UNIVERSITY OF OSLO

Faculty of Mathematics and Natural Sciences

Exam in: **FYS4630 and FYS9630**
Transport of radiation in the atmosphere
Day of exam: **December 1, 2016**
Exam hours: **09:00 – 13:00**
This examination paper consists of 4 pages
Appendices: None
Permitted materials: Calculator,
Rottmann: Matematisk formelsamling (all editions)

Make sure that your copy of this examination paper is complete before answering.

Problem 1

a) Define the spectral net flux, F_ν, and the spectral hemispherical fluxes, F_ν^+ and F_ν^-.

b) Define the spectral intensity, I_ν.

Derive the relationship between F_ν and I_ν.

Problem 2

a) Define the spectral directional emittance, $\varepsilon(\nu, \hat{\Omega}, T_s)$ for a surface.

Show that the spectral flux emittance can be written as

$$\varepsilon(\nu, 2\pi, T_s) = \frac{1}{\pi}\int_{+} d\omega \cos\theta \; \varepsilon(\nu, \hat{\Omega}, T_s)$$

b) What is a grey body?

A circular disk of radius R is a grey body. Show that the outward flux, at a point lying on the axis of the disk a distance *z* from the center of the disk is given by

$$F_\nu(z) = \frac{\pi \varepsilon B_\nu R^2}{z^2 + R^2}$$

where B_ν is the Planck function at frequency ν.

Problem 3

a) The azimuthally averaged radiative transfer equation is

$$u \frac{dI(\tau,u)}{d\tau} = I(\tau,u) - \frac{a}{2} \int_{-1}^{1} du' p(u',u)\, I(\tau,u')$$

The crudest way to handle the problem with strong forward scattering is to approximate the phase function as follows:

$$\hat{p}\,(u',u) = C \cdot f \cdot\ \delta(u'-u) + (1-f)$$

Where δ is a Dirac δ-function and f is the strength of the forward-scattering peak, $0 < f < 1$.

Show that the constant C = 2.

b) Show that the δ-N –scaled radiative transfer equation can be written as:

$$u\ \frac{dI(\hat{\tau},u)}{d\hat{\tau}} = I(\hat{\tau},u) - \frac{\hat{a}}{2} \int_{-1}^{1} du'\ I(\hat{\tau},u')$$

c) Compare $\hat{a}$ and a, and $d\hat{\tau}$ and $d\tau$.

What is the advantage of using δ-N-scaling from a computational point of view?

d) Give a physical interpretation of the δ-N scaled radiative transfer equation.

Problem 4

a) The azimuthally averaged radiative transfer equation with a thermal source function is:

$$u\,\frac{dI(\tau,u)}{d\tau} = I(\tau,u) - \frac{a}{2}\int_{-1}^{1} d\mu\, p(\tau,u)\, I(\tau,u') \; - \; (1-a)B(\tau)$$

How is a defined? Give a physical interpretation of a. What are possible values of a?

Write down the corresponding radiative transfer equations for the hemispherical intensities $I^{+}(\tau,\mu)$ and $I^{-}(\tau,\mu)$.

b) Assume an isothermal plane-parallel medium (slab). The vertical optical depth τ is as usual measured from the "top" of the medium (τ = 0) to the "bottom" (τ = τ*). Assume no scattering and that $I^{-}(\tau = 0,\mu) = 0$. Solve the radiative transfer equation and find the intensity at vertical optical depth τ: $I^{-}(\tau,\mu)$.

Problem 5

a) Define the bidirectional reflectance distribution function (BRDF): $\rho(\nu, -\widehat{\Omega}', \widehat{\Omega})$.

b) The incident intensity on a surface with a BRDF = $\rho(\nu, -\widehat{\Omega}', \widehat{\Omega})$ is $I_{\nu}^{-}(\widehat{\Omega}')$.

Find an expression for the reflected intensity $I_{\nu r}^{+}(\widehat{\Omega})$.

c) Assume that the incident intensity is uniform: $I_{\nu}^{-}(\widehat{\Omega}') = \text{constant} = I$. The surface is now Lambertian so that $\rho(\nu, -\widehat{\Omega}', \widehat{\Omega})$ = $\rho_L(\nu)$.

Show that the reflected flux is : $F_{\nu r}^{+} = \pi^2\,\rho_L(\nu)\, I$.

Problem 6

a) The two-stream equations for anisotropic scattering can be written as

$$\bar{\mu}\,\frac{dI^{+}}{d\tau} = I^{+} - a(1-b)I^{+} - abI^{-}$$

$$-\bar{\mu}\,\frac{dI^{-}}{d\tau} = I^{-} - a(1-b)I^{-} - abI^{+}$$

b is the backscattering coefficient:

$$b = \frac{1}{2}\sum_{l=0}^{\infty}(-1)^l \ (2l+1)\chi_l \left[\int_0^1 d\mu \ P_l(\bar{\mu})\right]^2$$

Assume a cloud to be a plane-parallel slab that scatters radiation conservatively (a = 1). Let $\bar{\mu} = \frac{1}{\sqrt{3}}$ and keep only the two first terms in the expansion series for b.

Find b.

Show that the two-stream equations can be written as:

$$\bar{\mu} \ \frac{d(I^+ \ - \ I^-)}{d\tau} = 0$$

$$\bar{\mu} \ \frac{d(I^+ \ + \ I^-)}{d\tau} = (1-g) \ (I^+ - \ I^-)$$

b) Solve the equations in a) and find $I^-(\tau^*)$. τ^* is the vertical optical depth at the bottom of the cloud (the ground). Use the boundary condition $I^-(\tau = 0) = \text{constant} = I$ and $I^+(\tau^*) = 0$.

c) The intensity of direct radiation at τ^* is $I_{dir}(\tau^*) = I \ e^{-\tau^*/\mu_0}$.

What is the downward *diffuse* intensity $I^-_{diff}(\tau^*)$?

When the cloud optical thickness (τ^*) increases from zero the intensity will increase, reach a maximum, and then decrease as the cloud becomes optically thick. Use the solution for $I^-_{diff}(\tau^*)$ to argue that such a maximum must exist. (Don't find the value of this maximum!)