

# Microfluidics

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FYS4230

7th nov. 2006

# Outline:

- General intro
- Mechanics of fluids
- Surface tension and related phenomena
- Electrokinetics
- Mixing

These topics are important for design of well-functioning fluidic microsystems.



# General introduction

# Microfluidics – what is it?

- Study and manipulation of fluid flows in microsystems
- Combination of fluid mechanics, thermodynamics, surface- and colloid chemistry, electrohydrodynamics (electrokinetics), magnetohydrodynamics, and more.....
- Also: fluid logics (like electronics)
  - Fluid logic circuits
- Typical reservoir/channel dimensions:  $\sim 5 - 500\mu\text{m}$
- Overall system dimensions from few millimeters to several centimeters

# Microfluidics

- Heat exchangers (also with boiling)
- Sensors, thrusters, microengines
- Two-phase flow systems
- Microreactors
- Microfluidics (refers to general flow manipulation and measurement techniques)
- Lab-on-a-chip (chemical)
- Optical systems
- BioMEMS

# Microfluidics

- Typically laminar flows (no turbulence)
- High surface – to – volume ratio ( $\sim 10^5 \text{ m}^{-1}$ )
  - ⇒ Surface forces dominate
  - ⇒ “Special” effects encountered on the microscale:
    - Flow generation by electrical fields, magnetic fields, and capillary forces
    - Bi-directional flow in a single channel
    - Complex flow patterns in simple geometries
    - Counter-current two phase flow without mixing
    - Precise generation and manipulation of droplets and liquid plugs
- Benefits of downscaling:
  - Ability to manipulate and detect small volumes
  - Low consumption of reagents
  - Quick system response
  - Manipulation and processing of biological material
  - High rates of heat transfer
  - Batch production => low prices

# Mechanics of fluids

# Viscosity

## Senturia 13.2.1

- Deformation of fluids in the presence of shear forces
- The property of a fluid that resists the action of a shear force
- $\eta$  [ Pa s ]
- Newtonian fluid:

$$\tau = \eta \frac{U}{h}$$

$$\tau = \eta \frac{\partial U_x}{\partial y}$$

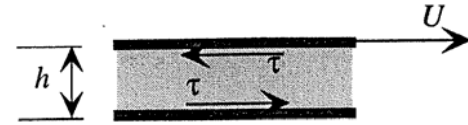
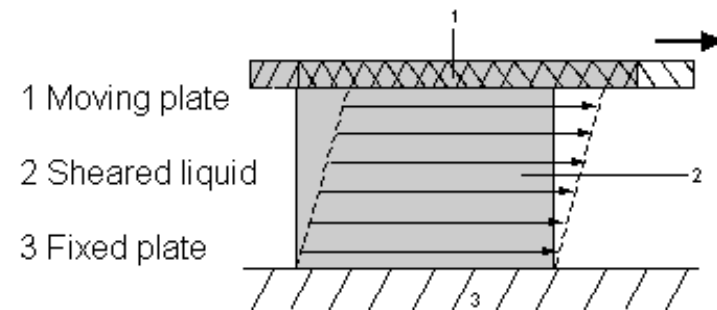


Figure 13.1. Fluid between two plates. The upper plate moves to the right with velocity  $U$ , setting up shear forces  $\tau$ .





# Navier-Stokes equations

- Conservation of mass

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0$$

- Newton's 2nd law for a fluid

$$\rho \left( \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right) = -\nabla p + \eta \nabla^2 \vec{v}$$

# Reynolds number

$$\text{Re} = \frac{\rho UL}{\eta}$$

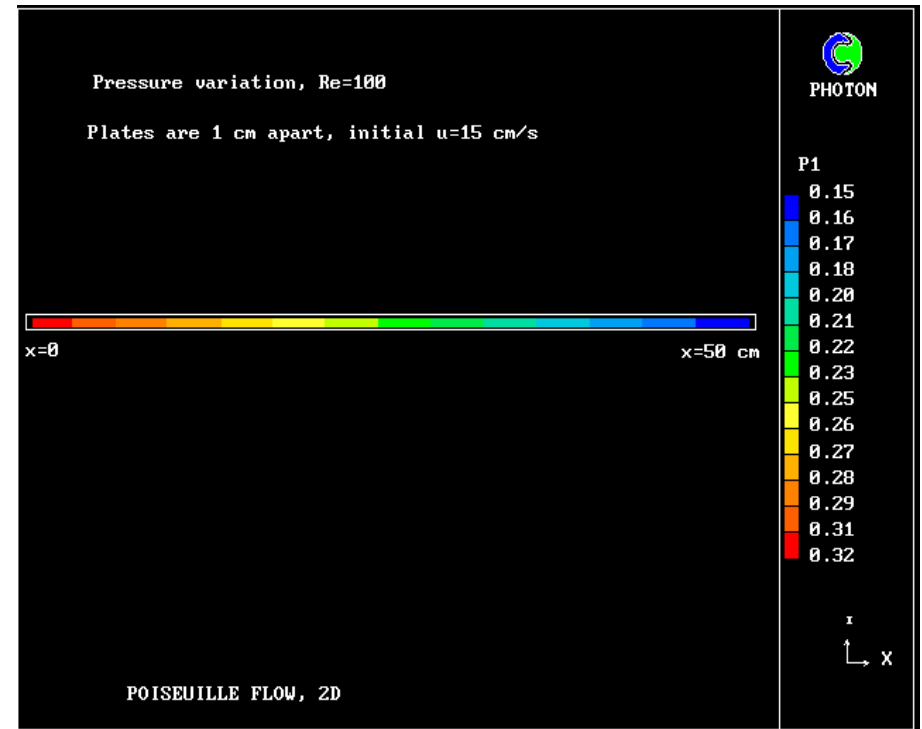
- Laminar or turbulent flow?
- Ratio of inertial forces to viscous forces
- Reynolds number:  
ratio of kinetic energy of a volume of fluid in the flow to the energy dissipated by the volume in the shear caused by interaction with its solid boundaries.

- Microchannel:
  - 1 cm long
  - 1 mm wide
  - 100  $\mu\text{m}$  deep
- $L=50 \mu\text{m}$
- $\rho=1000 \text{ kg/m}^3$
- $\eta=0.001 \text{ kg/ms}$

Laminar for flow speeds less than 10m/s

# Example: Poiseuille flow

- Pressure driven flow in channel
- Pressure drop along channel
- Steady flow
- Incompressible flow
- Flow in x-direction, only
- No-slip boundary condition



# Poiseuille flow

$$\eta \frac{\partial^2 U_x}{\partial^2 y} + \frac{\Delta p}{L} = 0$$

Integrate twice :

$$U_x(y) = -\frac{1}{2\eta} \frac{\Delta p}{L} y^2 + c_1 y + c_2$$

No slip boundary condition gives :

$$U_x(y) = \frac{1}{2\eta} \frac{\Delta p}{L} \left[ \left( \frac{a}{2} \right)^2 - y^2 \right]$$

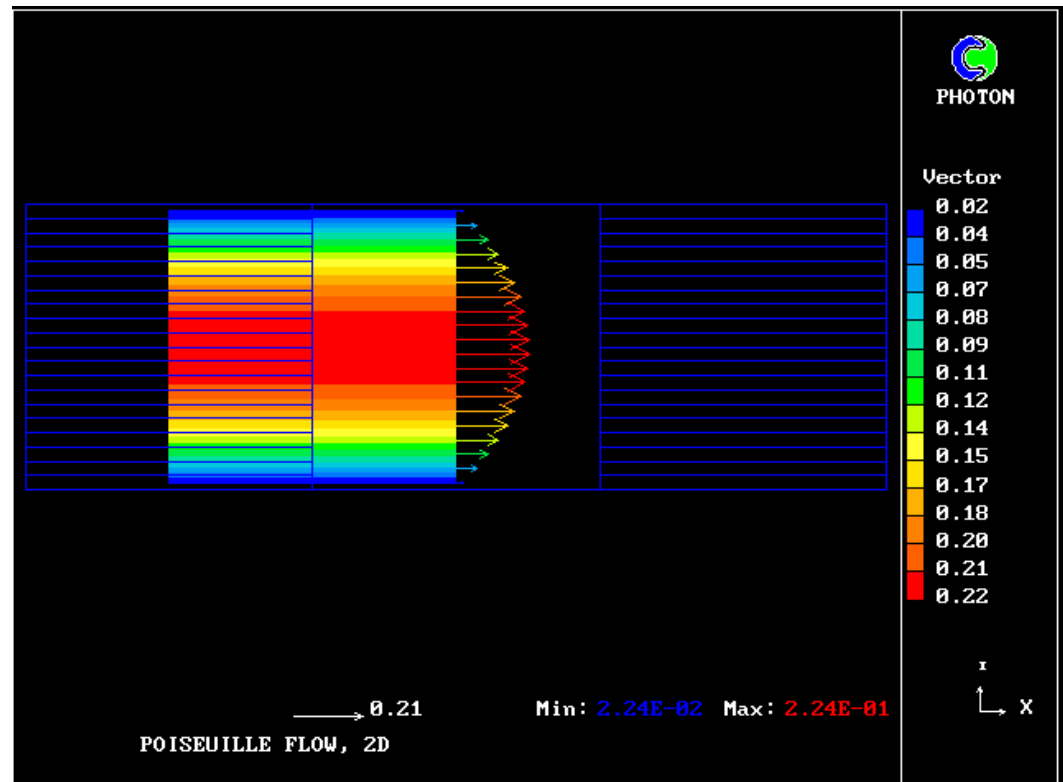
Flow rate :

$$Q = \int_0^{l_z} dz \int_{-a/2}^{a/2} U_x(y) dy$$

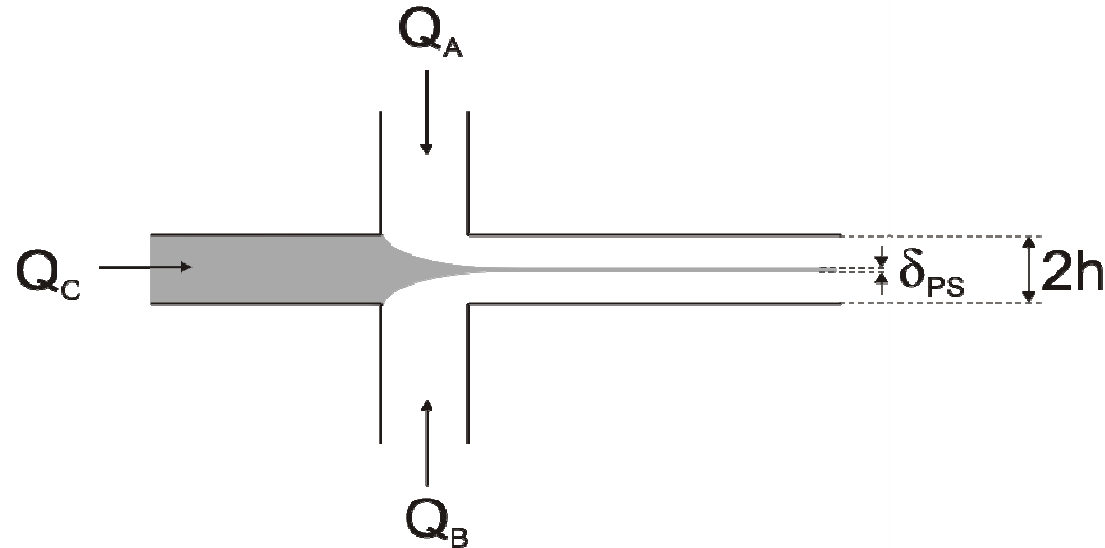
$$Q = \frac{l_z a^3}{12\eta} \frac{\Delta p}{L}$$

Circular pipe :

$$Q = \frac{\pi a^4}{8\eta} \frac{\Delta p}{L}$$



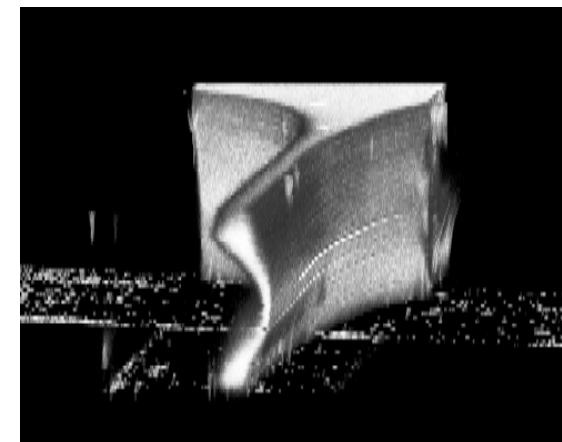
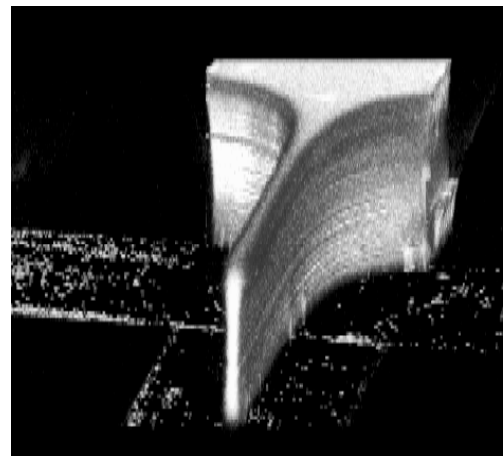
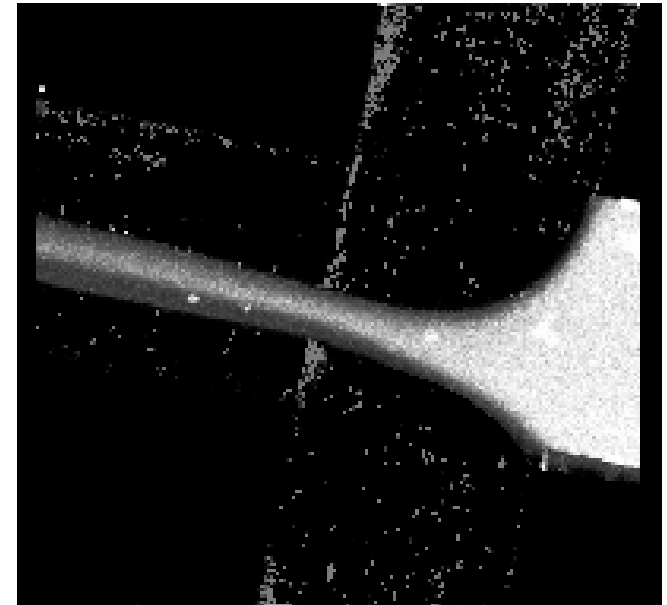
# Example: flow manipulation by hydrodynamic focusing



- $Q_A$  &  $Q_B$ : buffer streams
- $Q_C$ : stream seeded with fluorescent particles
- Sheet thickness:  $\delta_{PS}/2h \propto Q_C/(Q_A+Q_B)$
- Sheet position:  $f(Q_A/Q_B)$
- Requirement: stable flow conditions at focusing intersection

# Example: flow manipulation by hydrodynamic focusing

- Flow addressing
- Rapid mixing
  - Decreased diffusion length
- Manipulation of particles
  - Stretching of DNA strands
- Selective coating of particles
- Polymer membrane generation
- Multiple buffer layers possible



# New Micro Flow Rate Sensor for Standardized Industrial Production

3  $\mu\text{m}$



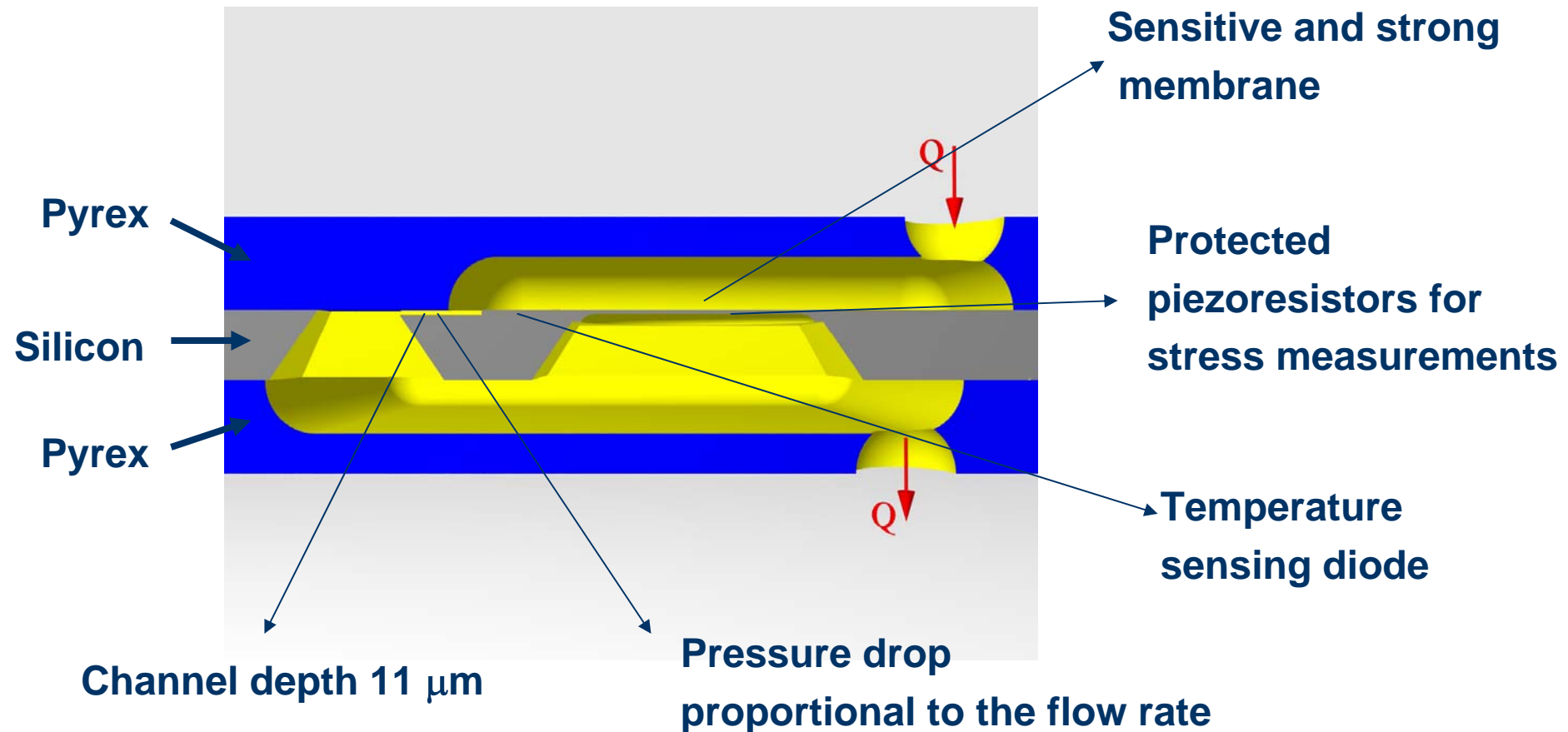
6 mm



Liv Furuberg  
Dag Wang  
Andreas Vogl

Microsystems and Nanotechnology  
SINTEF Information and Communication Technology

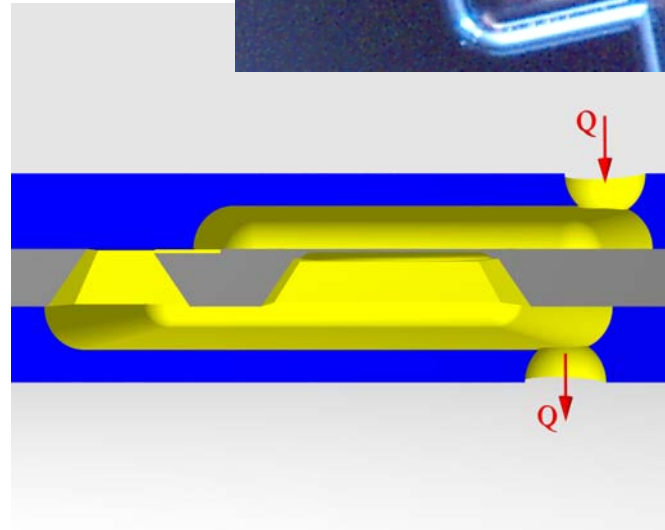
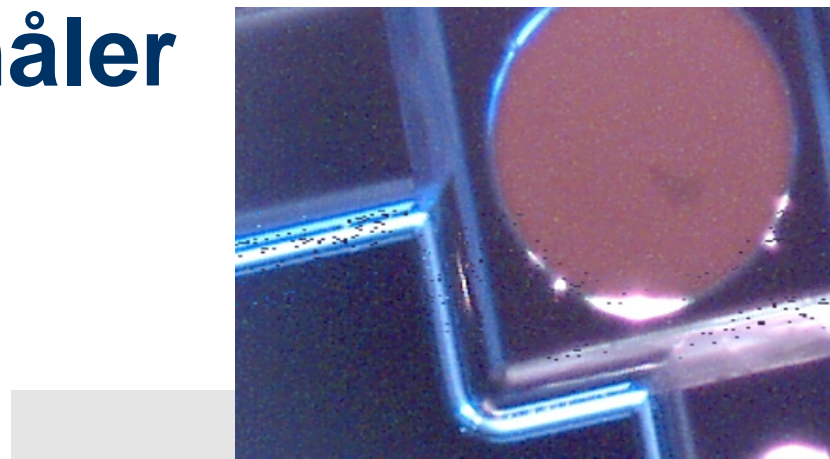
# The new design suggests a low-noise, mechanically robust flow sensor





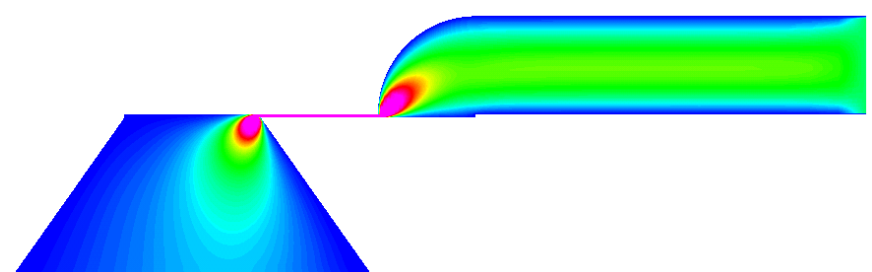
# Volum-strømningsmåler

- Applikasjoner: Dosering, tilføring av reagerer, måle flow gjennom analysesystem
- Væskestrøm gjennom brikken
- Glass-silisium-glass brikke
- Laminær strøm, lave Re tall
- Differensiell trykksensor (membran + piezomotstander)
- Trang kanal med trykkfall, Pouseille strøm
- Trykkfall ~ 100 -200 Pa
- Integrert temperaturmåler



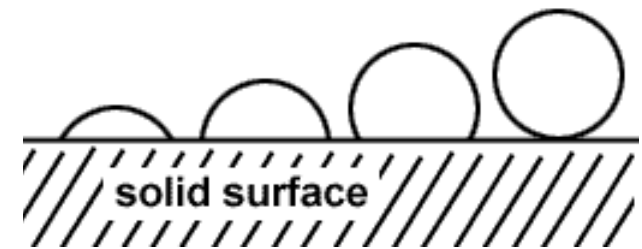
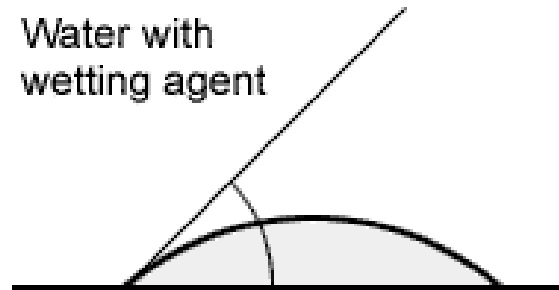
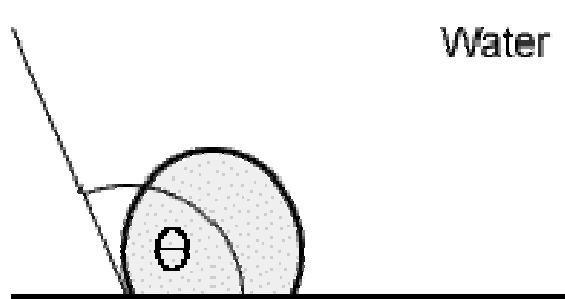
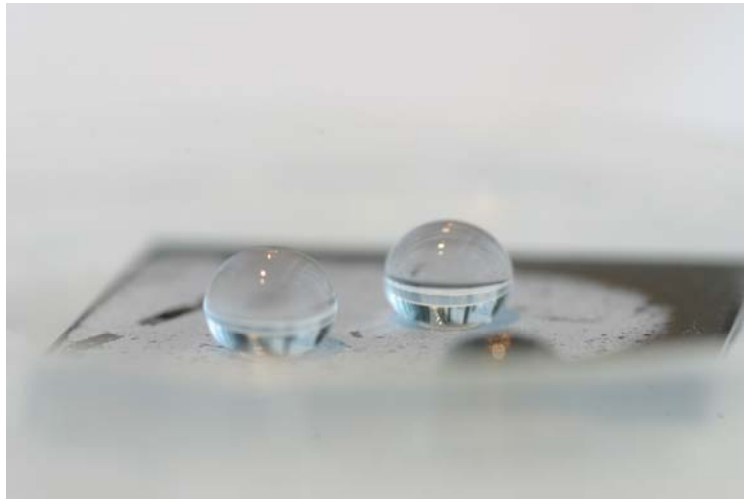
- Kanal: 800x1500x10  $\mu\text{m}$
- Flow rate 2  $\mu\text{l}/\text{min}$

$$\Delta p = \frac{12 \cdot \eta \cdot l \cdot Q}{w \cdot h^3}$$

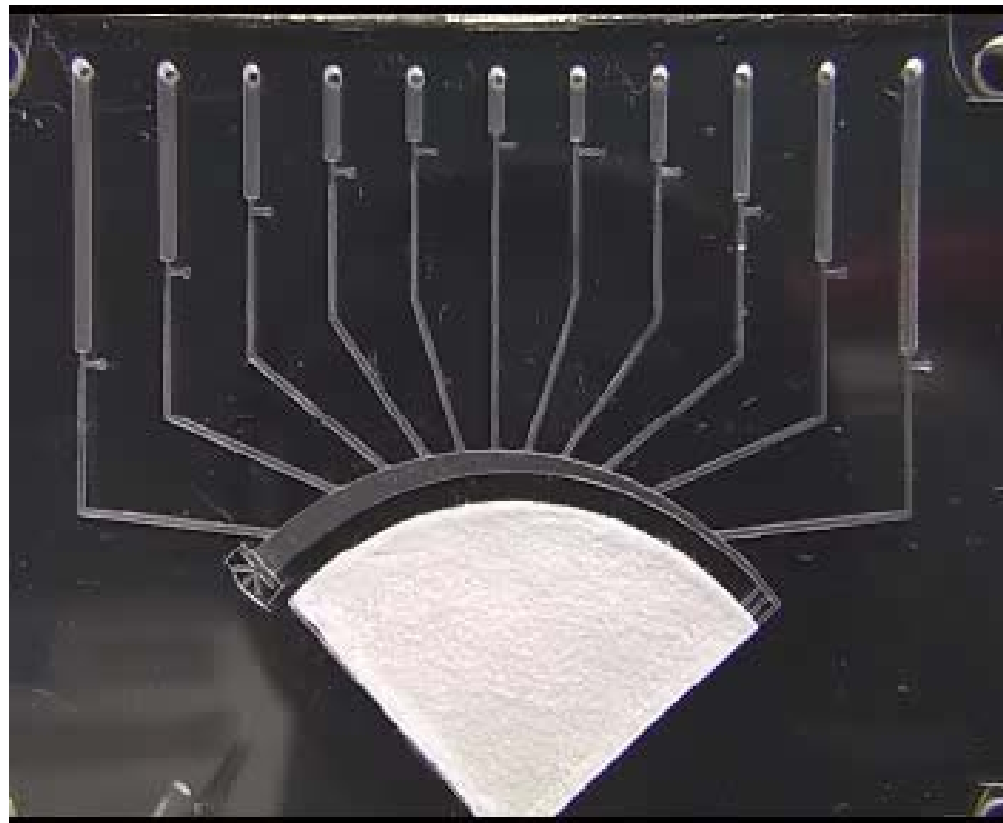


# Surface tension and related phenomena

# Wetting / Non-wetting

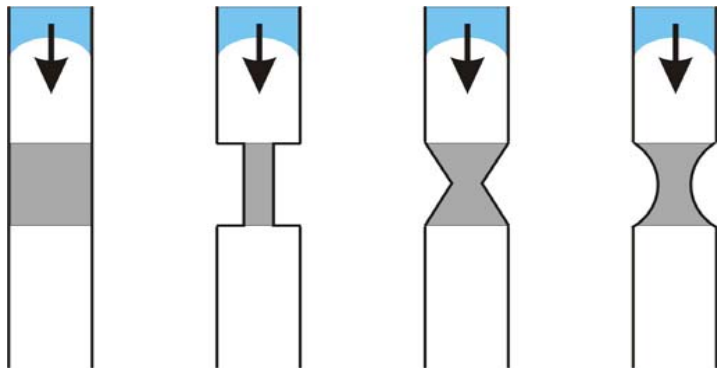


# Capillary filling

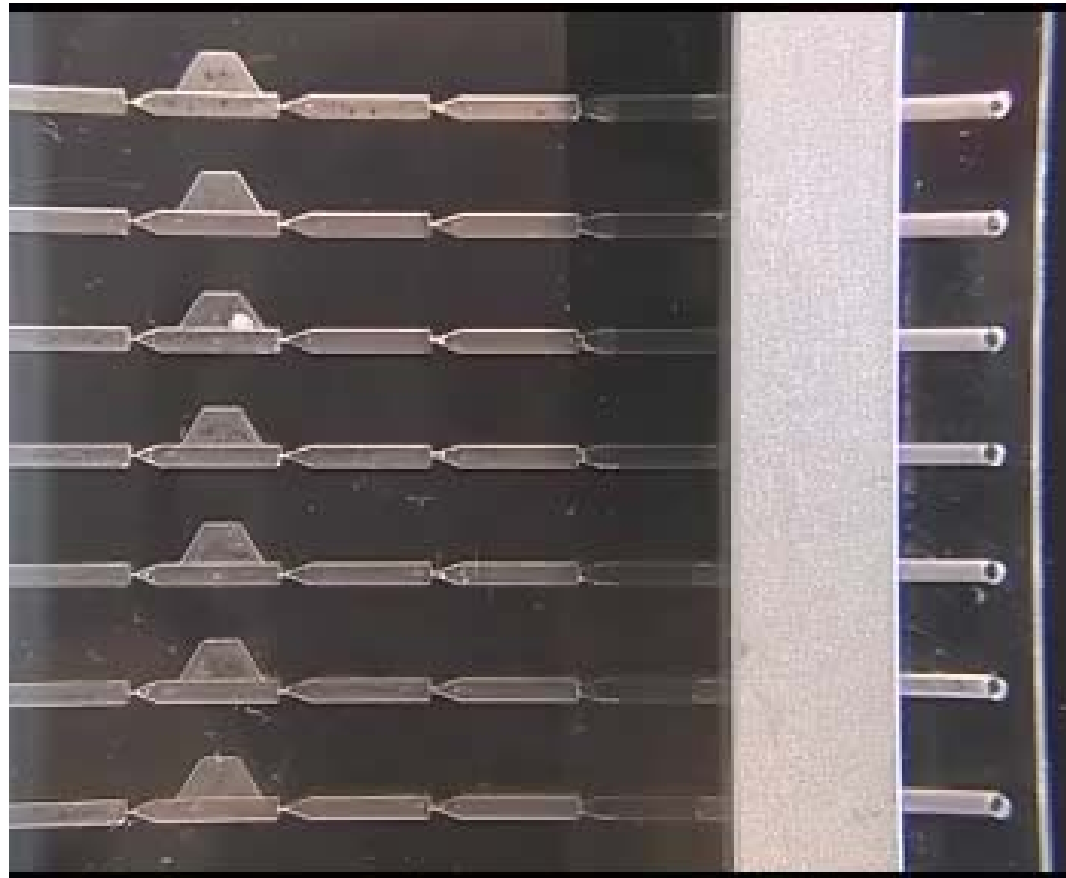


# Capillary valving (surface manipulation to change wetting properties)

- Hydrophobic valves
- Small restriction – high pressure required to “break” the valve
- Can efficient flow control using a single pressure source for many parallel channels be achieved?

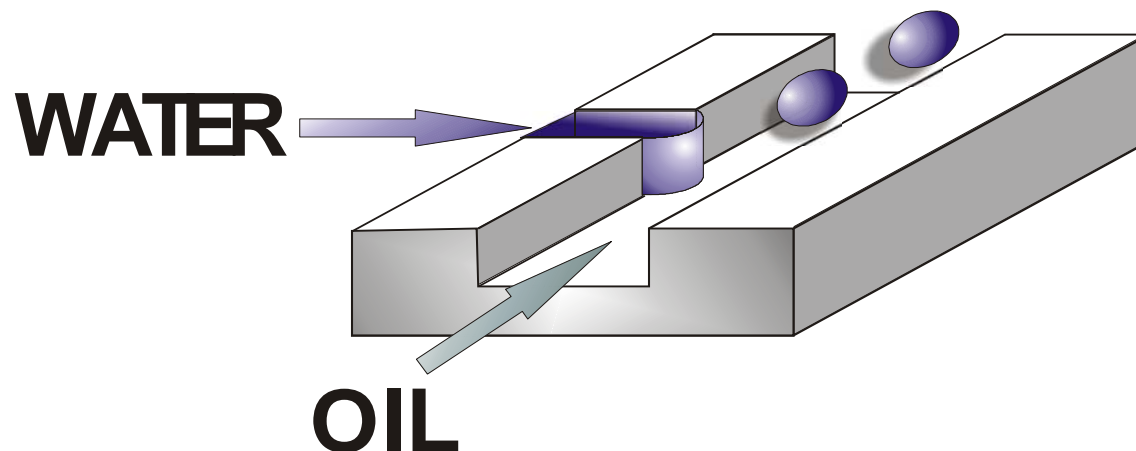


# Capillary valving (surface manipulation to change wetting properties)



# Two-phase (liquid-liquid) flows

- Simple T-junction geometry
- Two immiscible liquids
- Shear forces exerted by the oil deform, and eventually break off a water droplet

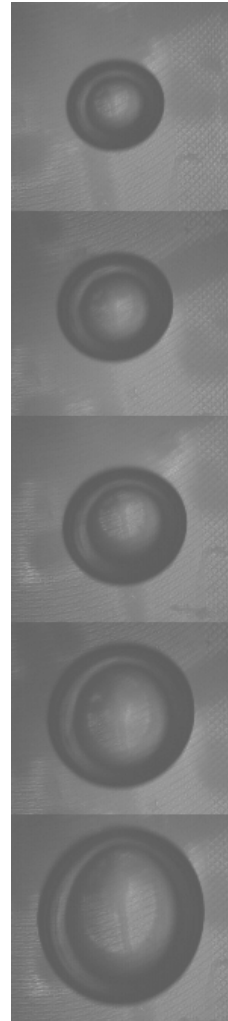
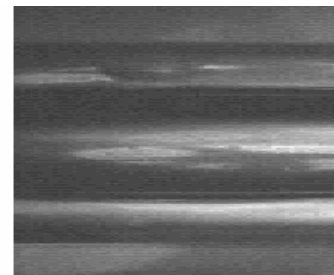
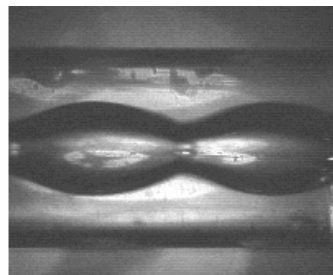
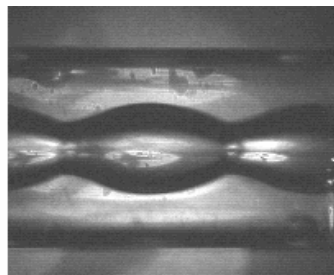
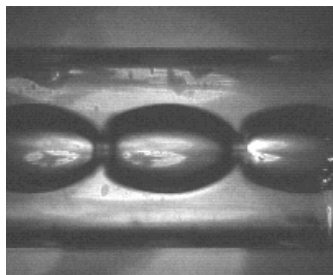
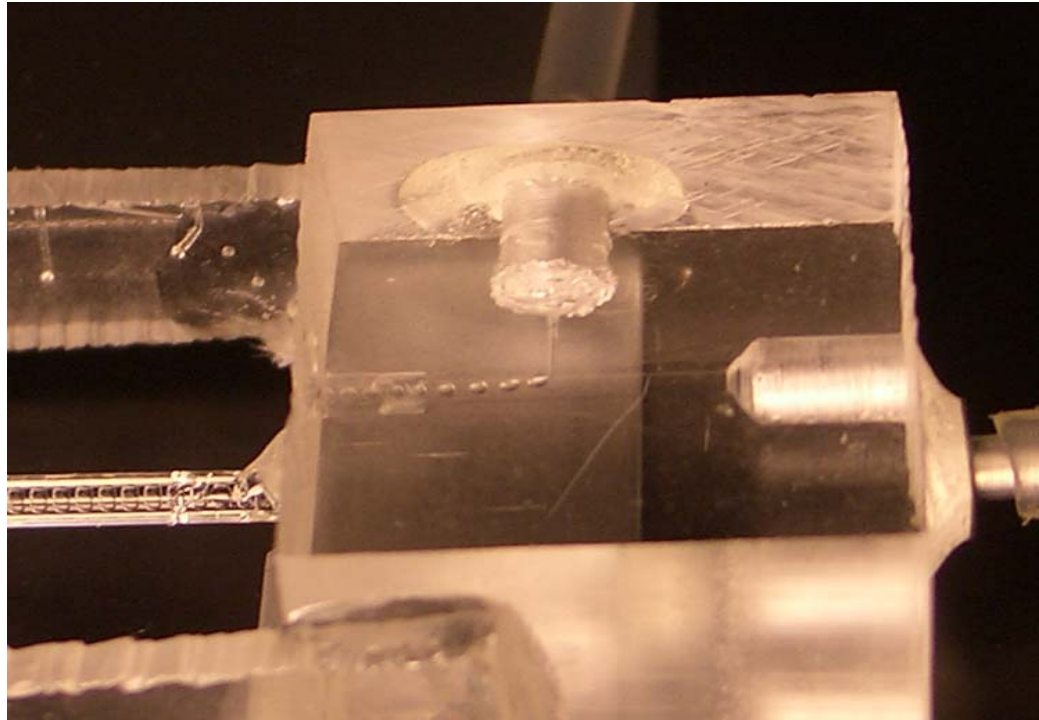


# Two-phase (liquid-liquid) flows

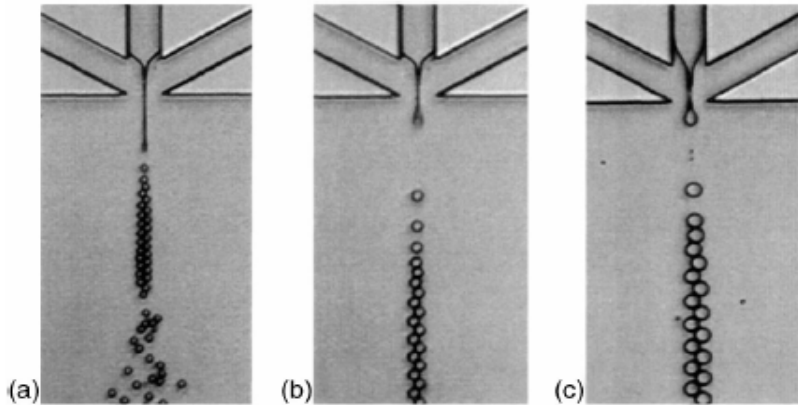




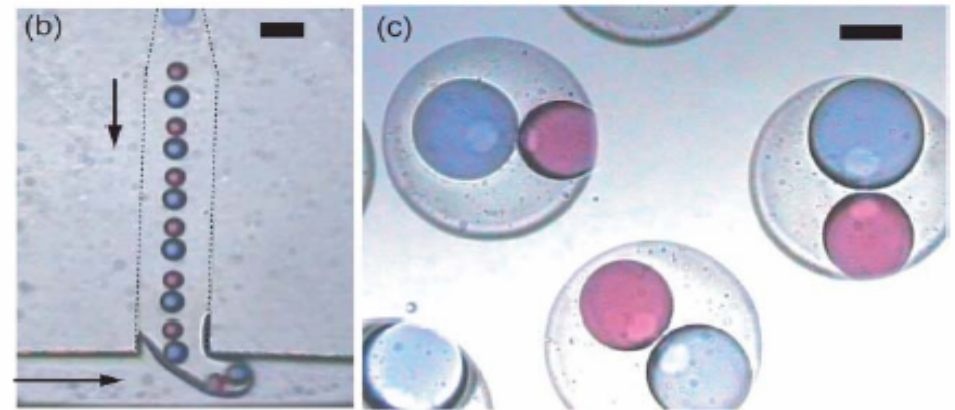
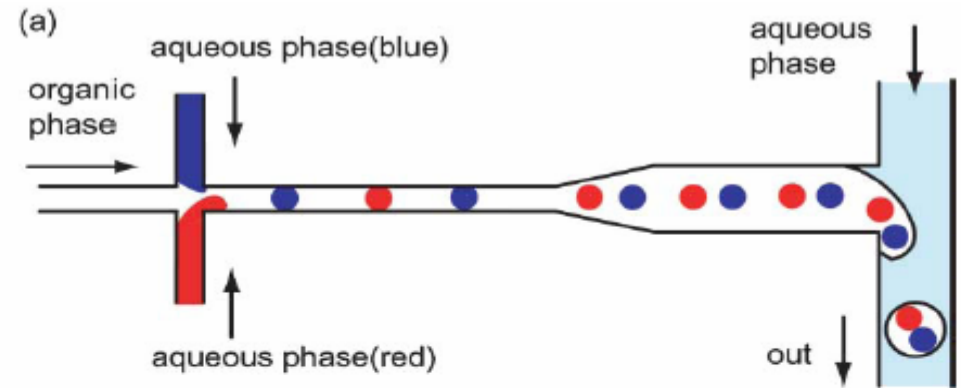
# Two-phase (liquid-liquid) flows



# Two-phase (liquid-liquid) flows



Xu et al. (2004)  
Droplet size: 9 $\mu$ m



Nisisako et al. (2005)

# Droplet generation - applications

- Monodisperse emulsions
- Bidisperse emulsions
- Production of polymer particles
- Chemical reactors
- Transportation of biological species
- Encapsulation of drugs, reagents, etc.

# Electrokinetics

# Electroosmotic Flow

- Flow driven by electric field
- Voltage applied between electrodes immersed in electrolyte
- Force on fluid near the boundaries, excess of charged particles
- Debye layer, typically 10nm - 100nm
- Disadvantages:
  - Sensitivity to impurities
  - Ohmic generation of heat
  - Need for high voltage
- Advantage:  
Plug flow

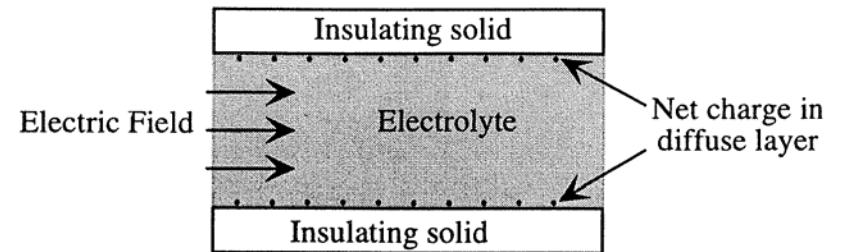


Figure 13.11. Illustrating electroosmotic flow

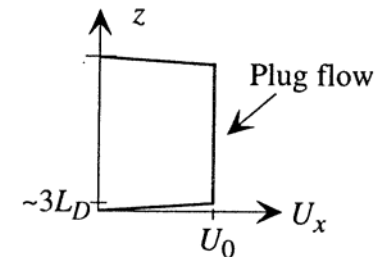
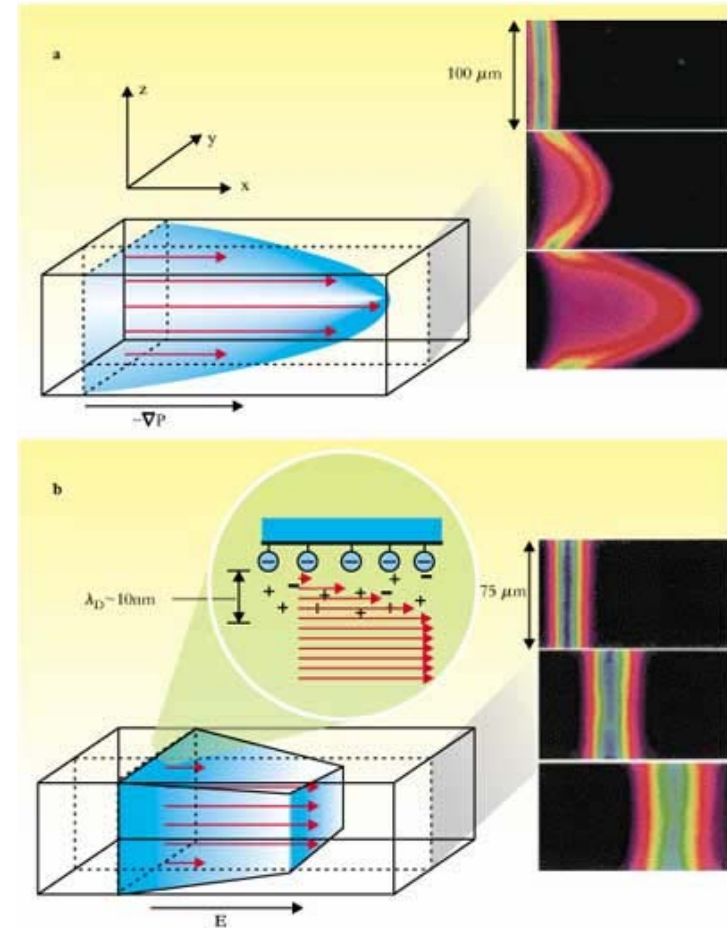


Figure 13.12. Electroosmotic flow profile.

# Electroosmotic flow

- Poiseuille flow vs. electroosmotic flow
- Advantage in 3D visualization/detection
- Three pictures after creation of fluorescent molecule:
  - 0s
  - 66ms
  - 165ms
- Separation based on charge-to-size ratio of molecules.
- Separated bands of species



# Electrophoresis

- Species carried along with electroosmotic flow
- Drift relative to the moving velocity:

$$v_{ep} = \mu_{ep} \mathcal{E}_x$$

- Electrophoretic mobility
- Apply voltages to channels
- Create controlled plug of species
- Separate molecules by charge and volume by electrophoresis

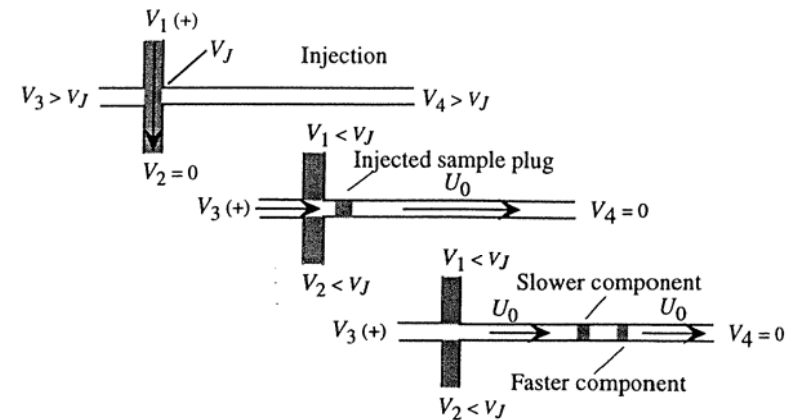
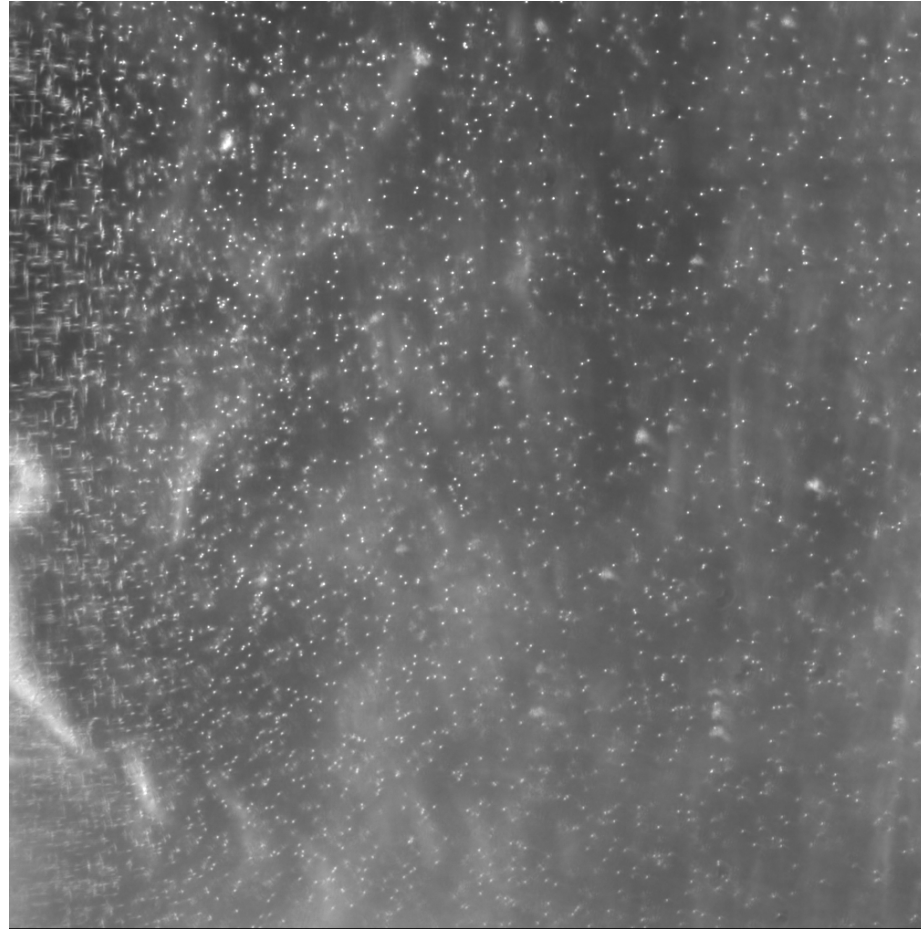


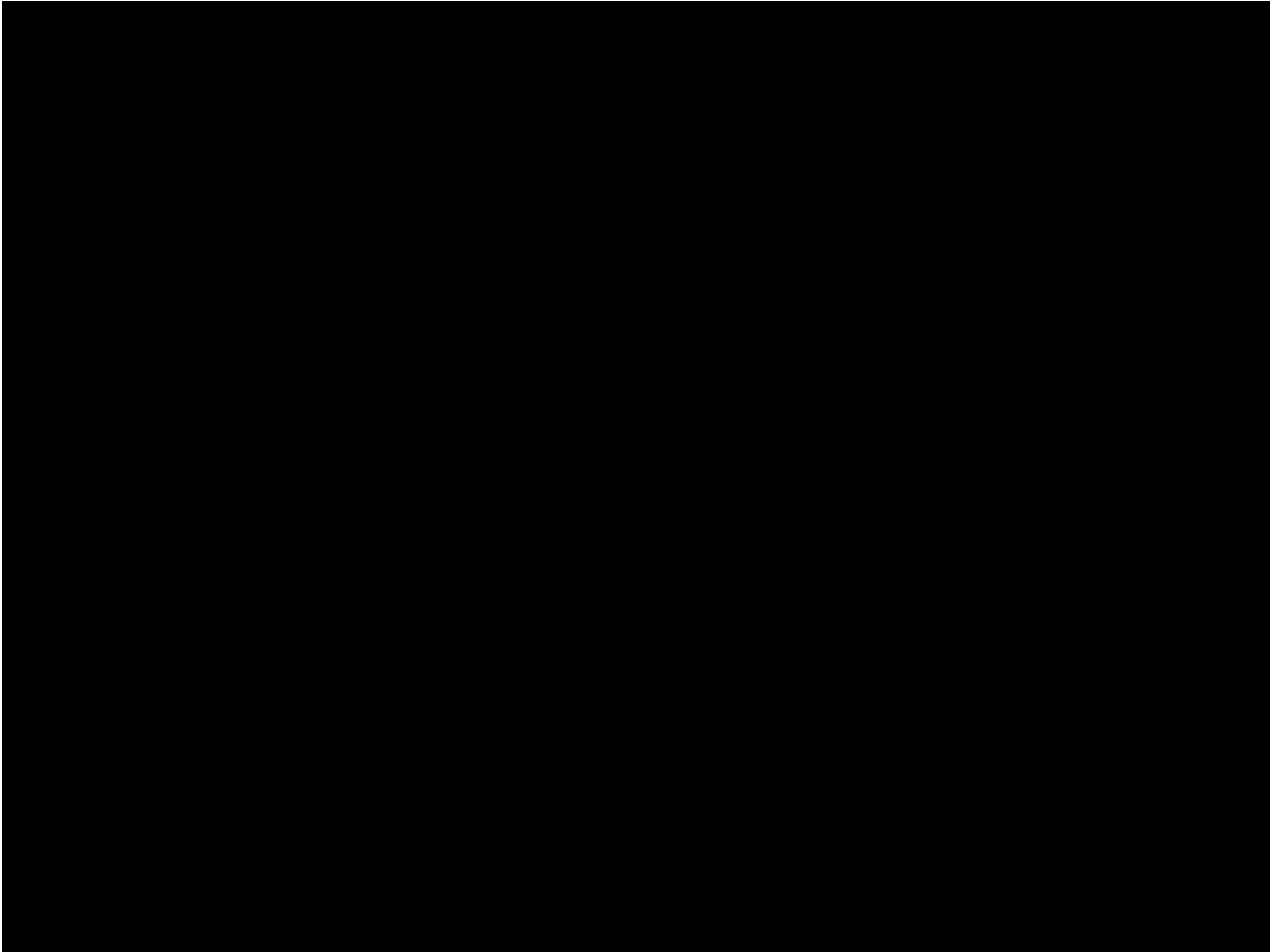
Figure 13.14. Illustrating electrophoretic separation with electroosmotic flow. The voltages used during the injection and separation sequence are described in the text.

# Oscillating electroosmotic flow





# Electrokinetic instabilities



# Mixing

# Mixing

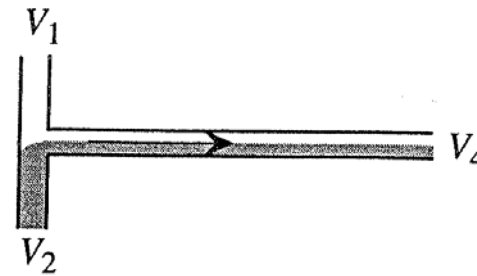
- Laminar flow
- Mixing by diffusion only

- Diffusion equation

$$\frac{\partial C(r,t)}{\partial t} = D\nabla^2 C(r,t)$$

- Average displacement of diffusing particle:

$$l = \sqrt{2Dt}$$



*Figure 13.16.* Illustrating laminar flow when two streams are combined. Mixing occurs only by diffusion.

# Mixing

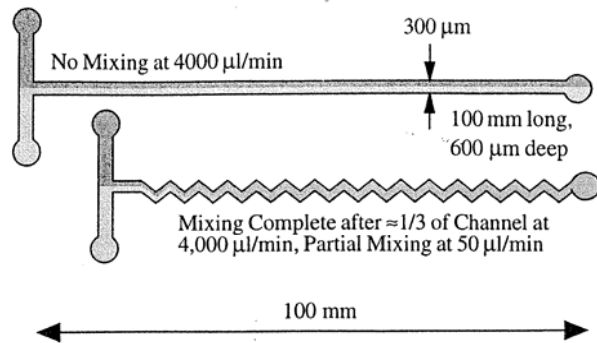


Illustration of miniature fluidic channels used to compare mixing in macroscopic and microscale fluidics. After Branebjerg, et al. (1994).

