FYS 3610

Solution Week 34

a) This is quite straight forward:

$$m_e \frac{d\vec{v}_{gc}}{dt} = -e(\vec{E} + \vec{v}_{gc} \times \vec{B})$$

$$m_e \frac{d}{dt}\left(\frac{\vec{E} \times \vec{B}}{B^2}\right) = -e\left(\vec{E} + \frac{\vec{E} \times \vec{B}}{B^2} \times \vec{B}\right)$$

Left side is zero because both the electric and the magnetic field are time constant. The right hand side is evaluated with a vector identity:

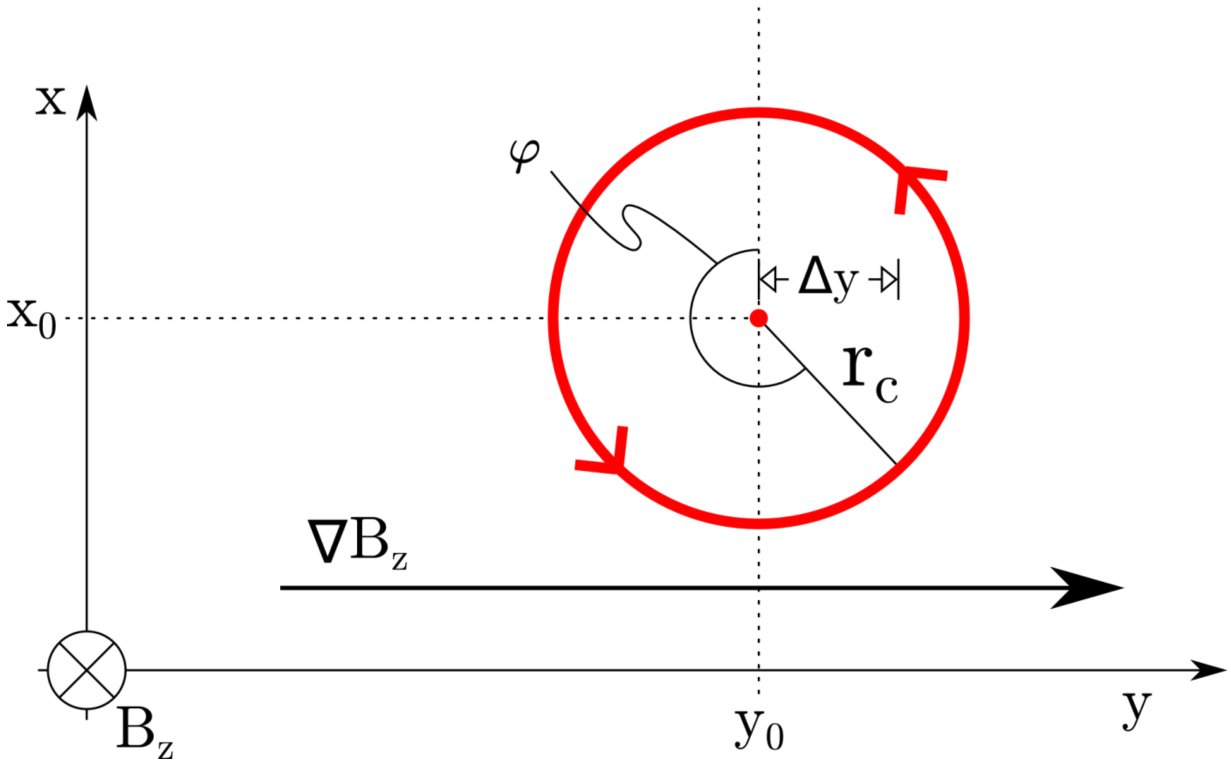
$$0 = -e\left(\vec{E} + \frac{(\vec{B} \cdot \vec{E})\vec{B} - (\vec{B} \cdot \vec{B})\vec{E}}{B^2}\right)$$

Because $\vec{E} \perp \vec{B}$ this gives

$$0 = -e\left(\vec{E} + \frac{-(\vec{B} \cdot \vec{B})\vec{E}}{B^2}\right) = 0$$

The same is easily shown for the ions, because the result does not depend on the sign of e.

b) Consider the following picture of a positive particle gyrating:



The magnetic field varies linearly with y, then

$$B_z(y) = c + my = B(y_0) + \frac{\partial B_z}{\partial y}(y - y_0) = B_{0z} - \frac{\partial B_z}{\partial y} r_c \sin\varphi$$

c) Evaluate the Lorenz-force $\vec{F} = e(\vec{v} \times \vec{B})$

$$\vec{F} = e(\vec{v} \times \vec{B}) = e\left[\begin{pmatrix} -v_\perp \sin\varphi \\ -v_\perp \cos\varphi \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ B_{0z} - \frac{\partial B_z}{\partial y} r_c \sin\varphi \end{pmatrix}\right]$$

$$\langle F_x \rangle = -\frac{ev_\perp}{2\pi}\int_0^{2\pi}\left(B_{0z}\cos\varphi - \frac{\partial B_z}{\partial y} r_c \sin\varphi\cos\varphi\right)d\varphi$$

$$\langle F_y \rangle = \frac{ev_\perp}{2\pi}\int_0^{2\pi}\left(B_{0z}\sin\varphi - \frac{\partial B_z}{\partial y} r_c \sin^2\varphi\right)d\varphi = -\frac{1}{2}\frac{mv_\perp^2}{B_{0z}}\frac{\partial B_z}{\partial y}$$

d) Integrate

$$\langle F_x \rangle = -\frac{ev_\perp}{2\pi}\int_0^{2\pi}\left(B_{0z}\cos\varphi - \frac{\partial B_z}{\partial y} r_c \sin\varphi\cos\varphi\right)d\varphi = 0$$

$$\langle F_y \rangle = \frac{ev_\perp}{2\pi} \int_0^{2\pi} \left(B_{0z} \sin\varphi - \frac{\partial B_z}{\partial y} r_c \sin^2\varphi \right) d\varphi = -\frac{1}{2} \frac{mv_\perp^2}{B_{0z}} \frac{\partial B_z}{\partial y}$$

Hence the drift velocity is

$$v_{gc,x} = \frac{1}{\omega_g} \left(\frac{\vec{F}}{m} \times \frac{\vec{B}}{B} \right) \vec{e}_x = -\frac{1}{2} \frac{mv_\perp^2}{eB_{0z}^2} \frac{\partial B_z}{\partial y}$$