UiO : Department of Physics
University of Oslo

FYS 3610

Exercise Week 35 due 07. September 2016

Questions as they might appear in the mid-term and/or oral exam

For each of the following situations, sketch the particle trajectories separately for electrons and protons. For each case clearly illustrate the direction of the magnetic and electric field as well as the coordinate axes. Sketch the trajectories in a plane that best illustrates the motion of the charged particles. Also describe their motion in the direction normal to that plane.

a) Assume a static uniform magnetic field oriented along the x-axis, with no electric field. The particles have an initial velocity of $v_{x,y}=0$ and $v_z=v_0$.
b) Assume a static uniform magnetic field oriented along the z-axis, with no electric field. Charged particles are initially moving with non-zero v_x and v_z.
c) Assume a static uniform magnetic field oriented along the y-axis, with a static electric field along the z-axis. Charged particles are initially at rest.
d) Assume a static uniform magnetic field oriented along the negative y-axis, with a static electric field along z-axis. Charged particles are initially moving in the x-axis.
e) Assume a magnetic field along the y-axis increasing in strength with increasing z. Charged particles initially have non-zero velocity v_x and v_z, with $v_x << v_z$.
f) Which of the situations above give rise to currents in plasmas with equal numbers of positive and negative charged particles?

Derive the Vlasov equation from first principles, i.e., assume that the phase space density is a conserved quantity.

Derive the induction equation and discuss the magnetic Reynolds number.

What is the frozen-in theorem and what consequences follow from it?

Exercises

The velocity distribution of particles in a gas in thermal equilibrium assumes the form of the well-known Maxwell distribution:

$$f(v)dv = A\exp\left(-\frac{m\frac{v^2}{2}}{k_BT}\right)dv$$

a) Determine the constant A.

Hint 1: Remember that the zeroth moment of a distribution function is the particle density.

Hint 2: $\int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}$

b) Determine the bulk flow velocity.

Hint: $\int_0^{\infty} xe^{-x^2}dx = \frac{1}{2}$

Combining Faraday's Law with Ampere's Law (ignoring the displacement currents) and Ohm's Law one can derive the induction equation. Use the resistivity $\eta = {}^{1}/_{\mu_0\sigma}$.

a) Write the induction equation in one dimension, assuming $B = B_z(x,t)$ and $v = v_x(x,t)$, i.e., assuming that all other components of the magnetic and the flow field are zero.

b) Initially assume that the flow is zero for all x and t. For the following initial magnetic field profile

$$B_z(x, t = 0) = f(x) = A_0\exp\left({}^{-x^2}/_{L^2}\right)$$

calculate the time-dependent $B_z(x,t)$ from the formal solution of the diffusion equation given by

$$B_z(x,t) = \frac{1}{2\eta\sqrt{\pi t}}\int_{-\infty}^{\infty} f(x-\lambda)\exp\left(-\frac{\lambda^2}{4\eta^2 t}\right)d\lambda$$

by substitution and remembering that $\int_{-\infty}^{\infty} e^{-\alpha x^2}dx = \sqrt{{}^{\pi}/_{\alpha}}$.

c) Using words, describe with the help of the solution to the diffusion equation what happens to the initial magnetic field profile as $t \to \infty$. What happens if the resistivity of the medium is very small?

d) What form does the velocity field $v = v_x(x,t)$ need to have such that the magnetic field profile derived in b) is constant in time? Sketch the magnetic field and flow profile in this situation.