

# UNIVERSITY OF OSLO

## Faculty of Mathematics and Natural Sciences

**Exam in** FYS3610  
**Day of exam:** 07.10.2015  
**Exam hours:** 3  
**This examination paper consists of 3 page(s).**  
**Appendices:** 1  
**Permitted materials:** pocket calculator

*Make sure that your copy of this examination paper is complete before answering.*

### PROBLEM 1 (10 points)

- Sketch the entire magnetosphere, both the dayside and the nightside. Label all regions and boundaries of interest! (6 points)
- Describe the frozen-in concept! What is magnetic reconnection and why does the frozen-in concept break down during magnetic reconnection? (3 points)
- Under what condition is magnetic reconnection most active on the dayside of the terrestrial magnetosphere? (1 point)

### PROBLEM 2 (10 points)

- Sketch the electron density profile in the ionosphere, both the dayside profile and the nightside profile! Label the different regions and give their altitude ranges! (4 points)
- Describe the physical mechanism that is responsible for the formation of the ionosphere! What mechanism is responsible for the difference between the dayside and nightside profiles? (3 points)
- At 500 km altitude, atomic oxygen ( $O$ ) is the dominant neutral, and hence  $O^+$  is the dominant ion. The production rate of  $O^+$  is given by

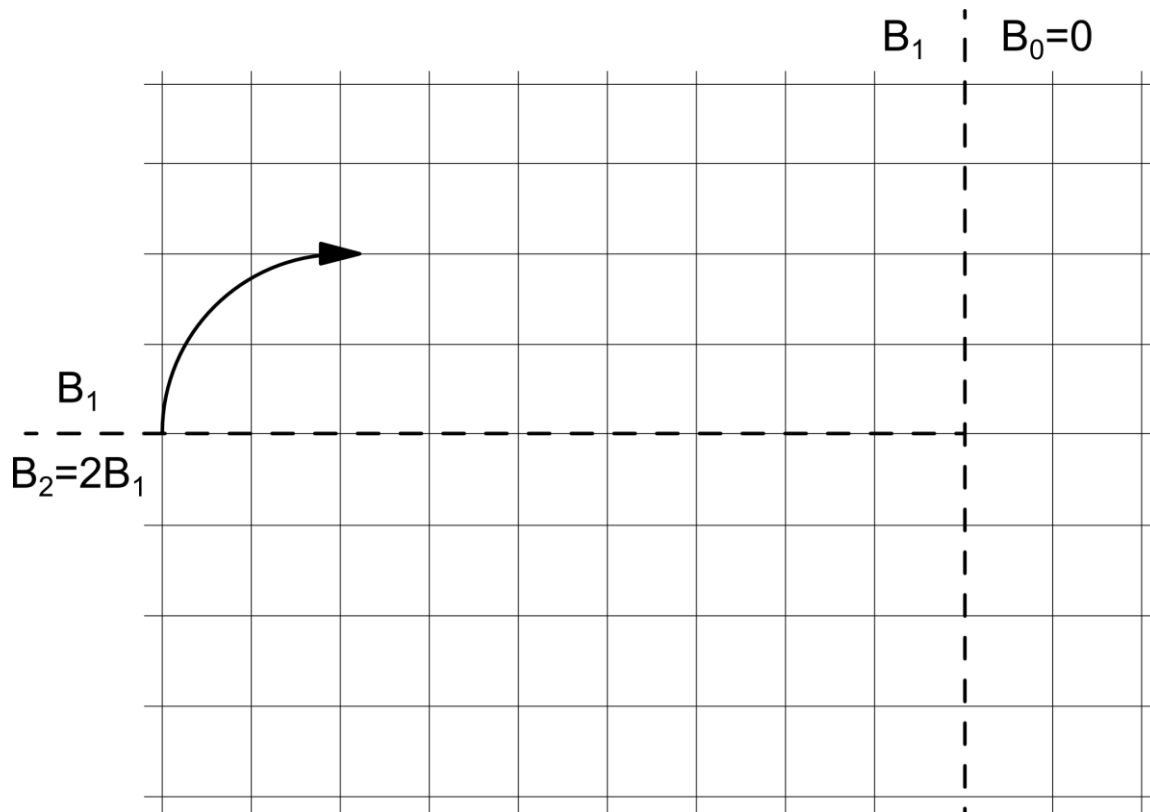
$$q_{O^+} = J_O n_O(h) e^{-\tau(h)}$$

where  $J_O$  is the ionization frequency of  $O$  (about  $2 \times 10^{-7} \text{ s}^{-1}$ ),  $n_O(h)$  is the density of the neutral  $O$  atom, and  $\tau(h)$  is the optical depth. For an oxygen density of  $10^{14} \text{ m}^{-3}$  and an optical depth of 0.06, what is the production rate of  $O^+$ ? (1 point)

- Assuming the production rate from c), what is the typical time needed to build up an electron density of  $n_e = 10^{11} \text{ m}^{-3}$ ? (2 points)

### PROBLEM 3 (10 points)

- Write down the equation of motion for a charged particle in a static electric and magnetic field! Name all parameters! (2 points)
- Assuming no electric field and a constant magnetic field in the z-direction, simplify the equation of motion and derive an expression for the gyrofrequency! (5 points)
- In the figure below, the magnetic field is pointing out of the sheet of paper. In the bottom left area, the magnetic is twice as strong as in the top left region; it is 0 right of the dashed line. Complete the trajectory of a charged particle as indicated! (3 points)



**PROBLEM 4 (10 points)**

- In the ionosphere, electrons typically have temperatures of  $T_e = 1000$  K. What velocity does this temperature correspond to when assuming that the thermal energy  $k_B T_e$  is equal to the electron's kinetic energy? (2 point)
- The magnetic moment  $\mu$  of a gyrating particle is given by

$$\mu = \frac{mv_{\perp}^2}{2B}$$

where  $m$ ,  $v_{\perp}$ , and  $B$  are the particle mass, the particle velocity perpendicular to the magnetic field and the magnetic field strength, respectively. What is the magnetic moment of an electron gyrating at the velocity calculated under a) at an altitude of 500 km exactly above the northern magnetic pole, where the magnetic field is vertical and given by

$$B(z) = \frac{\mu_0}{4\pi} \frac{2m_z}{z^3}$$

with  $z$  the vertical coordinate from the center of the Earth and  $m_z$  the magnetic moment of the terrestrial dipole field? (2 points)

- Because the magnetic field is converging toward the pole, there exists an upward force on the gyrating electron due to its magnetic moment given by

$$F_B = -\mu \frac{\partial B}{\partial z}$$

How big is this force for an electron with a temperature of  $T_e = 1000$  K? (2 points)

- What is  $v_{\perp}$  such that the force  $F_B$  pushing the electron upwards is exactly balanced by gravity pulling it downwards? (2 points)
- What temperature does the velocity calculated under c) correspond to? How does that compare to the average temperature of electrons in the ionosphere? Do you think there are many electrons where the magnetic moment force is balanced by gravity? (2 points)

**APPENDIX**

<b>Constant</b>	<b>Value</b>
Boltzmann's constant $k_B$	$1.38 \times 10^{-23} \text{J/K}$
Magnetic permeability of free space $\mu_0$	$4\pi \times 10^{-7} \text{H/m}$
Electron mass $m_e$	$9.1 \times 10^{-31} \text{kg}$
Earth's radius $R_e$	6372 km
Earth's magnetic moment $m_z$	$8 \times 10^{22} \text{Am}^2$
Gravitational acceleration at 500 km altitude $g$	$8.4 \text{m/s}^2$