Ocean Circulation

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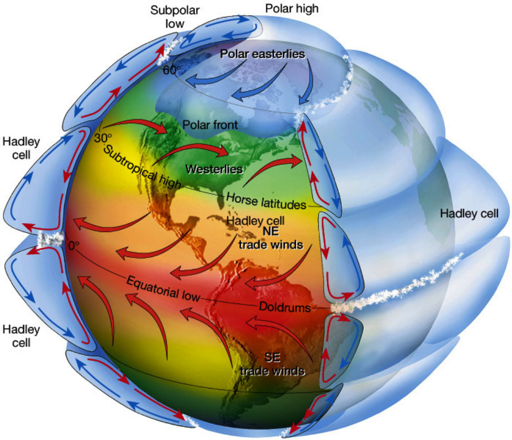
Section for Meteorology and Oceanography

November 18, 2016

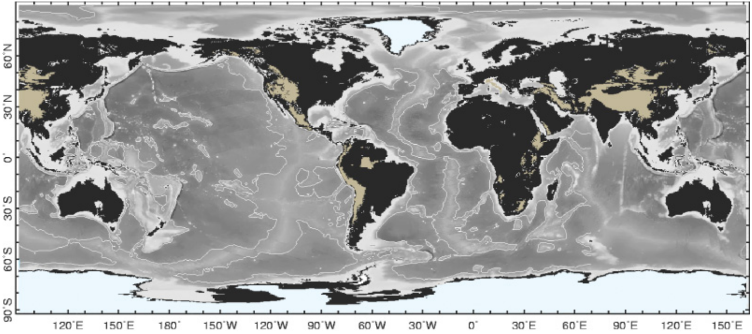
Outline

- Physical characteristics
- Observed circulation
- Geostrophic and hydrostatic balance
- Wind-driven circulation
- Buoyancy-driven circulation

Atmospheric geometry



Oceanic geometry



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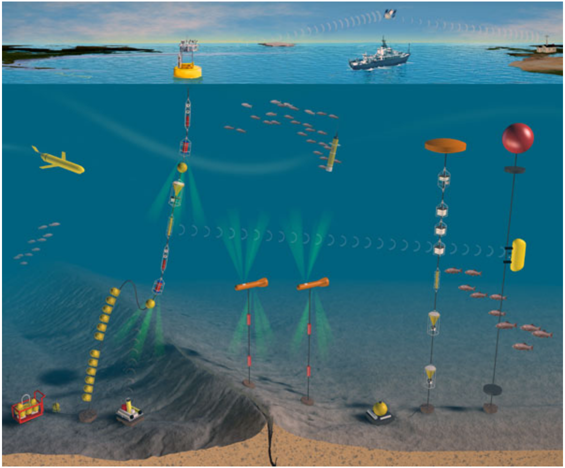
Ocean facts

- Covers 71 % of the earth's surface
- Average depth is 3.7 km
- Volume is 3.2 x 10^{17} m^3 = 1.3 x 10^{21} kg
- Heat capacity is 1000 time greater than atmosphere's
- Substantial fraction in ice sheets (Greenland, Antarctica)

Ocean observations



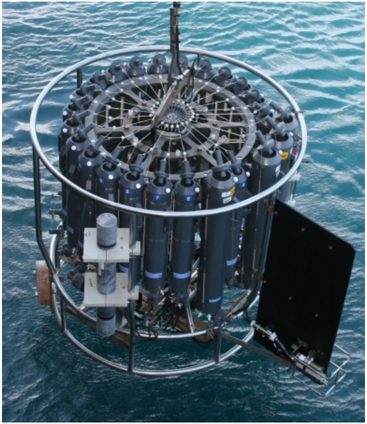
Woods Hole Oceanographic Inst.

Ships

University of Washington, Scripps Inst. of Oceanography

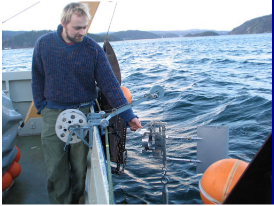
CTD

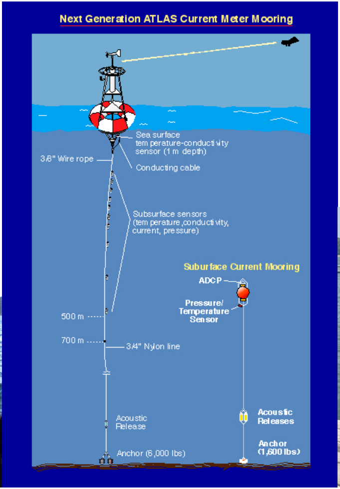
Conductivity, temperature and depth sensor



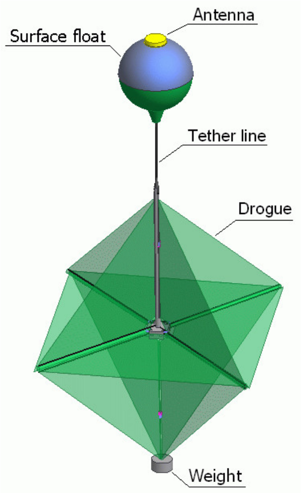
National Oceanographic Center, Southampton

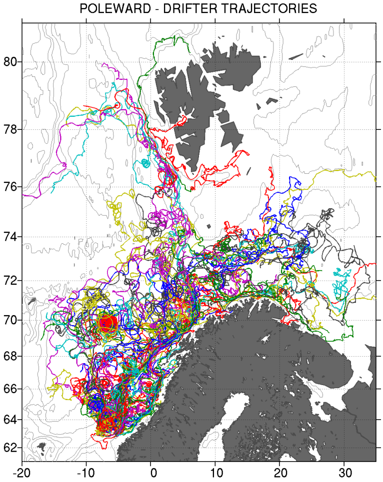
Current meters



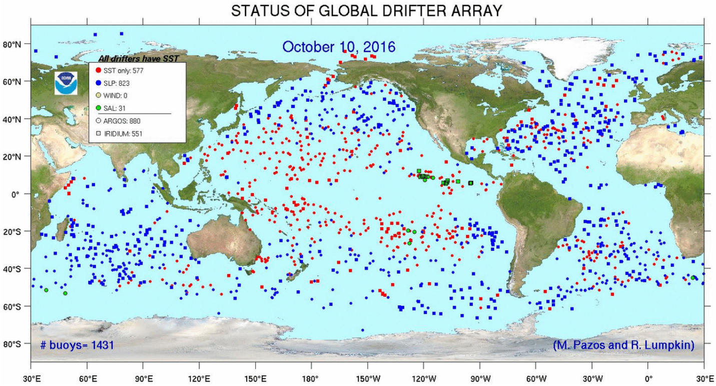


The surface drifter



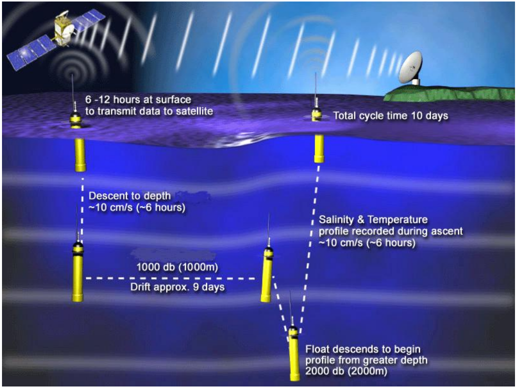


Surface drifter locations: Oct. 10, 2016



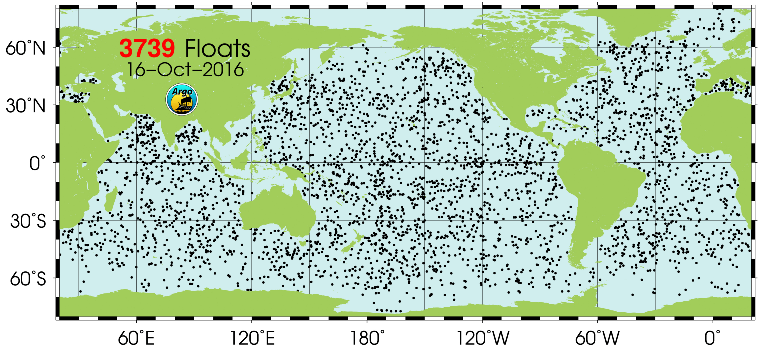
NOAA/AOML

ARGO Floats



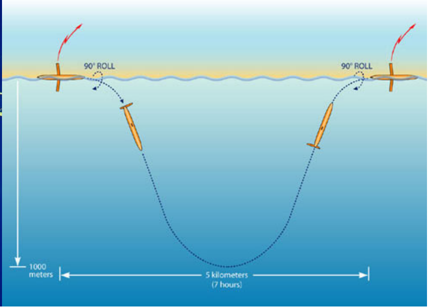
Scripps Inst. Oceanography

ARGO positions, 16 Oct. 2016



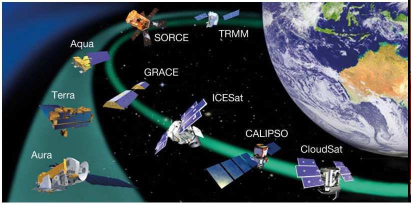
Scripps Inst. Oceanography

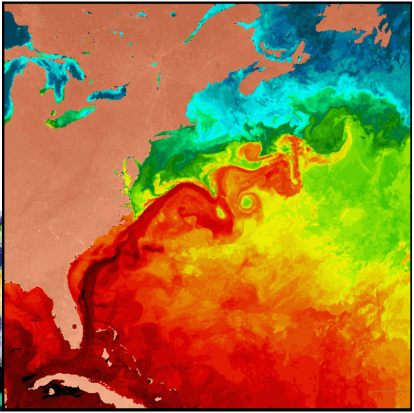
Gliders



Webb Research Inc., Woods Hole Oceanographic Inst.

Satellites





NASA

Observations: measurements

- **Surface temperature** and **salinity** (satellite)
- **Surface height** (satellite)
- **Subsurface temperature** and **salinity** (ships, floats, gliders)
- **Subsurface velocities** (current meters, floats)

Interpreting the observations

- Temperature, salinity and pressure → density
- Sea surface height → surface velocities
- Density profiles → velocity profiles

Calculating density

Density is determined from temperature, salinity and pressure:

$$\rho = \rho(T, S, p) = \rho_c[1 - \alpha_T(T - T_{ref}) + \alpha_S(S - S_{ref}) + ...]$$

where: $\rho_c = 1000 \text{ kg m}^{-3}$

- Warm water is lighter than cold water
- Salty water is heavier than fresh water

Seawater characteristics

Typical temperatures: $0 - 30$C

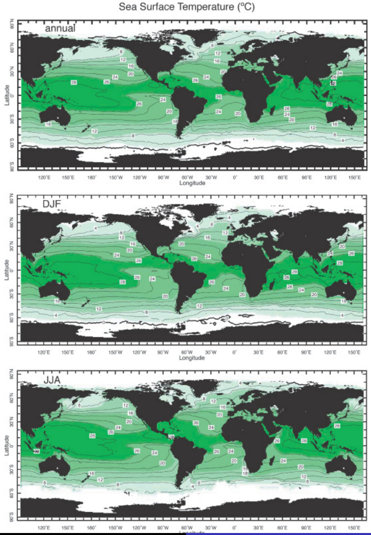
Typical salinities: $33 - 36$ psu (practical salinity units)

Compare:

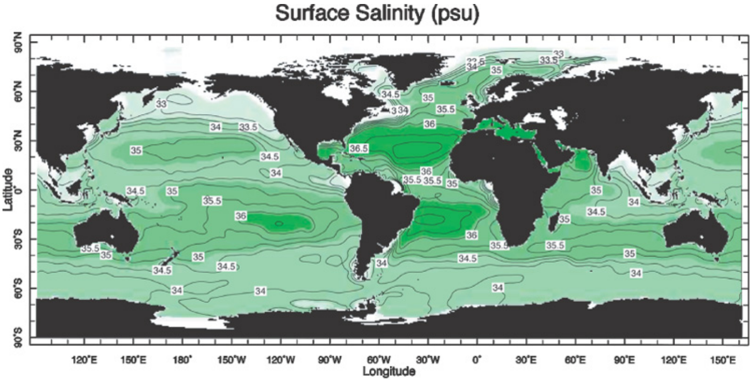
- Fresh water: 0 psu
- Dead Sea: 337 psu

$\rightarrow$ Usually temperature dominates changes in density

Sea surface temperature (SST)

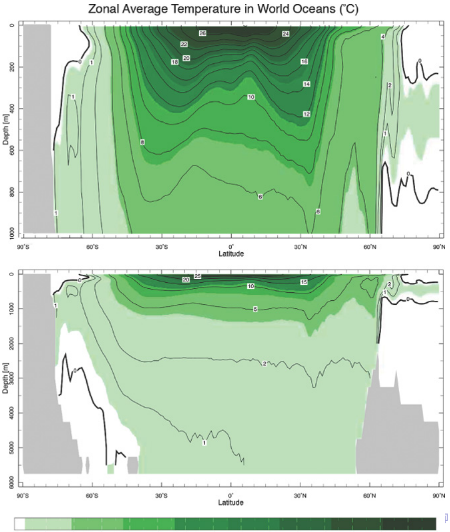


Sea surface salinity

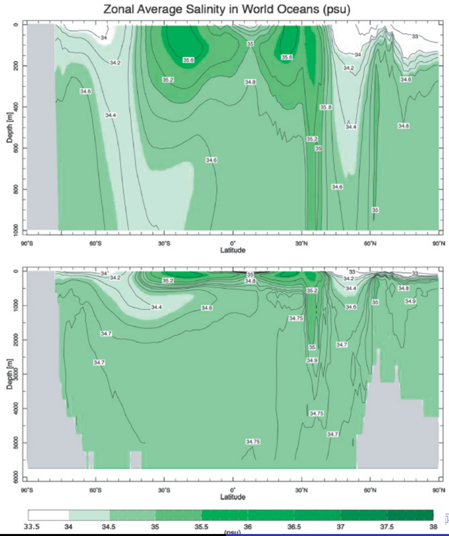


(Data from Levitus World Ocean Atlas (1994).)

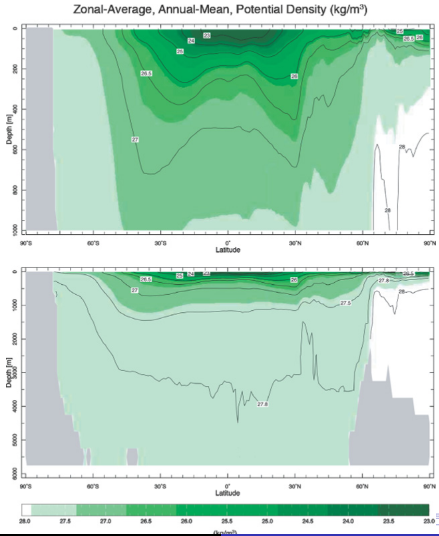
Zonally-averaged temperature vs. depth



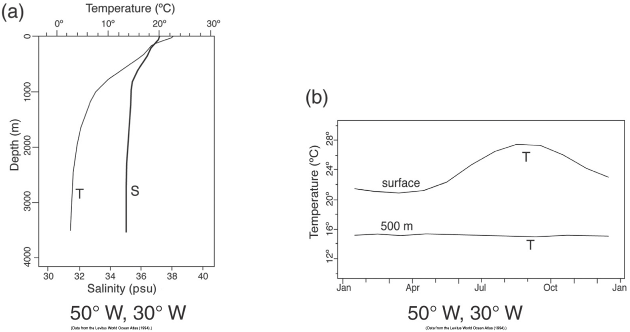
Zonally-averaged salinity vs. depth



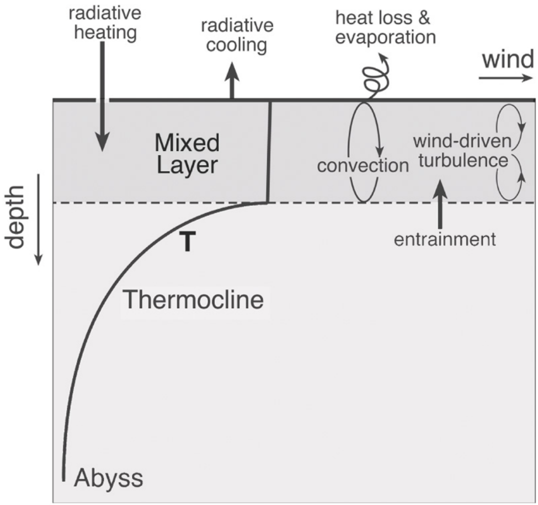
Zonally-averaged density vs. depth



Annual mean profiles, and seasonal variability



The mixed layer



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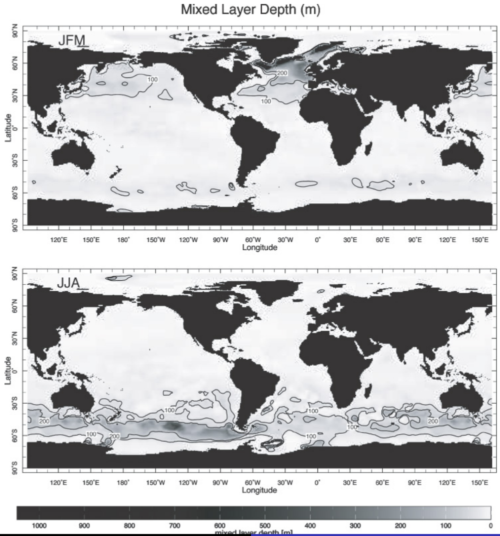
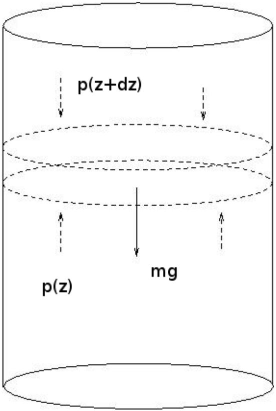
Mixed layer depth



Interpreting the observations

- ~~Temperature, salinity and pressure $\rightarrow$ density~~
- Sea surface height $\rightarrow$ surface velocities
- Density profiles $\rightarrow$ velocity profiles

Basic balances: hydrostatic



Basic balances: hydrostatic

Downward force: $-mg = -\rho V g = \rho (A\,dz) g$

Upward force: $(p(z+dz) - p(z))A$

The fluid is static if these are equal:

$$[p(z+dz) - p(z)]A = -\rho g A\,dz$$

or:

$$\frac{dp}{dz} = -\rho g$$

Basic balances: geostrophy

The momentum equations can be scaled:

$$\frac{d}{dt}\vec{u} + f\hat{k} \times \vec{u} = -\frac{1}{\rho_c}\nabla p$$

$$\frac{U}{T} \qquad fU \qquad \frac{\triangle p}{\rho_c L}$$

Divide by fU:

$$\frac{1}{fT} \qquad 1 \qquad \frac{\triangle p}{\rho_c fUL}$$

The temporal Rossby number is given by:

$$\epsilon \equiv \frac{1}{fT}$$

Basic balances: geostrophy

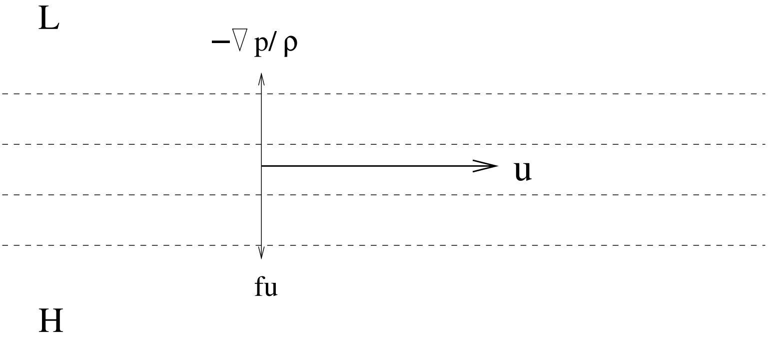
Large scale currents and eddies have a typical time scale of a week:

$$\epsilon = \frac{1}{10^{-4}(10^6)} = 0.01 \ll 1$$

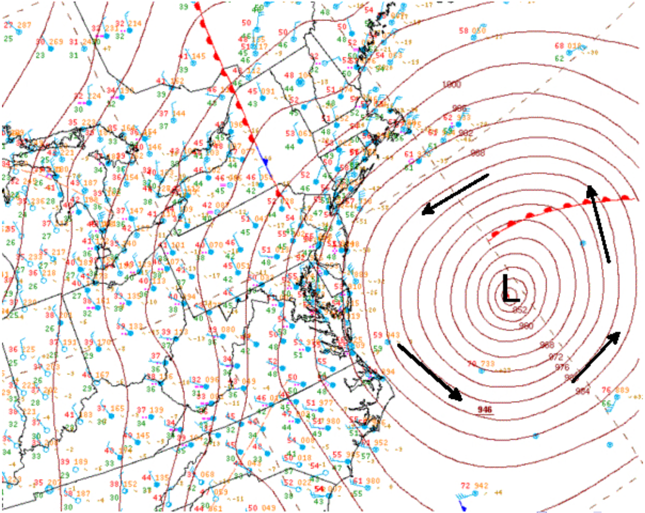
So the horizontal velocities are approximately in geostrophic balance:

$$fu = -\frac{1}{\rho_c}\frac{\partial}{\partial y}p, \qquad fv = \frac{1}{\rho_c}\frac{\partial}{\partial x}p$$

Basic balances: geostrophy



Hurricane Sandy



Combine geostrophic and hydrostatic balances

Integrate hydrostatic relation from z to the surface ($z = \eta$):

$$\int_{-z}^{\eta} \frac{\partial p}{\partial z}\, dz = p(\eta) - p(-z) = -g \int_{-z}^{\eta} \rho\, dz$$

The density is roughly constant and $p(\eta) = p_{atmos}$, so:

$$p(-z) = p_{atmos} + \rho_c g(\eta + z)$$

The atmospheric pressure has little effect:

$$\nabla p(-z) = \nabla p_{atmos} + \rho_c g \nabla \eta \approx \rho_c g \nabla \eta$$

Surface velocities

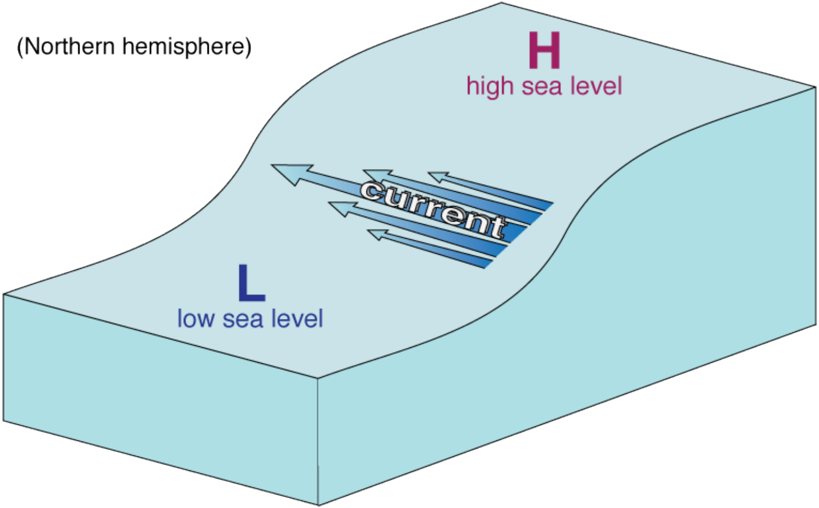
Thus the geostrophic surface velocities are:

$$u_g = -\frac{g}{f}\frac{\partial}{\partial y}\eta, \qquad v_g = \frac{g}{f}\frac{\partial}{\partial x}\eta$$

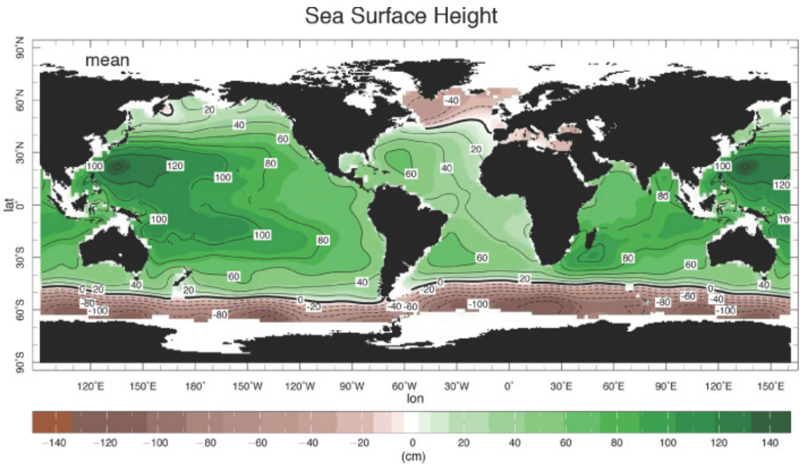
This is how satellite measurements of sea surface height can be used to estimate surface velocities

Can obtain global pictures of the surface velocity, with the same resolution as the satellite data (roughly 100 km)

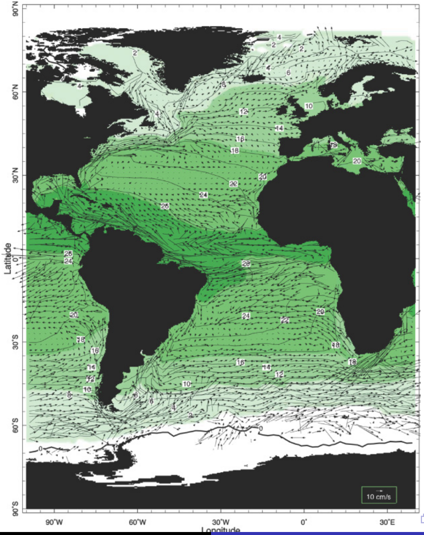
Surface geostrophic flow



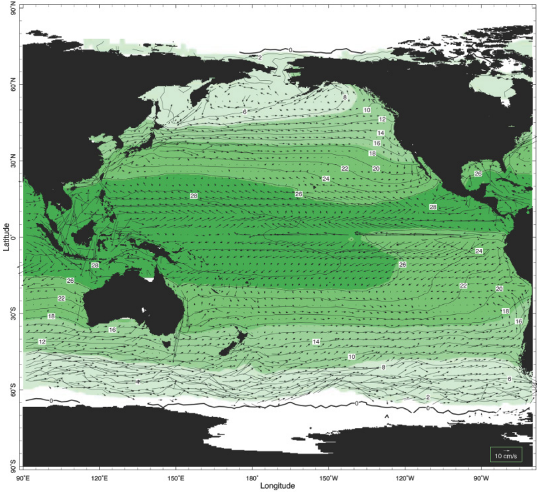
Mean sea surface height, global



Surface velocities, Atlantic

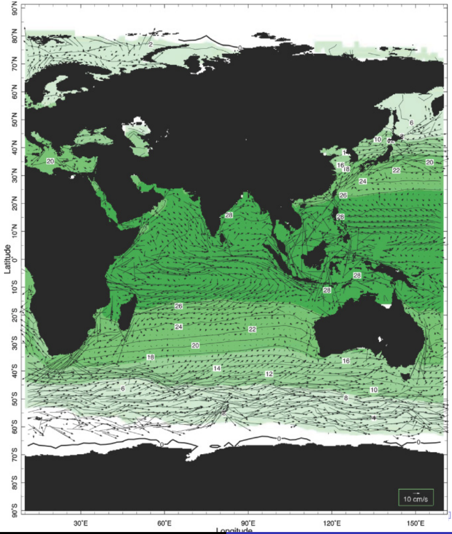


Surface velocities, Pacific

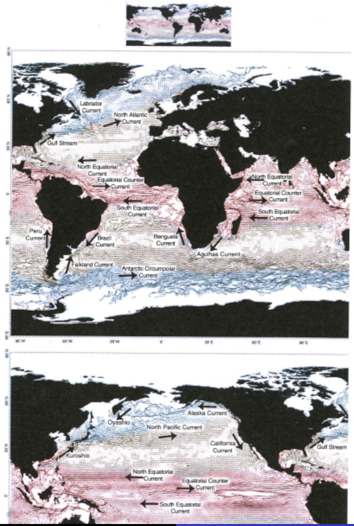


(Data courtesy of Maximenko and Niiler (personal communication, 2003).)

Surface velocities, Indian



Surface currents



Interpreting the observations

- ~~Temperature, salinity and pressure $\rightarrow$ density~~
- ~~Sea surface height $\rightarrow$ surface velocities~~
- Density profiles $\rightarrow$ velocity profiles

Basic balances: thermal wind

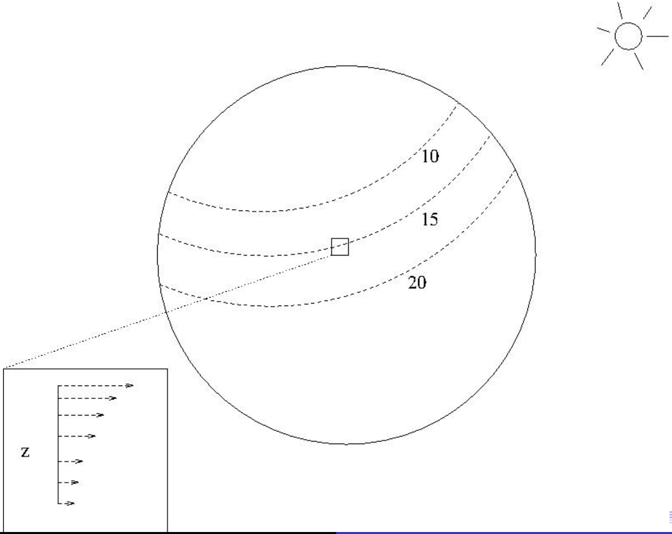
Combine the geostrophic and hydrostatic balances:

$$\frac{\partial}{\partial z} u_g = \frac{\partial}{\partial z}\left(-\frac{1}{\rho_c f}\frac{\partial p}{\partial y}\right)$$
$$= -\frac{1}{\rho_c f}\frac{\partial}{\partial y}\left(\frac{\partial p}{\partial z}\right)$$
$$= -\frac{1}{\rho_c f}\frac{\partial}{\partial y}(-\rho g)$$
$$= \frac{g}{\rho_c f}\frac{\partial}{\partial y}\rho$$

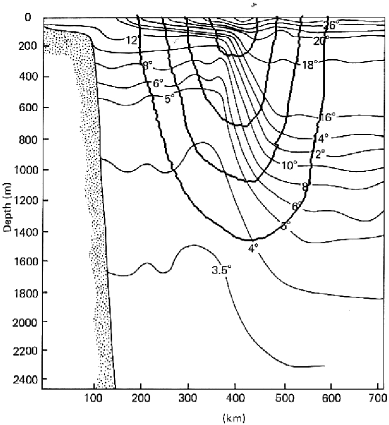
Likewise:

$$\frac{\partial}{\partial z} v_g = -\frac{g}{\rho_c f}\frac{\partial}{\partial x}\rho$$

Basic balances: thermal wind

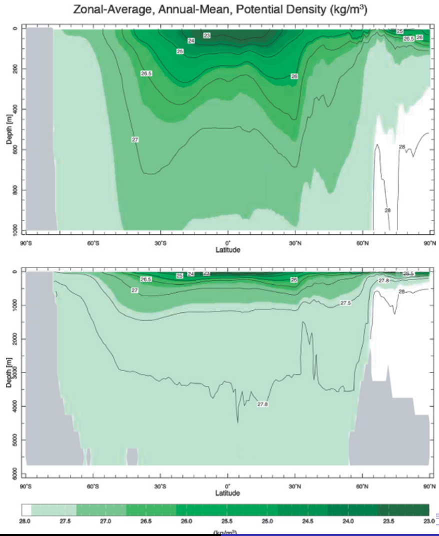


Basic balances: thermal wind



Gulf Stream velocity (dark) and temperature (thin) contours

Deducing the velocity



Deducing the velocity

Integrate the thermal wind relation vertically:

$$\int_{-zref}^{-z} \frac{\partial}{\partial z} u_g \, dz = \int_{-zref}^{-z} \frac{g}{\rho_c f} \frac{\partial}{\partial y} \rho \, dz$$

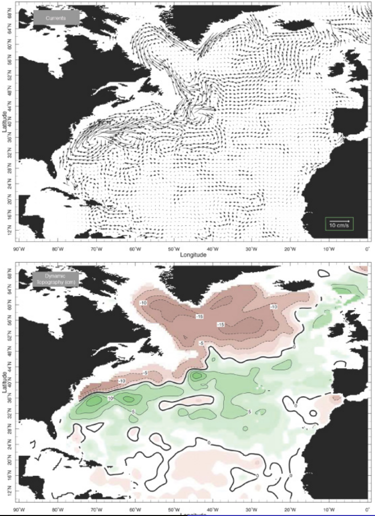
Or:

$$u_g(-z) = u_g(-z_{ref}) + \frac{g}{\rho_c f} \frac{\partial}{\partial y} \int_{-zref}^{-z} \rho \, dz$$

- Must specify the reference level velocity, $u_g(-z_{ref})$

Subsurface velocity

Currents And Pressure at 700m in The Atlantic



Basic balances: incompressibility

The full continuity equation is:

$$\frac{\partial}{\partial t}\rho + \vec{u} \cdot \nabla\rho + \rho(\nabla \cdot \vec{u}) = 0$$

In the ocean, $\rho \approx \rho_c =$ const. So:

$$\nabla \cdot \vec{u} = \frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v + \frac{\partial}{\partial z}w = 0$$

Ocean velocities are approximately incompressible

Basic balances: incompressibility

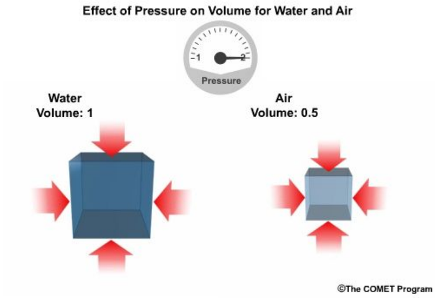
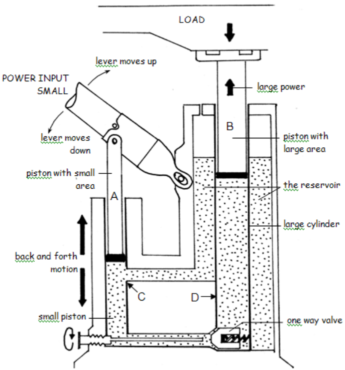


Figure 2: This is a cross-section of a simple hydraulic jack.



Basic balances: summary

- Hydrostatic balance

$$\frac{\partial p}{\partial z} = -\rho g$$

- Geostrophic balance

$$fu = -\frac{1}{\rho}\frac{\partial p}{\partial y}, \quad fv = \frac{1}{\rho}\frac{\partial p}{\partial x}$$

- Incompressibility

$$\nabla \cdot \vec{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Ocean forcing

The ocean is driven primarily by:

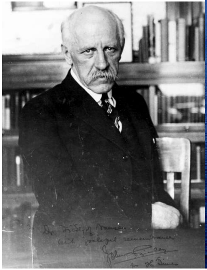
• Wind : forcing at the surface transfers momentum to the ocean, via waves and turbulent motion

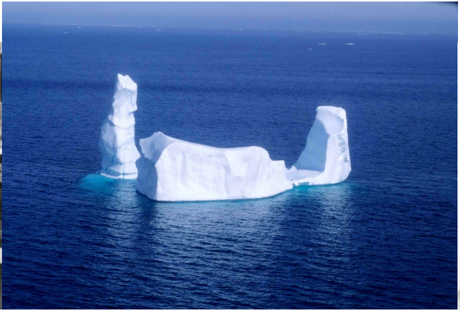
• Heating : the sun warms the low latitudes more than the high latitudes, creating a large scale density gradient at the surface

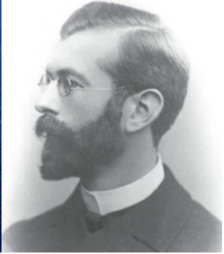
• Evaporation/precipitation : fresh water removal and input at the surface can also affect surface density

Background

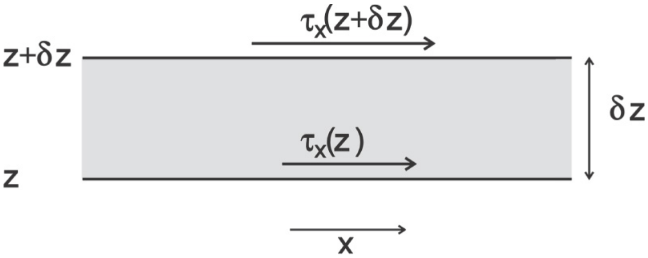
Nansen, icebergs, and Ekman (1905)







Applied stress



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$$\frac{d\vec{u}}{dt} = \frac{1}{\rho_c}\frac{\partial}{\partial z}\vec{\tau}$$

Planetary boundary layer equations

Add stress to geostrophic relations:

$$-fv = -\frac{1}{\rho_c}\frac{\partial}{\partial x}p + \frac{\partial}{\partial z}\frac{\tau^x}{\rho_c}$$
$$fu = -\frac{1}{\rho_c}\frac{\partial}{\partial y}p + \frac{\partial}{\partial z}\frac{\tau^y}{\rho_c}$$

Rewrite using the geostrophic velocities:

$$-fv = -fv_g + \frac{\partial}{\partial z}\frac{\tau^x}{\rho_c}$$
$$fu = fu_g + \frac{\partial}{\partial z}\frac{\tau^y}{\rho_c}$$

Planetary boundary layer equations

Collecting terms on the LHS:

$$-fv_a = \frac{\partial}{\partial z}\frac{\tau^x}{\rho_c}$$
$$fu_a = \frac{\partial}{\partial z}\frac{\tau^y}{\rho_c}$$

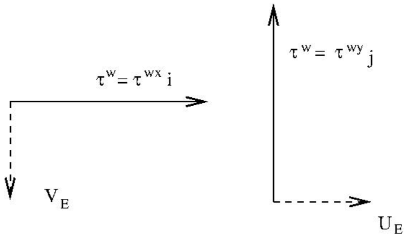
where $u_a = u - u_g$ and $v_a = v - v_g$ are the ageostrophic velocities, forced by the wind. These vary with depth.

Ekman transport

Integrate vertically over the depth of the layer:

$$-\int_{-\delta_E}^{0} f v_a \, dz \equiv -f V_E = \frac{1}{\rho_c} \tau^{wx}$$

$$\int_{-\delta_E}^{0} f u_a \, dz \equiv f U_E = \frac{1}{\rho_c} \tau^{wy}$$



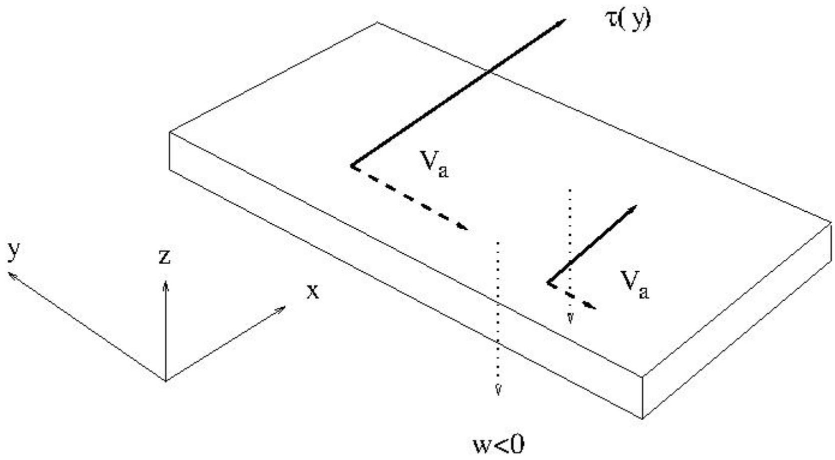
Ekman transport

$\rightarrow$ Net ageostrophic transport is 90° to the right of the wind

Transport is 90° to the left in the southern hemisphere

Accounts for the ice drift witnessed by Nansen

Ekman pumping



Vorticity equation

How does Ekman pumping affect the flow at depth? We can cross-differentiate the equations and subtract them:

$$\frac{\partial}{\partial x}[fu = -\frac{1}{\rho_c}\frac{\partial}{\partial y}p + \frac{\partial}{\partial z}\frac{\tau^y}{\rho_c}]$$

$$-\frac{\partial}{\partial y}[-fv = -\frac{1}{\rho_c}\frac{\partial}{\partial x}p + \frac{\partial}{\partial z}\frac{\tau^x}{\rho_c}]$$

This eliminates the pressure terms, leaving a vorticity equation:

$$f(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) + v\frac{df}{dy} = \frac{1}{\rho_c}\frac{\partial}{\partial z}(\frac{\partial \tau^y}{\partial x} - \frac{\partial \tau^x}{\partial y})$$

Vorticity equation

From incompressibility:

$$\frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v = -\frac{\partial}{\partial z}w$$

So the vorticity equation becomes:

$$\beta v = f\frac{\partial w}{\partial z} + \frac{1}{\rho_c}\frac{\partial}{\partial z}(\frac{\partial \tau^y}{\partial x} - \frac{\partial \tau^x}{\partial y})$$

where $\beta = df/dy$.

Integrated vorticity equation

Assuming a flat upper surface (which is reasonable over large scales) and a flat bottom (which is less reasonable, but there it is), we can integrate over the entire water column:

$$\int_{-H}^{0} \beta v \, dz \equiv \beta V_I = fw(z)|_{-H}^{0} + (\frac{\partial \tau^y}{\partial x} - \frac{\partial \tau^x}{\partial y})|_{-H}^{0}$$

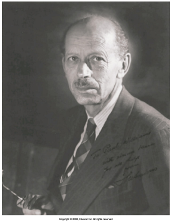
Both $w(0)$ and $w(-H)$ are zero, because of the flat surfaces. Furthermore we neglect the bottom stress (reasonable in deep water), and equate the surface stress with the winds.

Sverdrup relation

The result is:

$$\beta V_I = \frac{1}{\rho_c}\left(\frac{\partial \tau^{wy}}{\partial x} - \frac{\partial \tau^{wx}}{\partial y}\right) = \frac{1}{\rho_c}\hat{k} \cdot \nabla \times \vec{\tau}^{w}$$

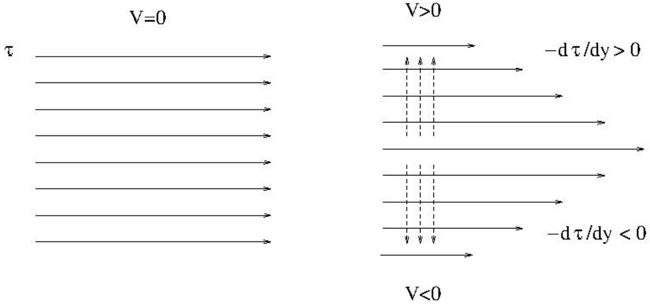
This is the Sverdrup balance



Sverdrup relation

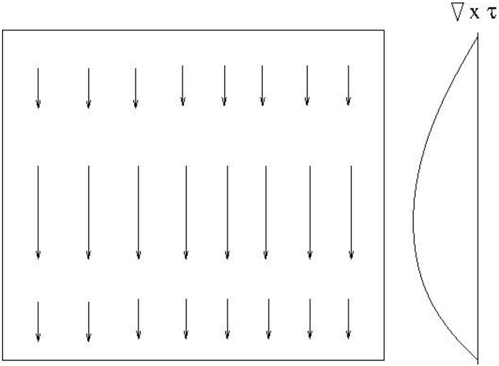
For example, if the wind blows east:

$$\beta V = -\frac{1}{\rho_c}\frac{\partial}{\partial y}\tau^{wx}$$



Boundary currents

Imagine a negative wind stress curl over a basin:



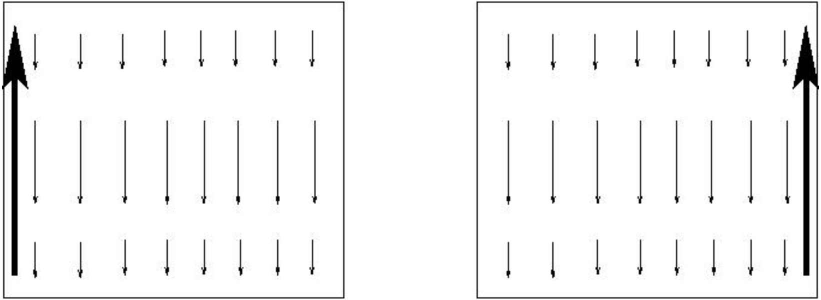
But how does the fluid return north?

Gulf Stream

Benjamin Franklin (1770)

Boundary currents

Actually two possibilities:



Boundary currents

Problem solved by Stommel (1948)

Stommel's Gulf Stream

Stommel realized need additional dynamics to allow a return flow. The simplest is linear bottom friction. In the interior ocean, geostrophy is replaced by:

$$-fv = -\frac{1}{\rho_c}\frac{\partial}{\partial x}p - ru$$
$$fu = -\frac{1}{\rho_c}\frac{\partial}{\partial y}p - rv$$

Friction breaks geostrophy, allowing ageostrophic return flow.

Stommel's Gulf Stream

Now the Sverdrup relation is:

$$\beta V = \frac{1}{\rho_c}\left(\frac{\partial \tau^{wy}}{\partial x} - \frac{\partial \tau^{wx}}{\partial y}\right) - r \int_H^0 \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) dz$$

For simplicity, assume the velocities are depth-independent:

$$\beta H v = \frac{1}{\rho_c}\left(\frac{\partial \tau^{wy}}{\partial x} - \frac{\partial \tau^{wx}}{\partial y}\right) - rH\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Stommel's Gulf Stream

We can split the velocity into two parts: $v = v_I + v_B$, with:

$$\beta v_I = \frac{1}{\rho_c H}\left(\frac{\partial \tau^{wy}}{\partial x} - \frac{\partial \tau^{wx}}{\partial y}\right)$$

in the interior, and in the boundary layer:

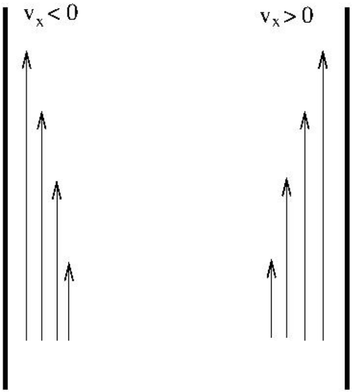
$$\beta v_B = -r\left(\frac{\partial v_B}{\partial x} - \frac{\partial u_B}{\partial y}\right)$$

The meridional velocities in the boundary current are large and the width is narrow so:

$$\beta v_B \approx -r\frac{\partial v_B}{\partial x}$$

Stommel's Gulf Stream

Now consider the shear in the boundary current:



Stommel's Gulf Stream

- Western boundary current

$$r\frac{\partial v_B}{\partial x} < 0 \quad \rightarrow \quad \beta v_B > 0$$

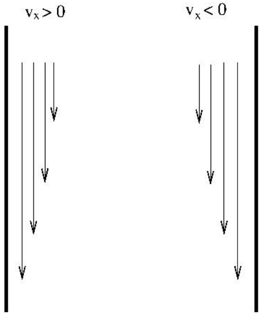
- Eastern boundary current

$$r\frac{\partial v_B}{\partial x} > 0 \quad \rightarrow \quad \beta v_B < 0$$

So the eastern boundary current is inconsistent with northward flow. Thus only a western boundary current works.

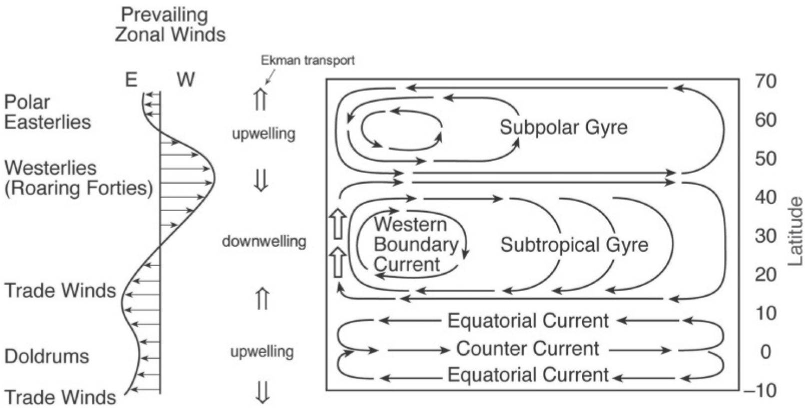
Stommel's Gulf Stream

Works the other way too. If the interior flow is northwards, the boundary currents go south:



$$\text{West}: \quad r\frac{\partial v_B}{\partial x} > 0 \quad \rightarrow \quad \beta v_B < 0$$

The mid-latitude gyres



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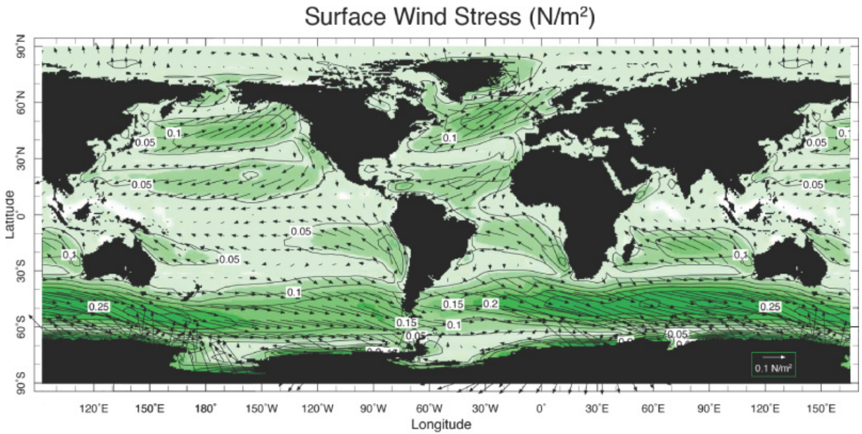
Observed wind stress



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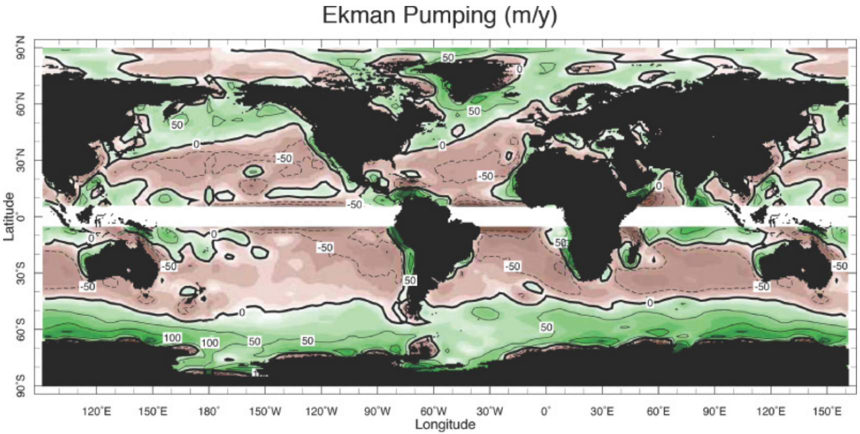
Ekman pumping



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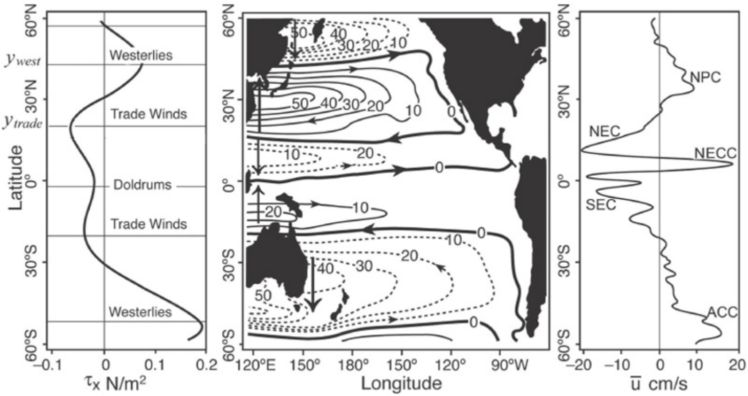
The Pacific gyres



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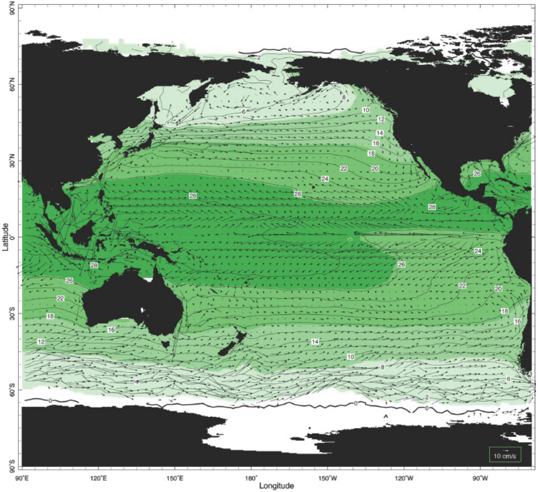
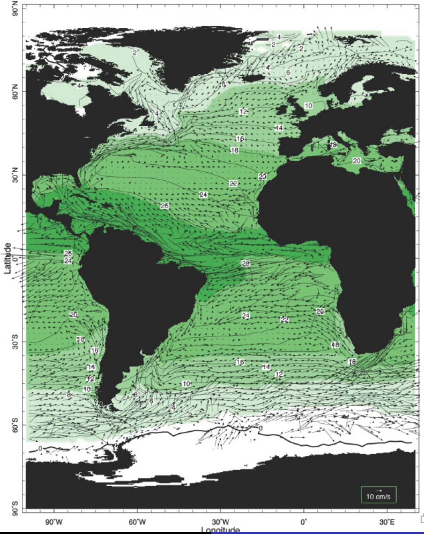
Observations: Pacific



(Data courtesy of Maximenko and Niiler (personal communication, 2003).)

Observations: Atlantic



Thermohaline circulation

The ocean is also heated by incoming shortwave radiation and cooled by outgoing longwave radiation and evaporation

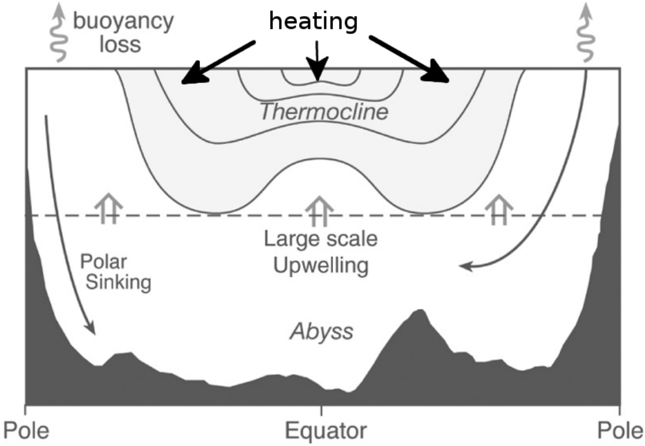
Drives large scale flow, the thermohaline circulation

Global "over-turning" circulation superimposed on wind-driven gyres

Important for redistribution of heat and CO2 in climate system

Hard to observe: time scales of 1000s of years and weak velocities

Buoyancy forcing



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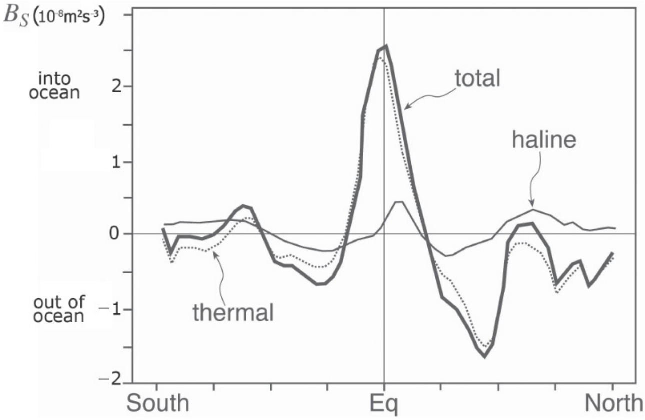
Zonally-averaged buoyancy forcing



(Data from Kalnay et al. (1996).)

Thermally-driven flow

What type of flow do we expect, with warming at low latitudes and cooling at high latitudes? From thermal wind:

$$\frac{\partial}{\partial z}u_g = \frac{g}{\rho_c f}\frac{\partial}{\partial y}\rho$$

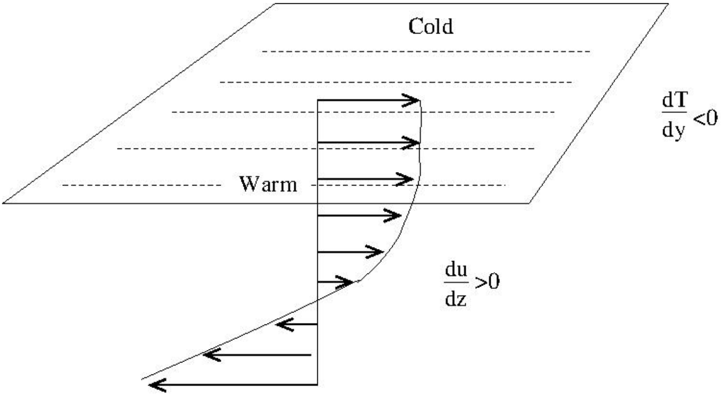
Use the equation of state, assume temperature dominates:

$$\frac{\partial}{\partial z}u_g = \frac{g}{\rho_c f}\frac{\partial}{\partial y}\rho_c(1-\alpha(T - T_{ref})) = -\frac{g\alpha}{f}\frac{\partial}{\partial y}T$$

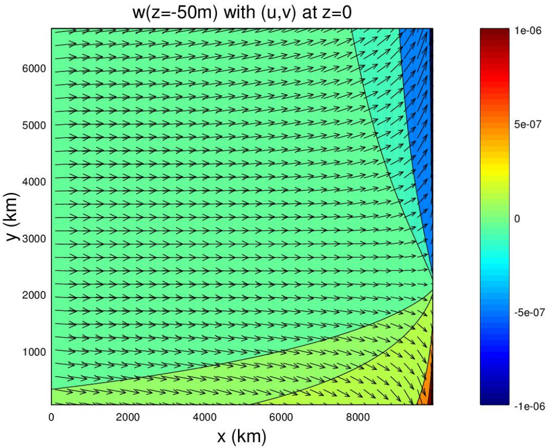
In the northern hemisphere:

$$\frac{\partial}{\partial y}T < 0, \quad f > 0 \quad \rightarrow \quad \frac{\partial}{\partial z}u_g > 0$$

Thermally-driven flow

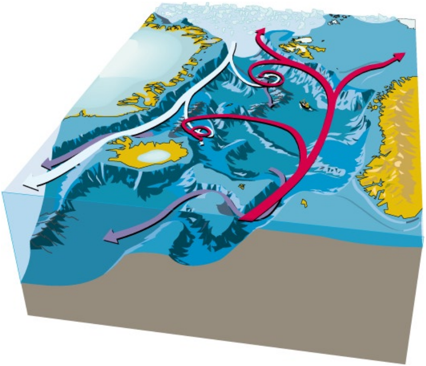


Surface velocities in a thermally-driven box



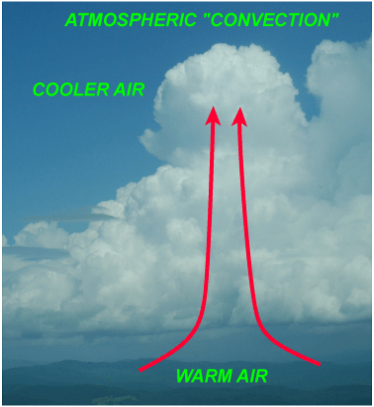
Gjermundsen and LaCasce (2015)

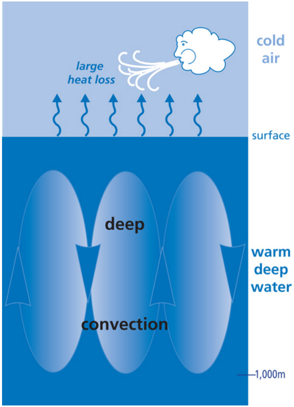
Circulation schematic, Nordic Seas



C. Mauritzen (1996), Bentsen et al. (2002)

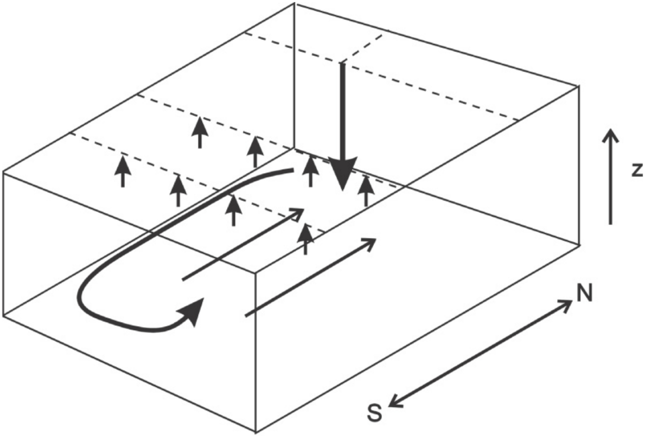
Oceanic convection





Stommel-Arons (1960)

Stommel-Arons abyssal layer



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Stommel-Arons model

Same equations as for Gulf Stream:

$$-fv = -\frac{1}{\rho_c}\frac{\partial}{\partial x}p - ru$$
$$fu = -\frac{1}{\rho_c}\frac{\partial}{\partial y}p - rv$$

Cross-differentiate to make a vorticity equation:

$$\beta v = f\frac{\partial w}{\partial z} - r(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})$$

Stommel-Arons model

In the abyssal layer, assume no vertical shear:

$$\frac{\partial}{\partial z}u = \frac{\partial}{\partial z}v = 0$$

Integrate vorticity equation vertically, from the (flat) bottom to the top of the abyssal layer:

$$\beta H_A v = f w_T - r H_a \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

where H_A is the layer depth and w_T is vertical velocity at top

Basin interior

Away from boundaries, Sverdrup balance:

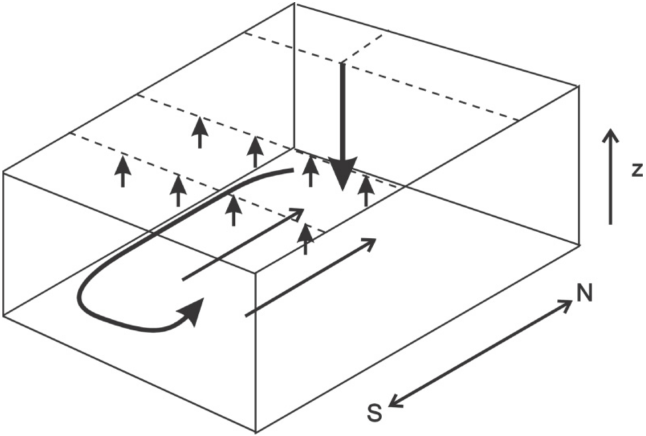
$$\beta H_A v = f w_T$$

We don't know w_T. Stommel assumed this was constant and upward in the interior: $w_T = W$. So:

$$v = \frac{fW}{\beta H_A} > 0$$

everywhere in the interior

Basin interior



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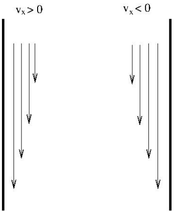
Boundary current

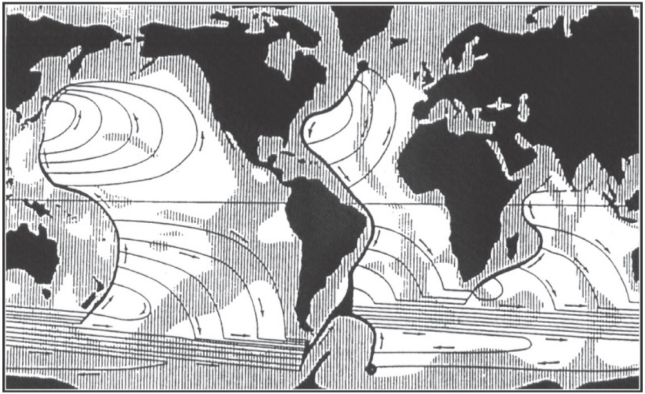
Flow returns in deep western boundary current

$$\beta v_B = -r(\frac{\partial v_B}{\partial x} - \frac{\partial u_B}{\partial y}) \approx -r\frac{\partial v_B}{\partial x}$$



$$\text{West}: \quad \frac{\partial v_B}{\partial x} > 0 \quad \rightarrow \quad v_B < 0$$

Stommel-Arons circulation



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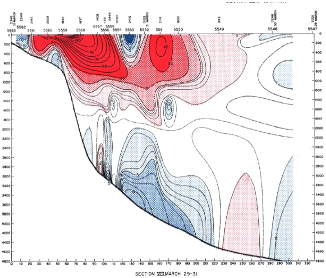
Swallow's observation of deep western boundary current



Summary

- Wind-forcing drives gyres with western boundary currents
- Buoyancy forcing drives a global circulation with deep western boundary currents and interior upwelling. Largely driven by surface heating/cooling, although sensitive to fresh water input (melting).
- How these two interact is not well understood
- Extremely important for the climate system