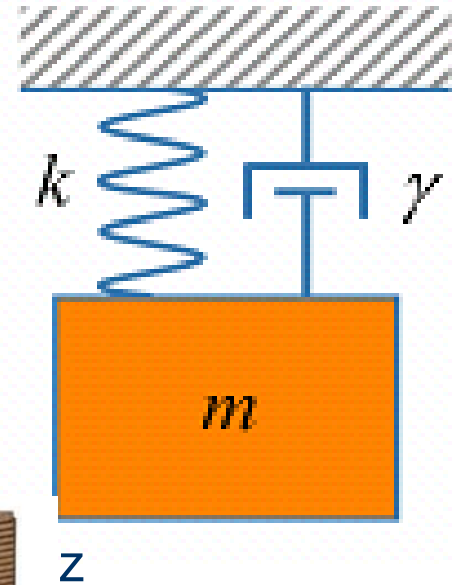
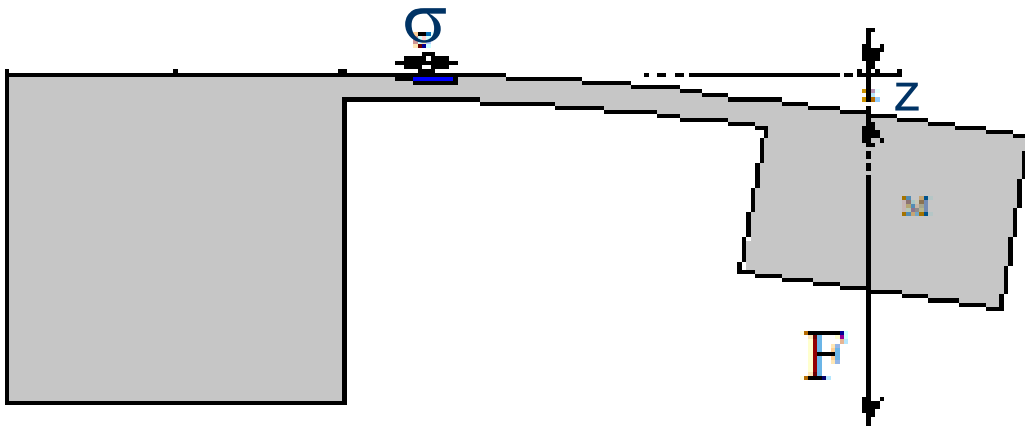


A bar walks into a man; oops, wrong frame of reference.

Hovedtyper

■ Mekanisk spenning

■ Utbøyning



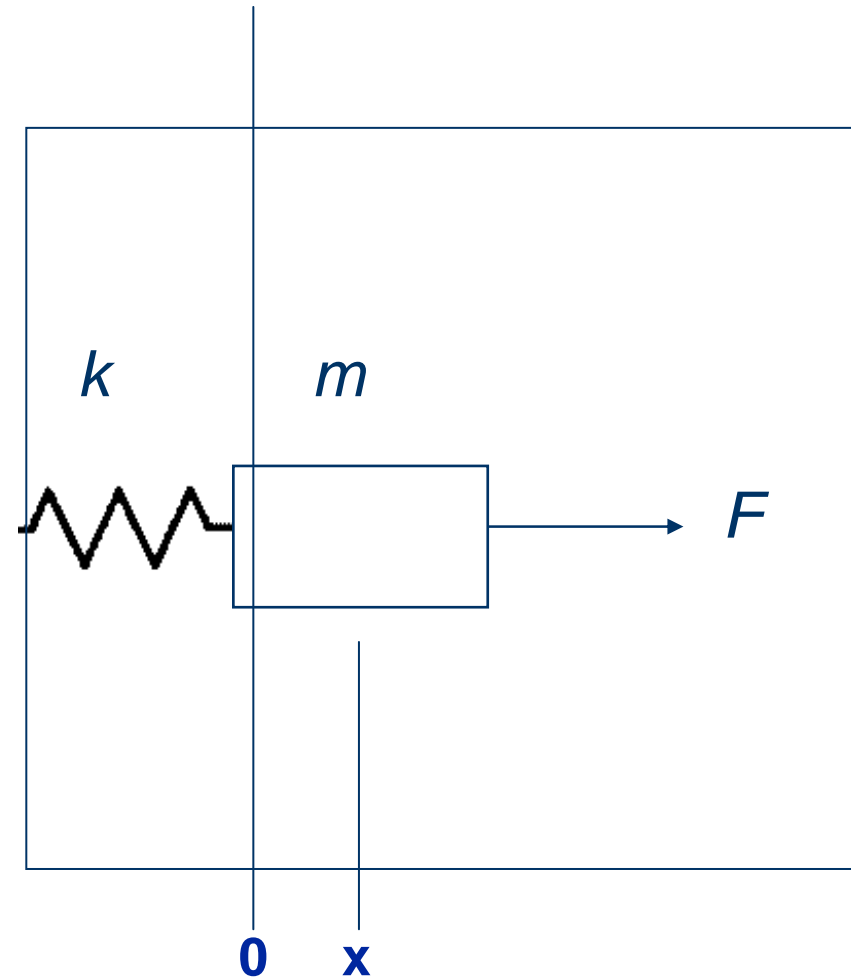
Dynamics

- Dynamic equation (without damping)

$$m \frac{d^2 x}{dt^2} + kx = F(t)$$

- Resonance frequency

$$\omega_0 = \sqrt{\frac{k}{m}}$$



Damping: Spring-mass-dashpot model

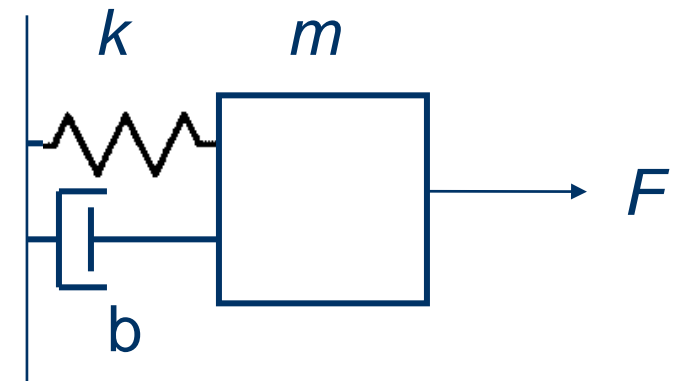
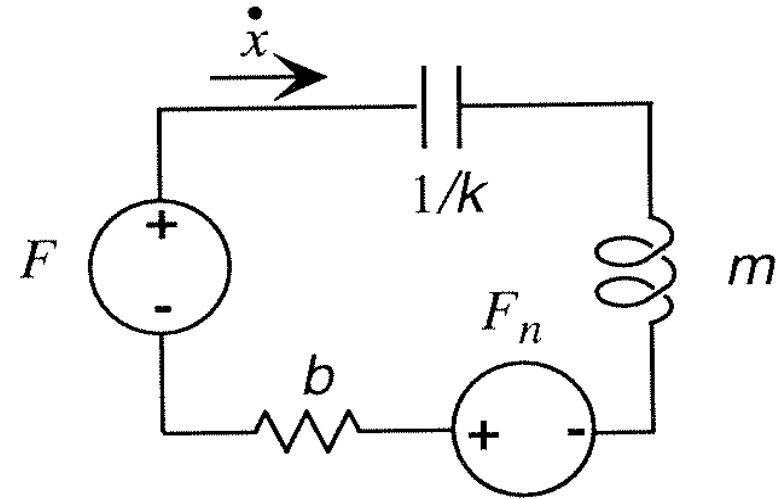
$$0 = m\ddot{x} + kx + b\dot{x} + F(t)$$

- Damped resonance frequency

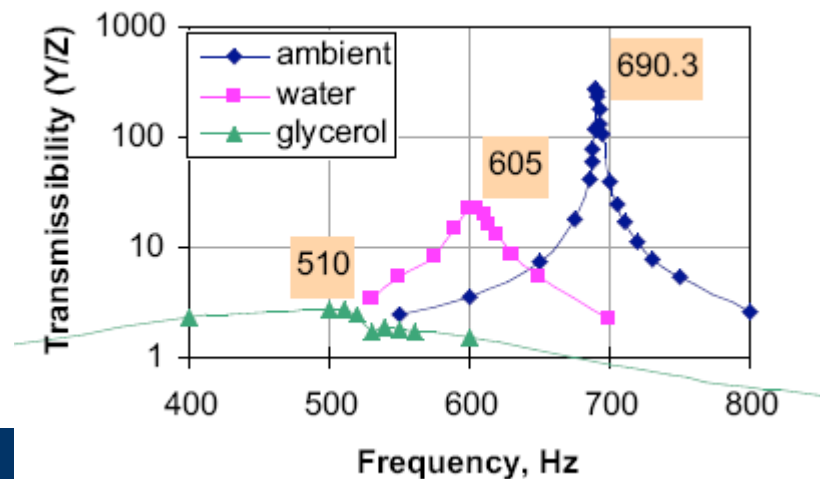
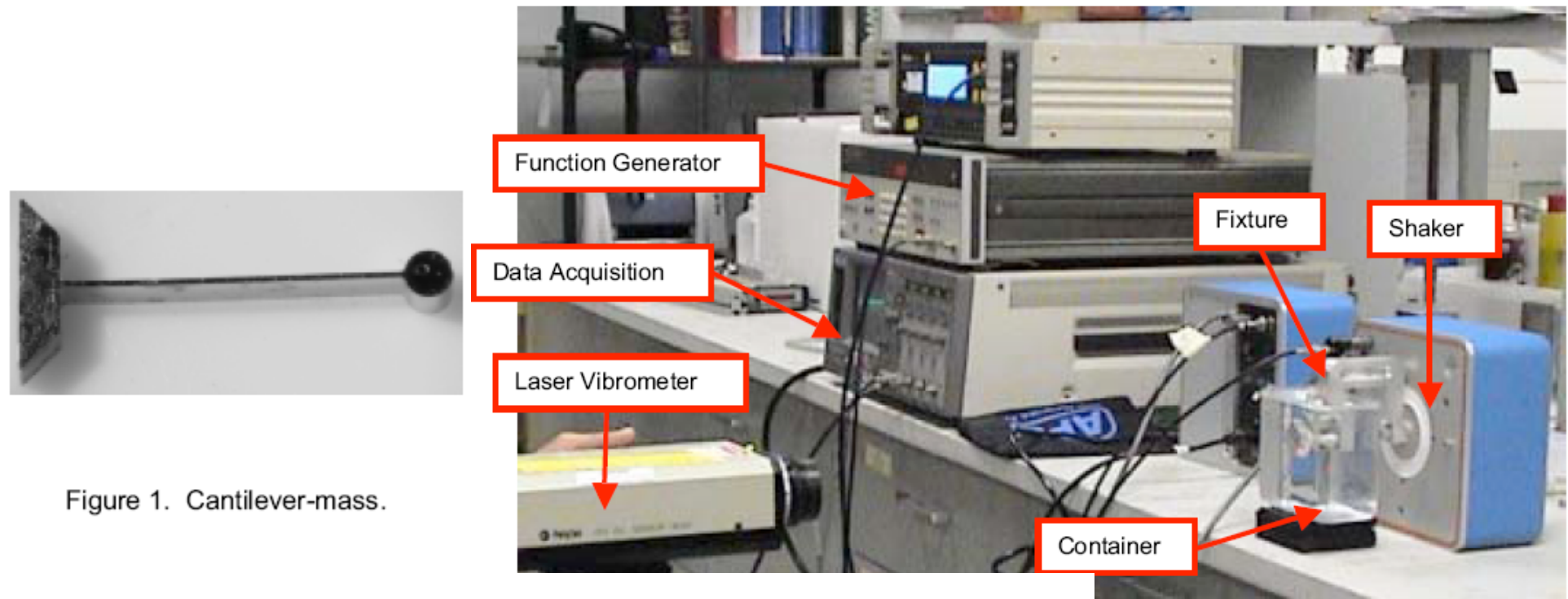
$$\omega_b = \sqrt{\frac{k}{m} - \frac{b^2}{2}}$$

- Quality factor

$$Q = m\omega_0 / b = \sqrt{km} / b$$



Frequency response (Sandia measurements)



ntal Setup

Q factor is a measure of the "quality" of a resonant system. Resonant systems respond to frequencies close to the natural frequency much more strongly than they respond to other frequencies. On a graph of response versus frequency, the bandwidth is defined as the 3 dB change in level besides the center frequency.

The Q factor is defined as the resonant frequency (center frequency) f_0 divided by the bandwidth BW:

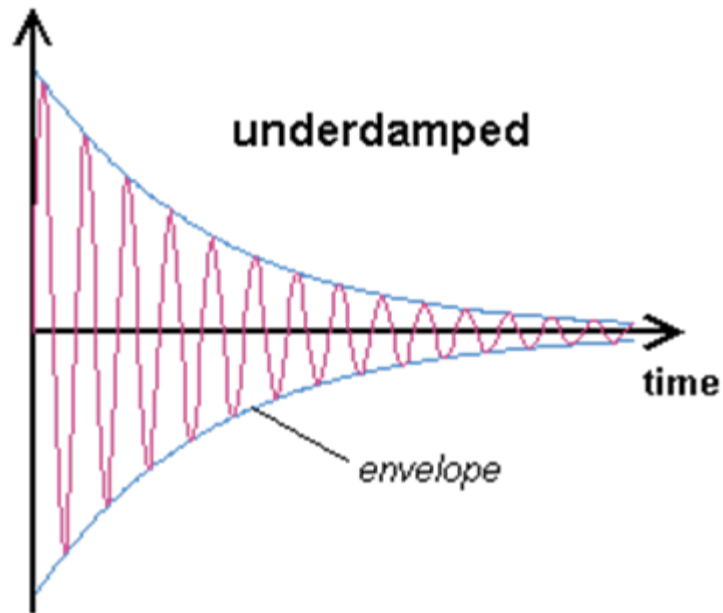
$$Q = \frac{f_0}{f_2 - f_1} = \frac{f_0}{BW}$$

Bandwidth $BW = f_2 - f_1$, where f_2 is the upper and f_1 the lower cutoff frequency.

In a tuned radio frequency receiver (TRF) the Q factor is:

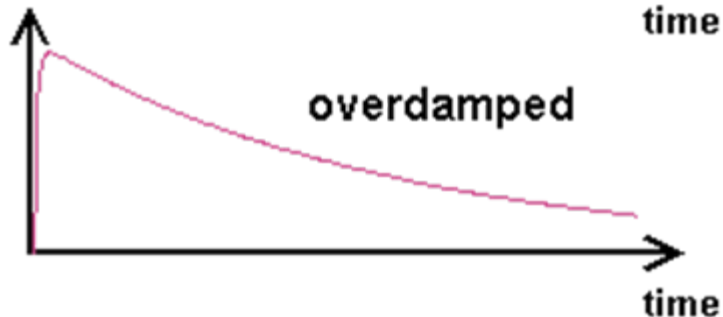
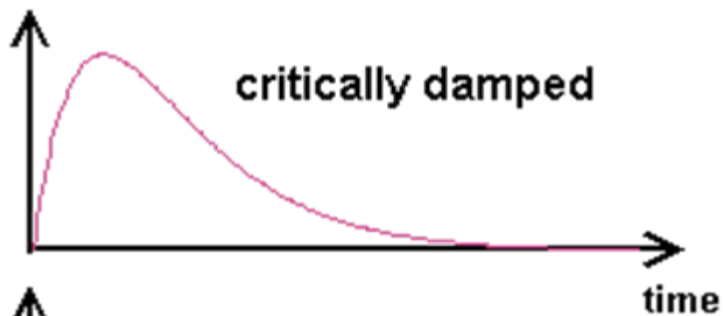
$$Q = \frac{1}{R} \sqrt{\frac{L}{C}}$$

Damping



Another equivalent definition of quality factor:

The number of oscillations before the amplitude reaches $1/e$ times the original amplitude



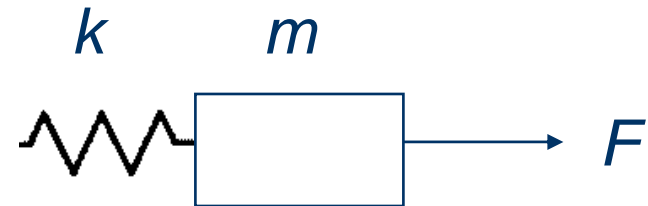
Quasi-static response

- Resonant frequency much larger than the expected maximum frequency component of the acceleration signal
- Quasi static response:

$$x = \frac{F + F_n}{k}$$

$$x = a / \omega_0^2$$

- Scale factor depends only on resonant frequency
- Quick response - small position sensitivity



Capacitor

Capacitance is typified by a parallel plate arrangement and is defined in terms of charge storage:

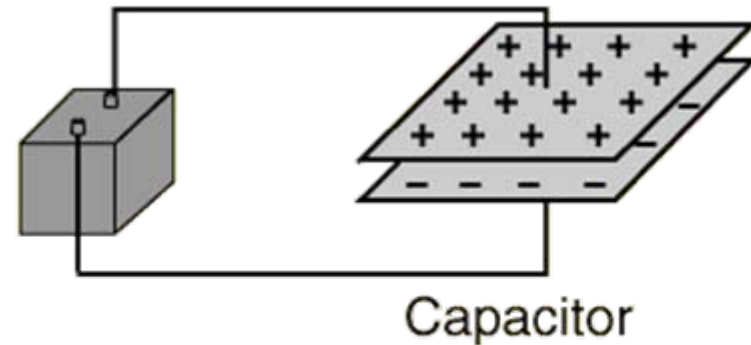
where

$$C = \frac{Q}{V}$$

Unit = $\frac{\text{coulomb}}{\text{volt}} = \text{Farad}$

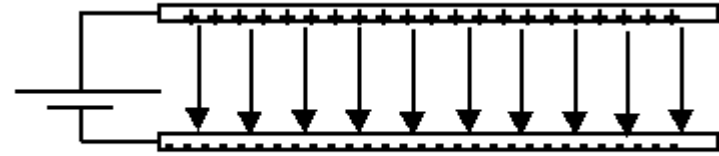
Q = magnitude of charge stored on each plate.

V = voltage applied to the plates.



A battery will transport charge from one plate to the other until the voltage produced by the charge buildup is equal to the battery voltage.

Parallel plate capacitor



- A parallel plate capacitor's effective capacitance is defined in terms of its geometry.
- $C = \epsilon A / d$
- ϵ , the permittivity of free space, is a constant equal to 8.85×10^{-12} F/m,
 A is the cross sectional area of ONE plate, and
 d is the distance between the plates.
- Dimensionally, capacitance is expressed in terms of a unit called a **farad**.

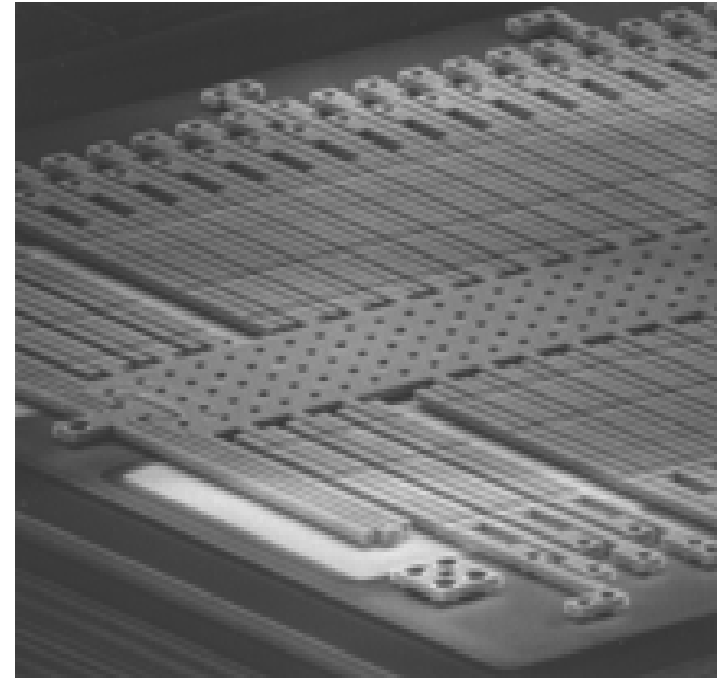
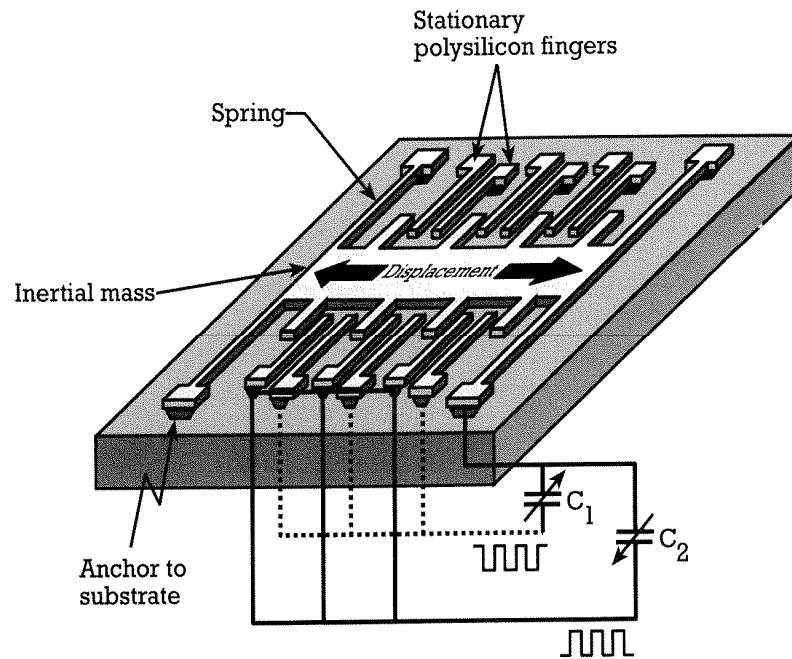
If the capacitors arranged in parallel then

$$Q_{\text{total}} = Q_1 + Q_2 + Q_3$$

$$C_{\text{total}} = C_1 + C_2 + C_3$$

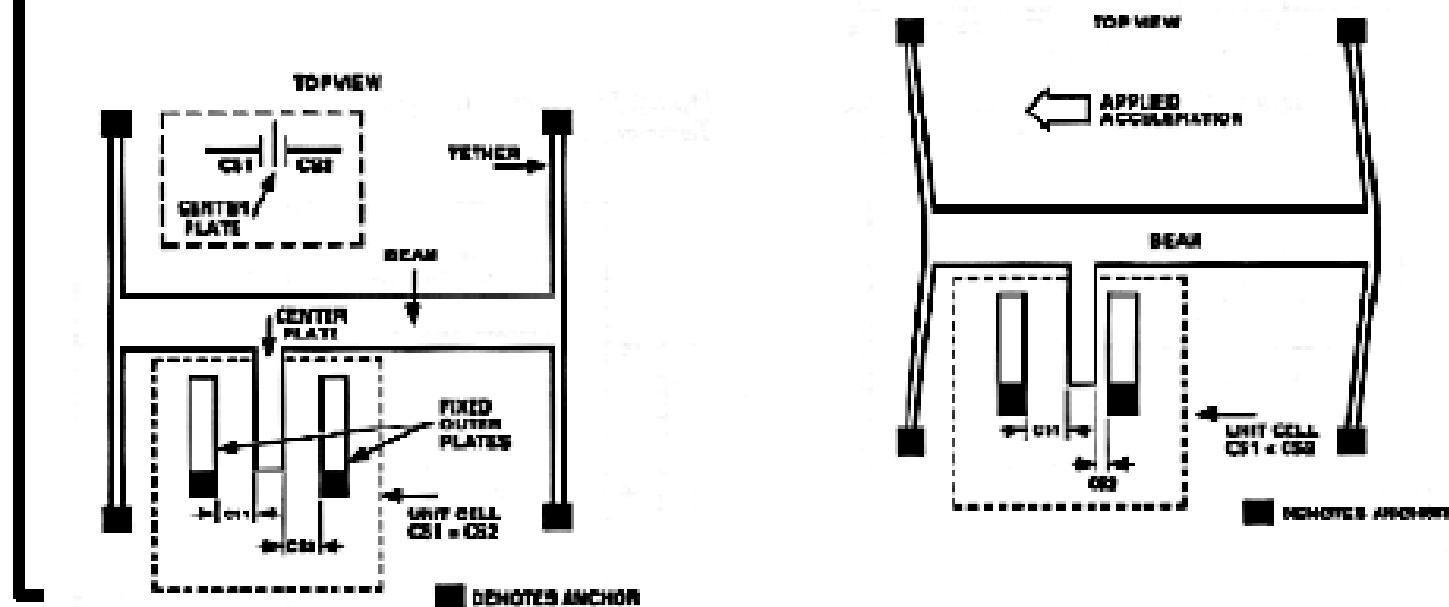
$$V_{\text{total}} = V_1 = V_2 = V_3$$

A Capacitive Accelerometer



ADXL50 Sensing Mechanism

- Balanced differential capacitor output
- Under acceleration, capacitor plates move changing capacitance and hence output voltage
- On-chip feedback circuit drives on-chip force-feedback to re-center capacitor plates.



ictp, 7/01

ADXL322 - Small, Thin, Low Power, Dual Axis $\pm 2g$ Accelerometer with Analog Output

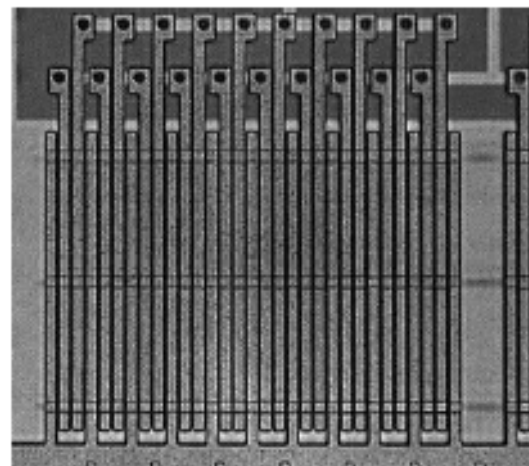
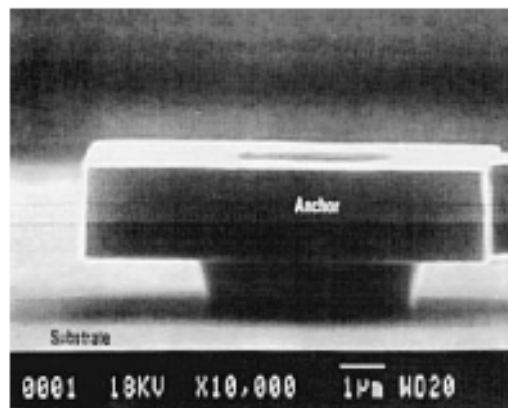
Product Description

The ADXL322 is an ultra small package (4 x 4 x 1.45 mm LFCSP) and low power (340 μA at $V_s = 2.4\text{V}$) $\pm 2g$ iMEMS[®] Accelerometer designed to accommodate the integration requirements of mobile phones and other portable devices for a variety of motion, tilt, and inertial sensing features (e.g., data entry, menu and display control, power management, situational awareness, navigation, and portrait vs. landscape display orientation). The ADXL322 also enables hard disk drive protection systems and security features in notebook computers, as well as position and tilt sensing for PC and gaming peripherals such as mouse and joystick devices.

Price: 3.75\$



Analog Devices Polysilicon MEMS



kspp-701

FEM solution to parallel plate capacitor

- Fringe fields effect
- Capacitance from parallel plate approximation: 53.1 pF

$$C = \frac{\epsilon A}{g}$$

- Capacitance calculated from FEM: 71.7 pF

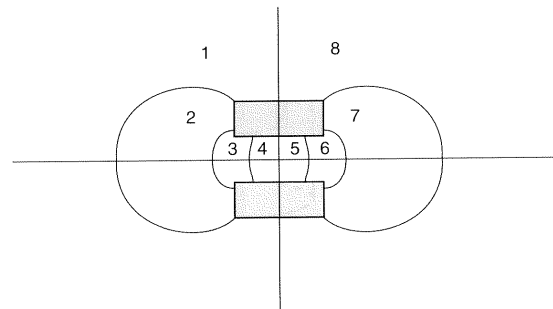
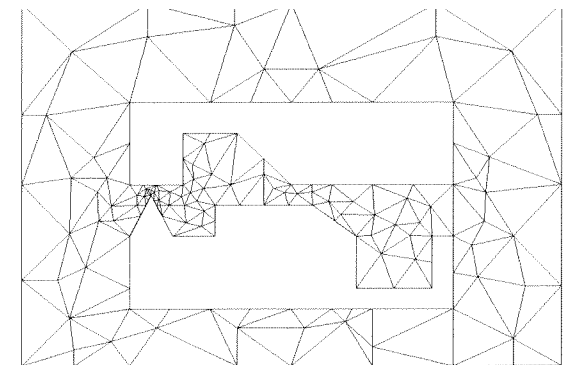
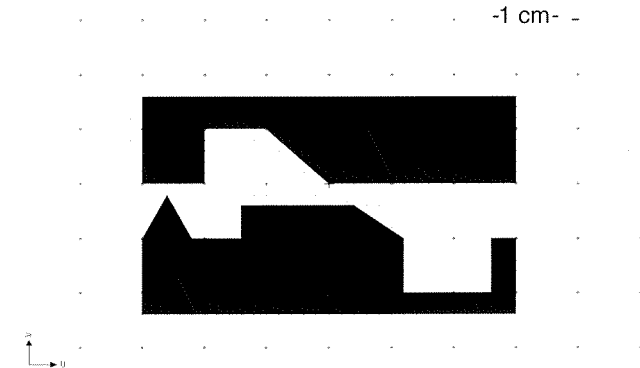


Figure 2.19 Two-dimensional field sketch



Position measurement with capacitance



Figure 19.3. Illustrating a variety of capacitor structures that can be used for position sensing.

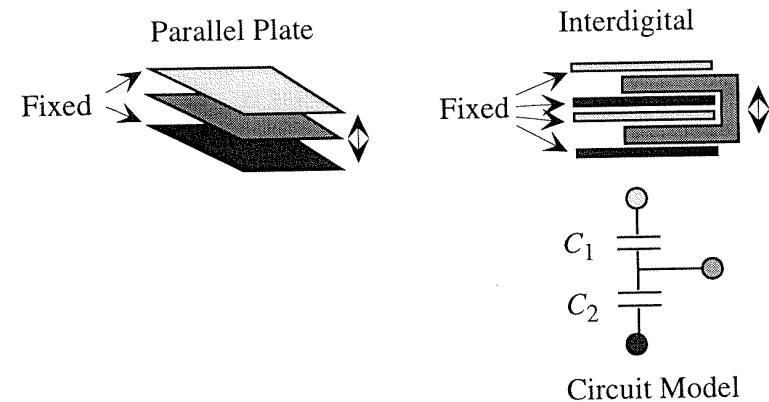


Figure 19.4. Examples of differential capacitors.

- Changes in capacitance: $<10^{-18}$ F
- Sensitivity; $\partial C/\partial x$

- Output: voltage
- Sensitivity: $\partial V/\partial x$

Capacitive comb-shaped accelerometer

- mass
- elastic spring
- capacitor (differential) for measurement of displacement

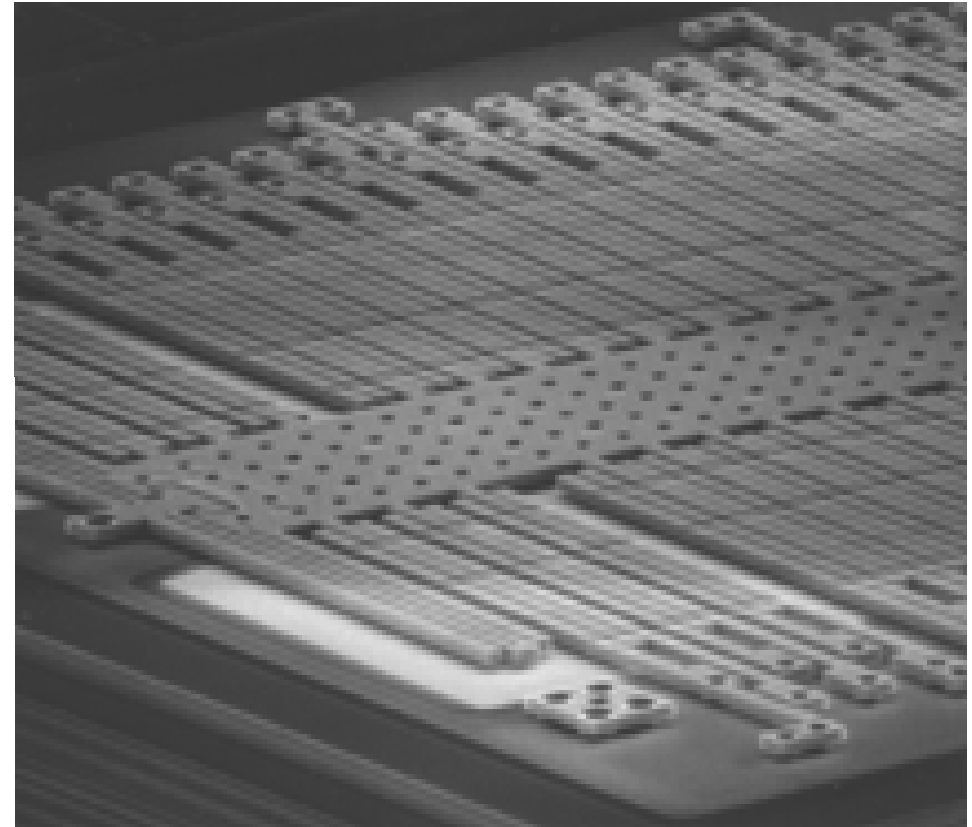
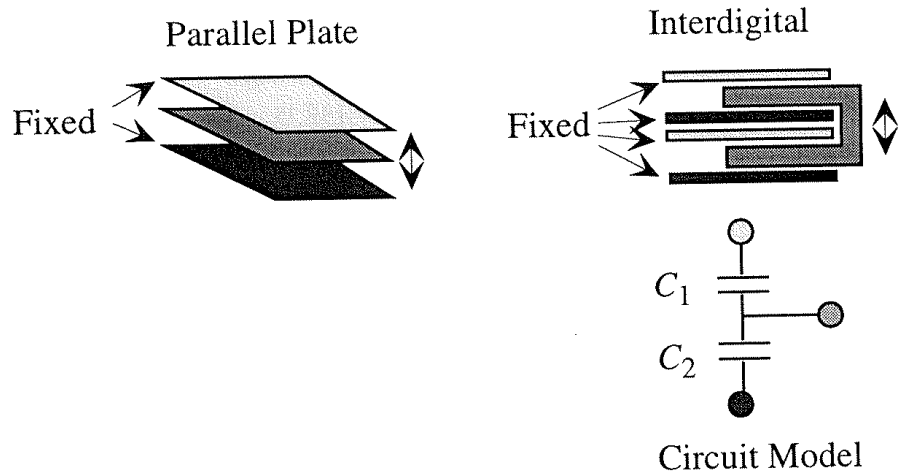


Figure 19.4. Examples of differential capacitors.

Differential capacitor

- Cancels many effects to first order
- Linearization about balance point
- Voltage that appears at output
- $G=d = \text{gap between capacitor plates}$

$$V_0 = -V_s + \frac{C_1}{C_1 + C_2} (2V_s) = \frac{C_1 - C_2}{C_1 + C_2} V_s$$

$$V_0 = \frac{G_2 - G_1}{G_1 + G_2} V_s$$

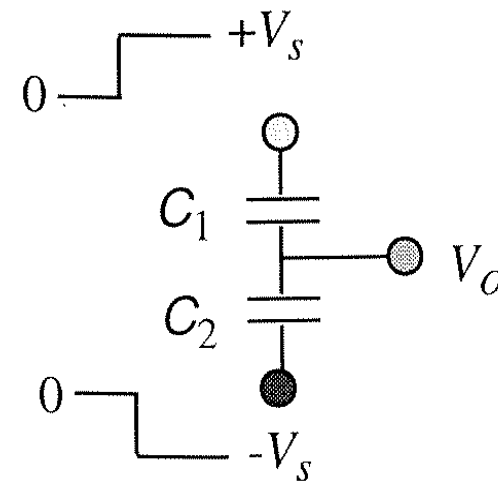


Figure 19.5. A typical circuit use of a differential capacitor

Circuits for capacitance measurement

- More complicated than for piezoresistive sensing
- Transfer from capacitance change to output voltage
- Open-loop – linear, large deflections ?
- Closed-loop

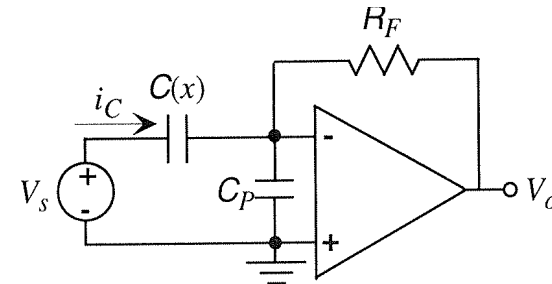


Figure 19.6. Use of a transimpedance amplifier to capture the capacitor current.

$$Q = C(x)V$$

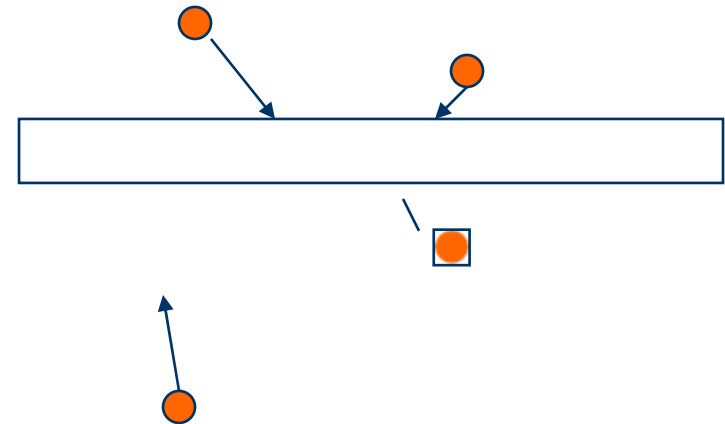
$$i_C = C(x) \frac{dV}{dt} + V \frac{\partial C}{\partial x} \frac{dx}{dt}$$

Brownian motion noise

- Noise related to the fluid damping
- Similar to Johnson noise in resistor
($4 \Pi k_B T R \Delta f$)
- Convert to an equivalent acceleration
- The root mean square force noise is

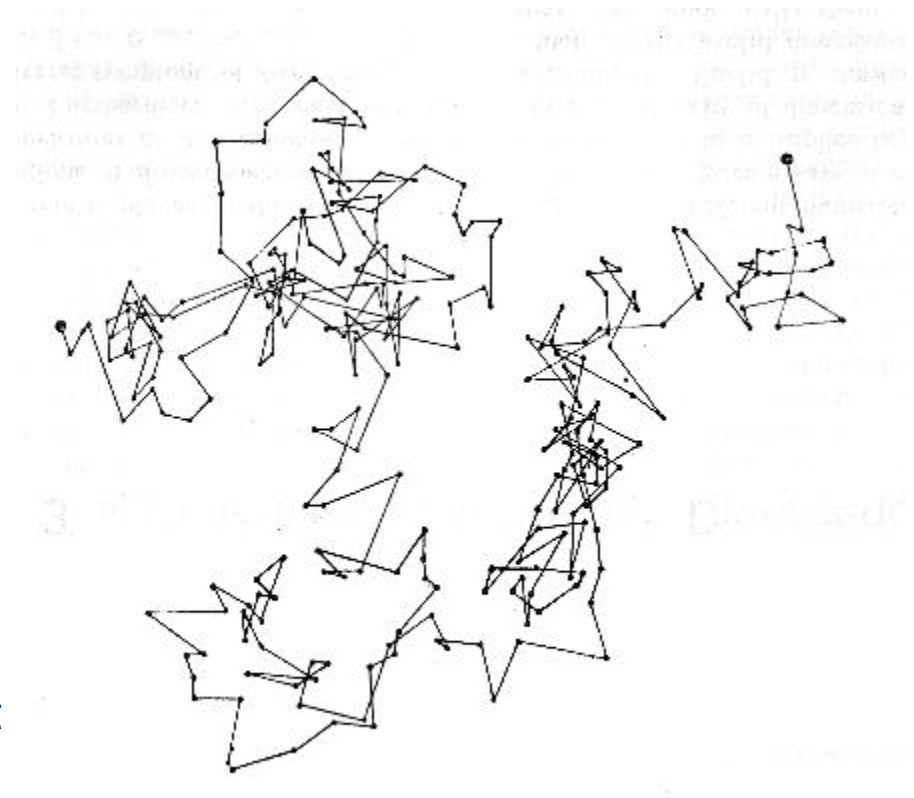
$$F = \sqrt{\frac{4k_b T \Delta f}{\omega_0 Q}}$$

$$a = \sqrt{\frac{4k_b T \omega_0 \Delta f}{mQ}}$$



Fluctuation dissipation theorem

Almost two centuries ago, the atomic nature of matter was elegantly revealed by brownian motion -- as exemplified by the random motion of pollen particles in water as they are bombarded by water molecules. In 1905, Albert Einstein (1879-1955)(2) pointed out a subtle consequence of the fluctuations in classical brownian motion: the same random forces that make a pollen particle jitter would also cause friction if the particle were dragged through the water. In other words, the fluctuation of the particle at rest has the same origin as the dissipation of the motion of a moving particle that is subject to an external force. Einstein's result is a general one, codified in the "fluctuation-dissipation theorem", which is one of the deepest results of thermodynamics and statistical physics.

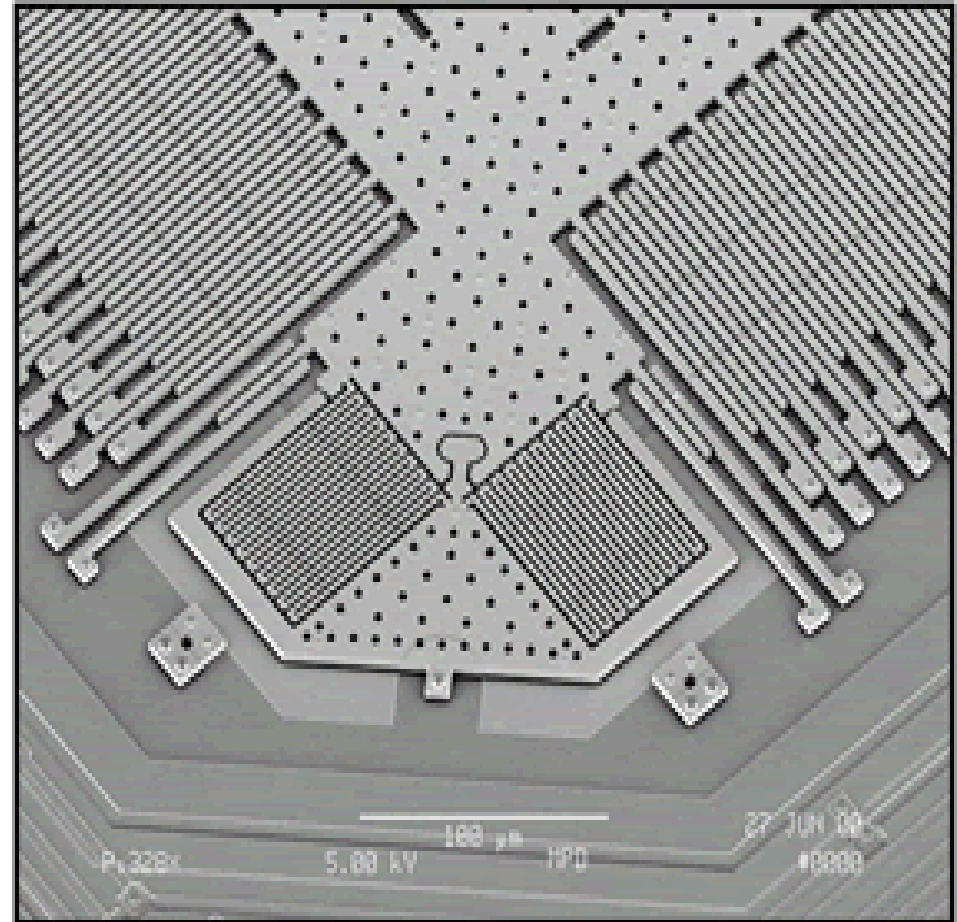


ADXL202

- Dual axis accelerometer

The bandwidth of the ADXL202 may be set from 0.01 Hz to 6 kHz via capacitors C_X and C_Y .

The typical noise floor is $200\mu\text{g}/\sqrt{\text{Hz}}$ allowing signals below 2 mg to be resolved for bandwidths below 60Hz.



Fabrication, Analog Devices

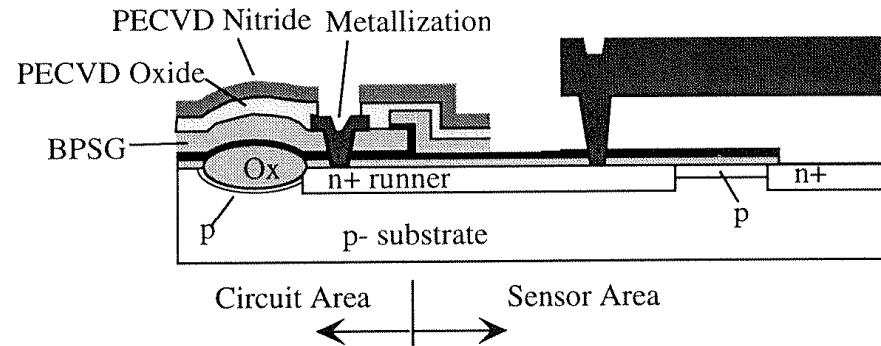


Figure 19.26. Schematic of the final released structure in the iMEMS process.

Sensor design ADXL150

- Shuttle, electrodes and folded spring, 2 μm thick
- Nominal gap between fingers 1.3 μm
- 42 fingers
- Length of overlap region 104 μm
- Capacitances:

$$C_{sense} = 42 \frac{\epsilon_0 H L_0}{G_0 \pm y}$$

$$C_{sense} \approx 60 \text{ fF} \left[1 \pm \frac{y}{G_0} \right]$$

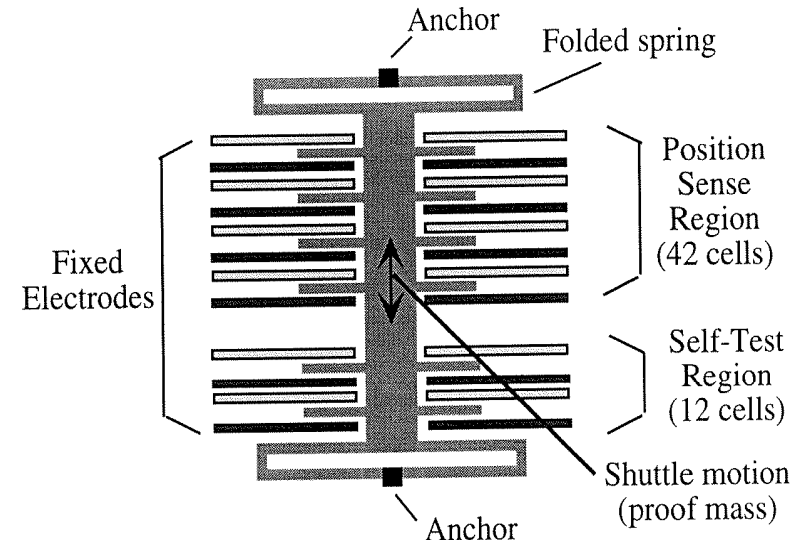


Figure 19.17. Schematic illustration of the design of the sensor portion of the ADXL150 accelerometer. Similarly shaded fixed electrodes in the sense region are connected together forming a lateral differential capacitor with a moveable proof-mass shuttle. Similarly shaded fixed electrodes in the self-test region are also connected together, forming a second differential capacitor that is used to apply electrostatic forces to displace the shuttle.

Doubly clamped beam with point load at midpoint

- Spring constant:

$$k = F / c = \left(\frac{\pi^4}{6} \right) \left[\frac{EWH^3}{(2L_1)^3 + (2L_2)^3} \right]$$

- c-central beam displacement
- W beam width (poly thickness)
- H beam thickness (lithography)
- Stiffness of folded spring: 2.8 N/m
- Stiffness of two springs: 5.6 N/m
- Spring softening due to applied voltage gives: 5.2 N/m

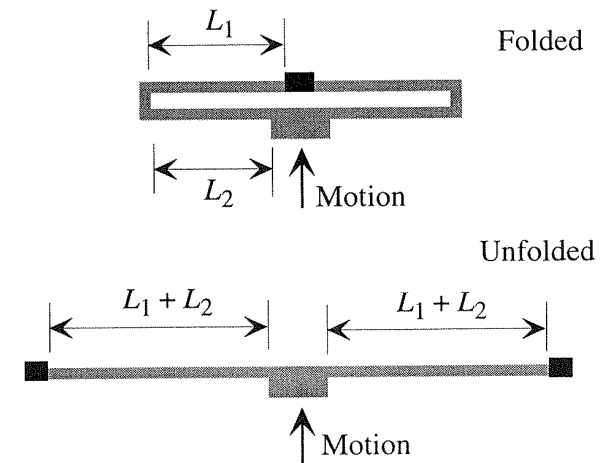


Figure 19.22. Illustration of a folded spring, and its unfolded equivalent.

Accelerometer specifications ADXL150

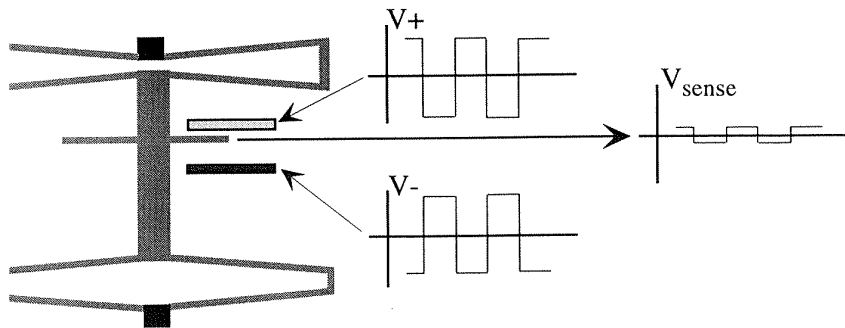


Figure 19.21. Highly exaggerated sketch of a displaced shuttle, due either to an external acceleration or to voltages applied to the self-test electrodes. The drive waveforms and resulting sense waveform are also shown.

$$V_{out} = V_s / 2 \pm \alpha + \beta a V_s$$

Table 19.1. Selected Specifications of ADXL150 Capacitive Accelerometer. (Source: Analog Devices Data Sheet.)

Property	Specification
Sensitivity	38mV/g
Full-scale range	$\pm 50 g$
Transfer function form	see text
Package type	14-pin cerpak
Temperature range	-40 to +85°C
Supply voltage	4 - 6 V
Nonlinearity	0.2 %
Package alignment error	$\pm 1^\circ$
Transverse sensitivity	$\pm 2\%$
Zero-g output voltage (Bias)	$V_s/2 \pm 0.35 V$
Temperature drift (from 25°C to T_{min} or T_{max})	0.2 g
Noise from 10 Hz to nominal bandwidth	1 mg/ $\sqrt{\text{Hz}}$
Clock noise	5 mV peak-to-peak
Bandwidth	400 or 1000 Hz, customer choice
Temperature drift of bandwidth	50 Hz
Sensor resonant frequency	24 kHz
Self test output change	400 mV
Absolute maximum acceleration	2000 g (unpowered) 500 g (powered)
Drop test	1.2 meters
Min/max storage temperature	-65 to 150 °C
Max lead temperature (10 seconds)	245 °C

Accelerometer specifications

- Mass of shuttle: $m=2.2 \times 10^{-10}$ kg
- Stiffness: $k=5.4$ N/m
- Undamped resonant frequency:
 $\omega=1.55 \times 10^5$ rad/sec = 24.7 kHz

- Operational bandwidth: 1000Hz

- Q-factor: 5
- Couette flow
- $b=\eta A/h$
- Q-factor from Couette: 120
- Squeezed film damping
- Q-factor from Couette: 34

General electrostatic forces

- Electric forces between charges
- Electric potential Φ
- Laplace equation + boundary conditions (dirichlet)

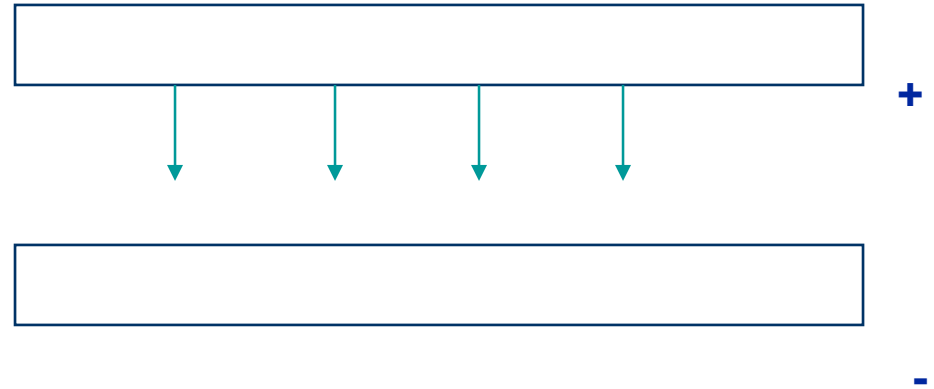
$$\nabla^2 \Phi = 0$$

- Electrostatic field: gradient of potential

$$\vec{\mathcal{E}} = -\nabla \Phi$$

- Electric force normal to conductor surface
- Charge distribution on surface conductors related to field

$$q(r) = \left| \vec{\mathcal{E}}(r) \right| \epsilon$$



- Force proportional to electric field

$$\vec{F} = q\vec{E}$$

- Forces between parallel plates in capacitor:

$$F = \frac{-\epsilon A V^2}{2g^2}$$

Linear elastic force

- Partial differential equation for force-elastic displacement (beam) + boundary conditions:

$$EI \frac{\partial^4 u}{\partial x^4} = F$$

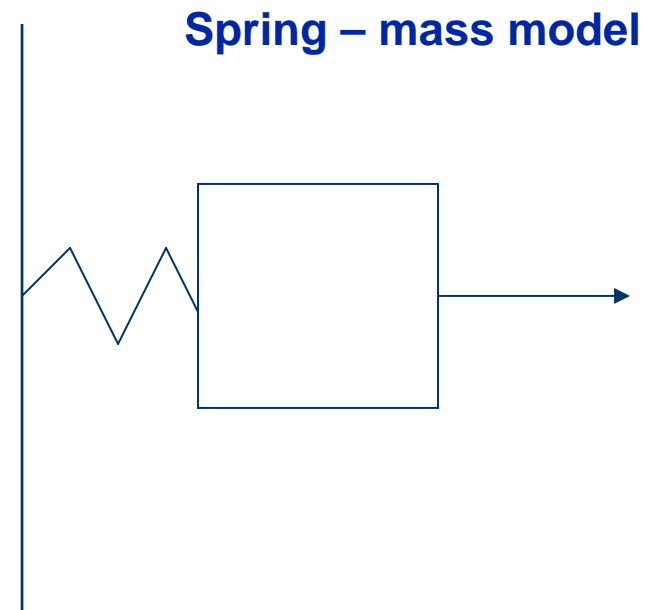
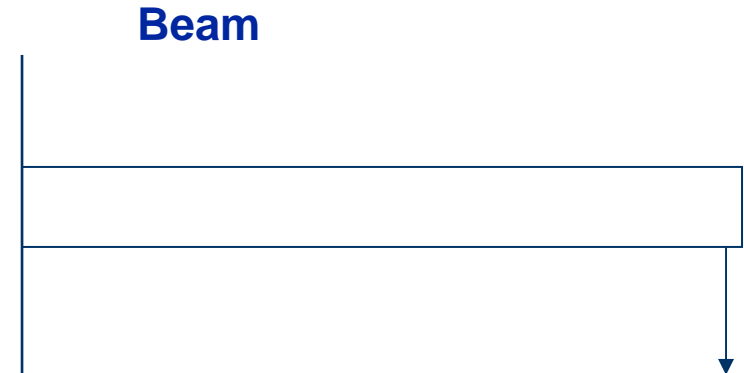
- Approximation e.g.

$$F = ku_{\max}$$

- Linear relation stress-strain (Always true for single-crystal silicon):

$$\sigma = E\varepsilon$$

- Strain is derivative of displacement. May give non-linear relation displacement-strain for large deflections (geometric effect)



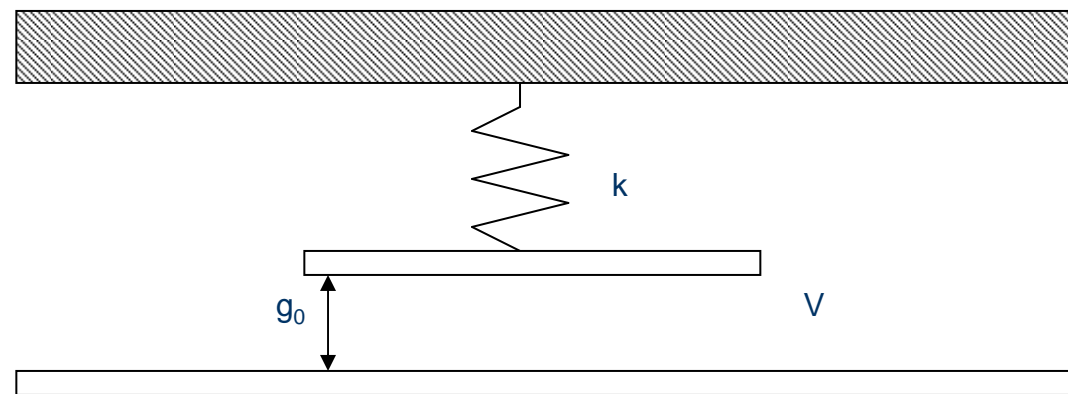
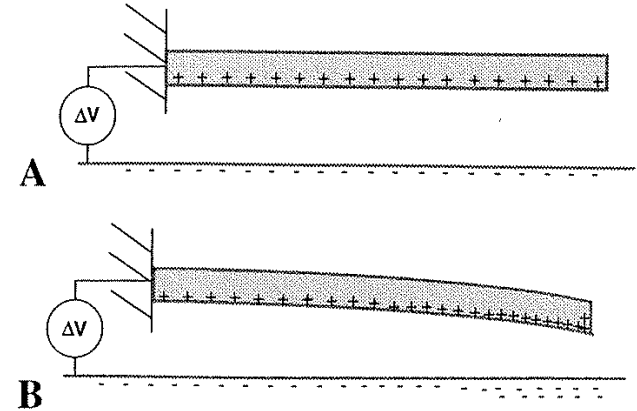
Electrostatic bending of beam, simplest model: plate suspended in spring

- Set up voltage ΔV between beam and substrate
- Beam bend due to electrostatic forces
- Elastic forces tend to pull beam back

- Total force:

$$F_{net} = \frac{-\epsilon A V^2}{2g^2} + k(g_0 - g)$$

- Equilibrium: $F_{net} = 0$



Pull-in of parallel plates with linear spring

- Increase ΔV , reach pull-in distance and voltage

$$g_{PI} = 2/3 g_0$$

$$V_{PI} = \sqrt{\frac{8kg_0^2}{27\varepsilon A}}$$

- If voltage is larger than pull-in voltage
=> no stable solution except $g=0$

- $\zeta = 1 - g/g_0$

- Senturia section 6.4.3

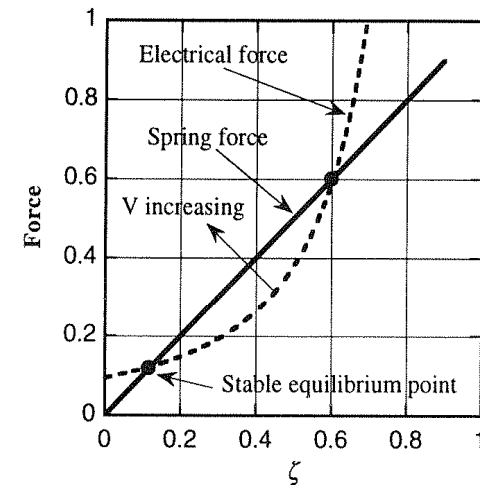


Figure 6.7. Electrical and spring forces for the voltage-controlled parallel-plate electrostatic actuator, plotted for $V/V_{PI} = 0.8$.

Pull-in of parallel capacitor plates with linear spring

- Stable and unstable equilibrium:

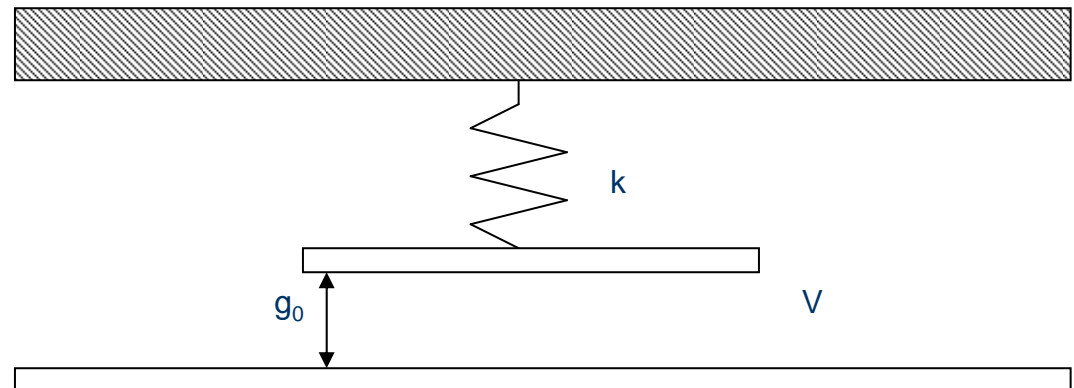
$$\frac{\partial F_{net}}{\partial g} = \frac{\epsilon A V^2}{g^3} - k$$

- STABLE**

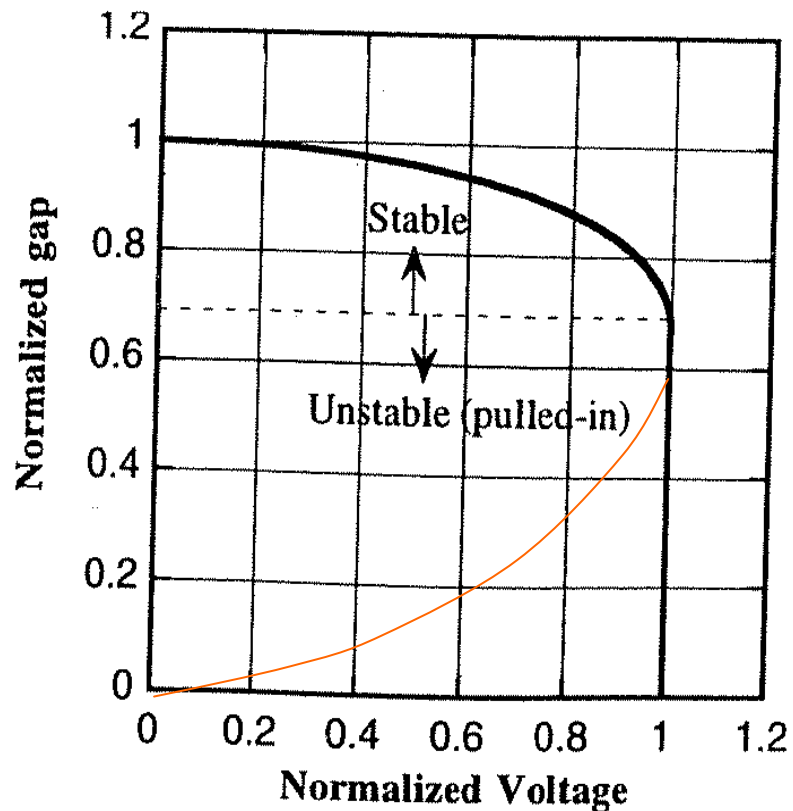
- $\partial F/\partial g$ negative: gap perturbed to be larger, negative forces pull back to equilibrium position
- $\partial F/\partial g$ negative: gap smaller, positive forces pull back to equilibrium position

- UNSTABLE**

- $\partial F/\partial g$ positive: gap perturbed to be larger, positive forces increase gap, pulled to other stable equilibrium
- $\partial F/\partial g$ positive: gap smaller, negative forces decrease gap, pulled to other stable equilibrium



Gap vs. voltage



- Parallel plates, linear spring elastic force
- Normalized gap g/g_0
- Normalized voltage V/V_{PI}

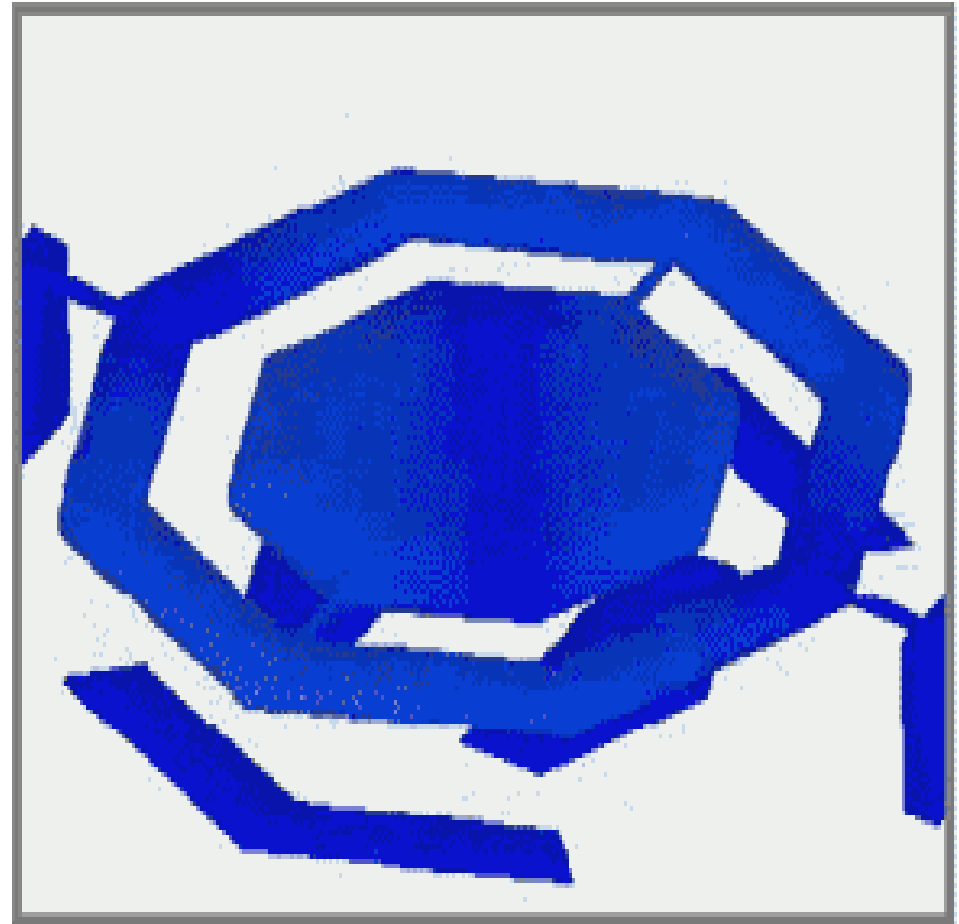
Mechanical and electrostatic equations

- Naviers equation for elastic forces:
(isotropic version)

$$(\lambda + \mu)\nabla\nabla \cdot u + \mu\nabla^2 u = 0$$

- Poisson equation for electrostatic
field:

$$\nabla^2 \Phi = -\frac{\rho}{\varepsilon}$$



Numerical electrostatic-elastic solvers

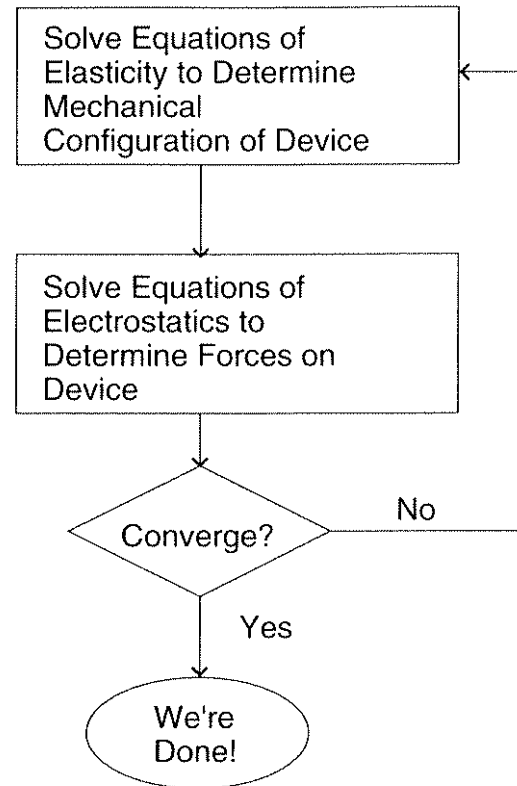


FIGURE 7.10: Flowchart of the basic iterative method for numerical electrostatic-elastic solvers.

Coventor – pull in and release voltage

Figure T3-9 Hysteresis in a Fixed-Fixed Beam

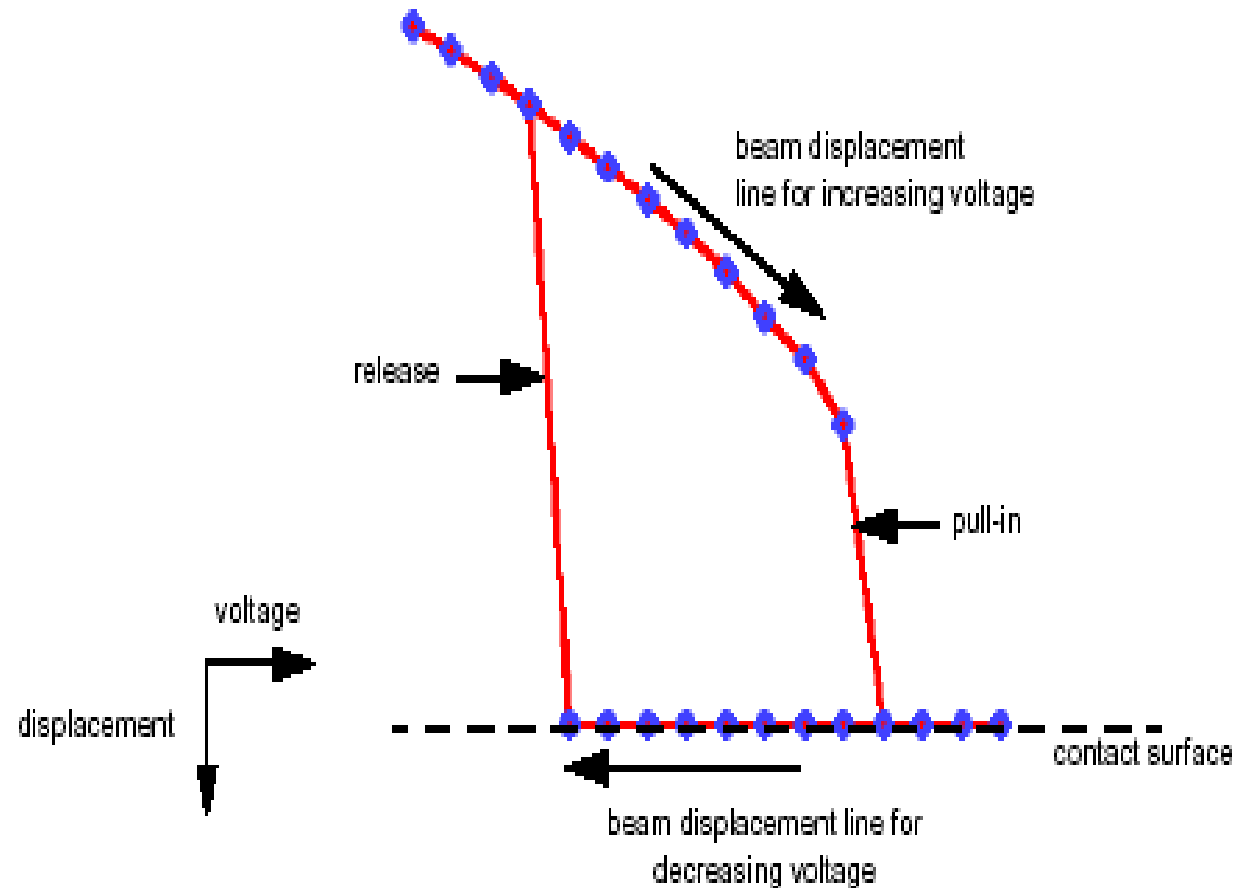


Figure T3-10 Displacement at Pull-in and Release Voltages

