Solution mid-term exam 2014

Problem 1

	1pt: correct gradients 1/2pt: Troposphere, stratosphere, mesosphere, thermosphere with explanation
	1pt: sketch stable 1pt: sketch unstable
	1pt: dayside profile, F-layer, E-layer 1pt: nightside profile, only F-layer
	1pt: production, loss, transport 1pt: photo-ionization, charge exchange, charge exchange, recombination, diffusion 1pt: dissociative recombination in E-layer is fast, radiative recombination in F-layer is slow.

Problem 2

	1/2pt: artistic skill 1/2pt for: bow shock, magnetosheath, magnetopause, lobes, plasma sheet
	1pt: supersonic solar wind is shocked at bow shock. 1pt: flows around Earth's magnetic field in magnetosheath 1pt: SW compresses dayside (SW dynamic pressure equals magnetic pressure), Chapman-Ferraro current
	1pt: solar wind dynamic pressure, Mach number 1pt: bow shock 15 Re, magnetopause 10 Re
	1pt: High temperature corona is not stable, plasma streams off into space 1pt: solar magnetic field is frozen into flow 1pt: top view (spiral) 1pt: side view (heliospheric current sheet)

Problem 3

$m\frac{d\vec{v}}{dt} = q(\vec{v} \times \vec{B})$	1pt: write down equation 1pt: naming mass, velocity, charge, magnetic field
$\frac{dv_x}{dt} = \frac{q}{m} v_y B_z \leftrightarrow \frac{d^2 v_x}{dt^2} = \frac{qB_z}{m}\frac{dv_y}{dt}$	1pt: working out the vectors 1pt: differentiate and substitute 1pt: solve both velocities

$$\frac{dv_y}{dt} = -\frac{qB_z}{m} v_x$$ $$\frac{d^2 v_x}{dt^2} = -\left(\frac{qB_z}{m}\right)^2 v_x$$ $$v_x(t) = v_\perp \sin(\omega_g t + \phi_0)$$ $$v_y(t) = v_\perp \cos(\omega_g t + \phi_0)$$	1pt: write down gyrofrequency
	1pt: positive 1pt: negative
	1pt: positive 1pt: negative
	1pt: positive 1pt: negative 1pt: geomagnetic tail, in the plasma sheet, cross tail current

Problem 4

$$\frac{dp}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} = qvB$$ $$P = \frac{1}{6\pi\epsilon_0}\frac{q^4 v^2 B^2}{m^2 c^3}$$	1pt: work out dp/dt 1pt: insert Lorentz-force
$$P = \frac{1}{6\pi\epsilon_0}\frac{q^4 v^2 B^2}{m^2 c^3} = -\frac{dE_{kin}}{dt}$$ $$= -\frac{1}{2} m \left(2v\frac{dv}{dt}\right)$$ $$\frac{dv}{v} = -\frac{1}{6\pi\epsilon_0}\frac{q^4 B^2}{m^3 c^3} dt$$ $$[\ln v]_{v_0}^{v(t)} = \left[-\frac{1}{6\pi\epsilon_0}\frac{q^4 B^2}{m^3 c^3} t'\right]_0^t$$ $$v(t) = v_0 \exp\left\{-\frac{t}{\tau}\right\}, \tau = \frac{6\pi\epsilon_0 m^3 c^3}{q^4 B^2}$$	1pt: write down kinetic energy 1pt: differentiate kinetic energy 1pt: separate variables 1pt: solve differential equation
$$\tau_a \approx 7.7 \times 10^{23} s \; or \; 2.5 \times 10^{16} a$$ $$\tau_b \approx 3.1 \times 10^{10} s \; or \; 990a$$ $$\tau_c \approx 3.1 \times 10^{-6} s$$	3pts: for each value
	1pt: Once the protons are heated they need a long time to cool down, hence in a fusion reactor you do not need to heat the gas constantly – unless the thermal coupling to the electrons is good, because the loose their energy 2000^3-times faster