FYS 3610: Week 39

Midterm exam type of exercises

Exercise 1 (17 points)

a) Draw a scheme of the temperature profile of the Earth's atmosphere with the altitudes of the boundaries and label the different layers (1+1+1+1+1+1 points)

b) What is the typical value of the temperature on the ground and at 500 km altitude? (1+1 points)

c) The altitudinal density profile of oxygen density can be calculated starting from the momentum balance equation for static equilibrium given by

$$\frac{dp}{dz} = -\rho\, g$$

What are the physical parameters p and ρ? (1+1 points)

d) Solve this equation to obtain an expression for the altitudinal density profile of oxygen assuming constant g and constant temperature T. (1+1+1+1 points)

Help: use the ideal gas law.

e) Calculate the scale height for oxygen at 500 km altitude using $g = 8.8\ m\ s^{-2}$ $m_O = 16\ m_p$ and the temperature you wrote in b). By how much does the density decrease for an altitude increase of one scale height? (1+1+1 points)

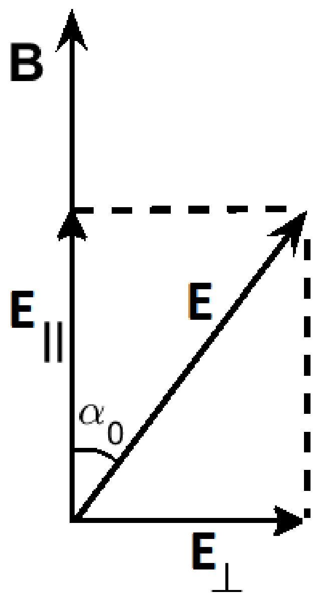
Exercise 2 (12 points)

a) Write the equation of motion of a charged particle in a static electric and magnetic field (use vector notation!). Name all the physical parameters. (1+1 points)

b) Decompose the equation you wrote in a) into one equation for the motion along the magnetic field and one for the motion perpendicular to the magnetic field. Find an expression for the velocity along the magnetic field. (1+1+1+1 points)

c) It can be shown that the perpendicular motion of the particle is a superposition of a gyration with a constant drift of the guiding center (gyrocenter). What is the mathematical expression for the drift velocity $\vec{v}_D^E$ of the guiding center in the case of a constant electric field? (1 point)

d) Using the scheme below. Calculate $\vec{v}_D^E$ using $E = 0.05\ V/m$ and $B = 50\ \mu T$ and $\alpha_0 = 30°$. (1+1+1 points)



e) Draw a scheme of another (different) perpendicular motion of your choice for electrons and protons (1+1 points)

Exercise 3 (10 points)

a) Draw a scheme of the Earth's magnetosphere and label three different regions or boundaries (1+1+1+1).

b) How far does the magnetosphere of the Earth typically extend on the dayside? (1 point)

c) Consider that magnetosphere of Mercury has a similar shape as the Earth's magnetosphere. The distance of the magnetopause from Mercury's center on the dayside can be calculated assuming that the pressure due to the solar wind and the internal magnetic pressure of the magnetosphere balance at the magnetopause (at the stagnation point). The pressure exerted by the solar wind at the stagnation point is given by $p_{sw} = K\, n_{sw} m_p\, v_{sw}^2$, where n_{sw} and v_{sw} are the solar wind density and velocity, respectively, and the magnetic pressure is given by $p_B = \frac{2B^2}{\mu_0}$.

Calculate the distance of Mercury's magnetopause on the dayside assuming that Mercury's dipole moment is $M = 2.7 \cdot\ 10^{19}\ \text{A m}^2$, its radius is $R_M = 2437$ km, $K = 1$, $n_{sw} = 5 \cdot 10^6\ \text{m}^{-3}$ and $v_{sw} = 400\ \text{km/s}$. Give your answer in units of R_M. (1+1+1+1+1 points)

Help: Use the Appendix.

Exercise 4 (8 points) (more difficult)

In magnetohydrodynamics, the equation of motion is given by

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\,\vec{v}\right) = -\nabla p + \rho\,\vec{g} + \,\vec{j}\times\,\vec{B}$$

a) In the Parker model (fluid gas dynamic) two terms of this equations are neglected. Which ones? (1+1)

b) Assuming no currents, no gravitational force, and adiabatic changes in the state of the plasma, the equation of motion can be rewritten as

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\,\vec{v}\right) = -\frac{\gamma p}{\rho}\nabla\rho$$

where γ is the constant adiabatic exponent. The density balance equation with no production nor losses can be written as

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0$$

Assuming only x dependency, those two equations can be linearized to:

d.1) Momentum equation: $\rho_0 \frac{\partial v_1}{\partial t} = -\frac{\gamma p_0}{\rho_0} \nabla \rho_1$

d.2) Density balance equation: $\frac{\partial \rho_1}{\partial t} + \rho_0 \frac{\partial v_1}{\partial \mathrm{x}} = 0$

Assuming that $\rho_1(x)$ and $\vec{v}_1(x)$ are of the form

$$\rho_1(x) = \rho_{1'} \exp\big(i(kx - \omega t)\big)$$

$$v_1(x) = v_{1'} \exp\big(i(kx - \omega t)\big)$$

insert those two expressions into equations d.1) and d.2) and solve the system of two equations to find the dispersion relation, i.e, find $\omega(k)$. (1+1+1+1+1 points)

e) Draw a graph of the frequency ω with respect to wavenumber k using $\frac{\gamma p_0}{\rho_0} \equiv 1$. (1 point)

APPENDIX:

1. The proton mass is $m_p = 1.67 \cdot 10^{-27}$kg

2. The elementary charge is $q = 1.60 \cdot 10^{-19}$ C

3. The magnetic permeability in vacuum is $\mu_0 = 4\pi \cdot 10^{-7}$T m/A

4. Elektronvolt til Joule: $1 \text{ eV} = 1.60 \cdot 10^{-19}$ J

5. Magnetic field intensity

$$B = \frac{\mu_0\, M}{4\pi\, r^3}\, \sqrt{1 + \sin^2\phi}$$

where ϕ is the magnetic latitude ($\phi = 0$ at magnetic equator), M is the magnetic dipole moment and r is the distance from the center.

6. Boltzmann konstant: $k_B = 1.38 \cdot 10^{-23}$ J/K

7. $1\ T = 10^6\ \mu T$