{\mu_0 \sigma}$.

a) Write the induction equation in one dimension, assuming $B = B_z(x,t)$ and $v = v_x(x,t)$, i.e., assuming that all other components of the magnetic and the flow field are zero.

b) Assume that the flow is zero for all x and t. For the following initial magnetic field profile

$$B_z(x, t=0) = f(x) = A_0 \exp\left({}^{-x^2}/_{L^2} \right)$$

calculate the time-dependent $B_z(x,t)$ from the formal solution of the diffusion equation given by

$$B_z(x,t) = \frac{1}{2\eta\sqrt{\pi t}} \int_{-\infty}^{\infty} f(x-\lambda) \exp\left(-\frac{\lambda^2}{4\eta^2 t}\right) d\lambda$$

by substitution and remembering that $\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{{}^{\pi}/_{\alpha}}$.

c) Using words, describe with the help of the solution to the diffusion equation what happens to the initial magnetic field profile as $t \to \infty$. What happens if the conductivity of the medium is very large?

d) What form does the velocity field $v = v_x(x,t)$ need to have such that the magnetic field profile derived in b) is constant in time? Sketch the magnetic field and flow profile in this situation.