FYS 3610

Exercises Week 45
(Relevant for the final exam)

Exercise 1

In the magnetosphere above the auroral oval, extreme low plasma densities are often observed. A satellite in this part of space have measured an electron plasma-frequency (f_p) of 20 kHz and an electron gyro-frequency (f_g) of 100 kHz.

1. Calculate the electron plasma density and the Earth magnetic field strength at the satellite.

2. Determine at the satellite, 1) the equations for cutoff-frequencies for electromagnetic waves, and 2) calculate the upper hybrid frequency.
   Hints:
   1. A cutoff occurs when the index of refraction n goes to zero ($n = 0$)
   2. The definition of the upper hybrid frequency is: $f_h^2 = f_g^2 + f_p^2$

3. Draw a graph for the refractive index as function of frequency for longitudinal and transversal waves, respectively.

Exercise 2

The daytime maximum electron density in the ionosphere of the planet Mars at equator is $1 \cdot 10^{11}$ m^{-3}. In the same height, the magnetic field is 100 nT.

1. Find the characteristic frequencies for electromagnetic waves propagating along the magnetic field in the ionosphere of Mars.
   Hint:
   Characteristic frequencies are defined for conditions where the refractive index either becomes zero or infinite.

2. Make a graph of the refractive index as function of the wave frequency for electromagnetic wave (propagating along the magnetic field).

Exercise 3

An Earth magnetic dipole field-line, starting at a geomagnetic latitude of 55 degrees, has a magnetic field strength in the equator plane of 1000 nT. The electron density here is observed to be $1 \cdot 10^9$ m^{-3}.
Electromagnetic whistler waves, having frequencies lower than the gyro-frequency and much lower than the plasma-frequency, are the focus of the exercise.

1. Use the Appleton-Hartree equation and show that the refractive index to a good approximation can be given by the equation:

$$n^2 = \frac{f_p^2}{f\left(f_g \cdot cos\theta - f\right)}$$

Hint:
θ is the angle between the wave propagation vector $\boldsymbol{k}$ and the Earth magnetic field $\boldsymbol{B_0}$.

2. Calculate the plasma-frequency and the gyro-frequency.

3. Calculate the phase velocity of whistler waves with a frequency of 2 kHz (propagating along the magnetic field lines).