0 Basic plasma physics

Depending on where you look, you will be told that between 95% and 99.9% of the visible universe consists of plasma, the fourth state of matter. In the context of space plasma physics, these plasmas are tenuous gases consisting of ionized particles displaying, on average, no net charge.

0.1 Plasma oscilations

Consider the situation where the electrons of a plasma (with density n) have been collectively displaced by a certain amount x, as shown in Figure 1.

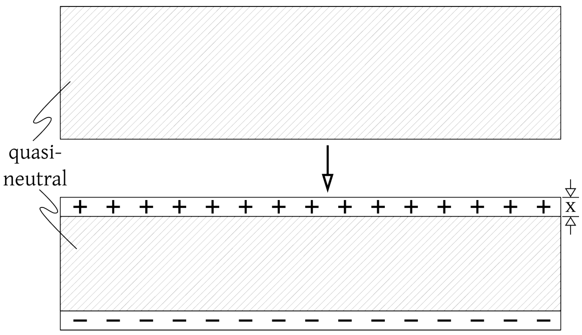

Figure 1: Origin of the plasma oscillations.

That creates an electric field E (Gauß's Law)

$$E = \frac{en}{\epsilon_0} x \tag{0.1}$$

and hence the equation of motion is given by

$$m_e \frac{d^2 x}{dt^2} = F_E = -eE = -\frac{e^2 n}{\epsilon_0} x \tag{0.2}$$

which is the equation of a simple harmonic oscillator with frequency $\omega_{pe}^2 = e^2 n/\epsilon_0 m_e$. Other constituents (ions) have other (lower, $m_i \gg m_e$) ω_p. If we observe plasma phenomena at timescales $\tau < 1/\omega_p$, we observe individual particle motion and not the collective (quasi-neutral) behavior that we are interested in.

0.2 Debye shielding

Just as the plasma frequency defines a *time scale* below which the plasma looses it quasi-neutrality, there exists a *spatial scale* below which the collective behavior of the plasma disappers. Consider again an ion surrounded by electrons. Because $m_i \gg m_e$, the electrons are much more mobile and are attracted to the ion. Within a certain sphere around the ion, the electron density n_e is higher that the background density n_0 and an electric potential is set up, according to the Poisson equation

$$\nabla^2 \phi = -\frac{e(n_i - n_e)}{\epsilon_0}. \tag{0.3}$$

In equilibrium, the ion density n_i is just the background density n_0 while the electron density is given by a Boltzman distribution function

$$n_e = n_0 \exp\left[\frac{e\phi}{k_B T_e}\right]. \tag{0.4}$$

Furthermore, the problem is spherically symmetric, i.e., the potential only varies as a function of radius r and hence

$$\frac{\partial^2 \phi}{\partial r^2} = -\frac{e\left(n_0 - n_0 \exp\left[\frac{e\phi}{k_B T_e}\right]\right)}{\epsilon_0} \tag{0.5}$$

Using a Taylor expansion for the electron density ($\exp x \sim 1 + x$) one finds

$$\frac{d^2 \phi}{dr^2} = \frac{e^2 n_0}{\epsilon_0 k_B T_e} \phi, \tag{0.6}$$

the solution to which is an exponentially decaying potential as

$$\phi(r) = \phi_0 \exp\left[-\frac{r}{\lambda_D}\right] \tag{0.7}$$

where

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_e}{e^2 n_0}} \tag{0.8}$$

is called the Debye length.

For distances $L \gg \lambda_D$ we see that the potential of the ion is decaying fast, i.e., the electrons shield the electric potential of the ions at larger spatial scales. In other words, at spatial scales much larger than the Debye length, the plasma appears quasi-neutral.

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