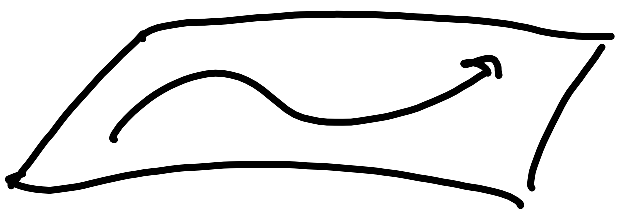
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \rightarrow \psi(x,y,t)$$

$$\frac{U}{L} \sim \frac{U}{L} \sim \frac{W}{H} \qquad W \sim \frac{H}{L} U$$

$U \sim 10$ m/s $\quad H \sim 10$ km $\quad L \sim 1000$ km

$$\frac{H}{L} = \frac{1}{100} \qquad W \sim \frac{U}{100}$$

500 hPa



$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv = \frac{-1}{\rho_c}\frac{\partial p}{\partial x}$$

$$\frac{U}{T} \quad \frac{U^2}{L} \quad \frac{U^2}{L} \quad \frac{VW}{H} \quad fU \quad \frac{\Delta p}{\rho_c L}$$

$$\frac{1}{fT} \quad \frac{U}{fL} \quad \frac{U}{fL} \quad \frac{W}{fH} \quad 1 \quad \frac{\Delta p}{\rho_c f U L}$$

$$R_o = \frac{U}{fL} = \frac{10}{10^{-4}\,10^{6}} = 0.1 \quad (0.01)$$

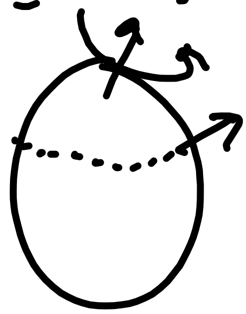
$$-fv_g = \frac{-1}{\rho_c}\frac{\partial p}{\partial x}$$ geostrophic balance

$V \approx V_g \quad u \approx u_g$

$$v = \frac{1}{\rho_0 f_0}\frac{\partial p}{\partial x} \qquad u = \frac{-1}{\rho_0 f_0}\frac{\partial p}{\partial y}$$

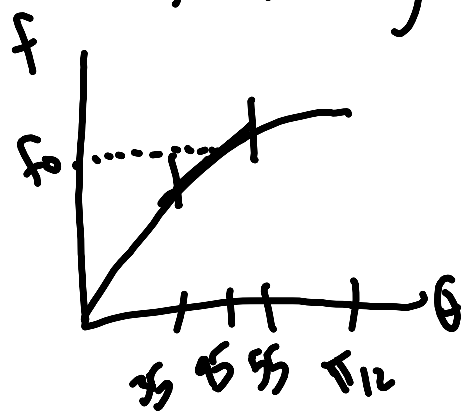
$$v = \frac{\partial}{\partial x}\psi \qquad u = -\frac{\partial}{\partial y}\psi$$

$\Omega = 2\pi/\text{day}$



$$f = 2\Omega \sin\theta = 2\Omega \sin\left(\frac{y}{R_e}\right)$$

$f = f_0 + \beta y$ $\quad\beta$-plane approximation



$$|\beta y| \ll f_0$$

2D flow! $\psi = \frac{p}{\rho_0 f_0}$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{2D cont.}$$

$$\frac{\partial}{\partial x}\left(-\frac{\partial p}{\partial y}/\rho_c f_0\right) + \frac{\partial}{\partial y}\left(\frac{\partial p}{\partial x}/\rho_c f_0\right) = 0$$

2D turb

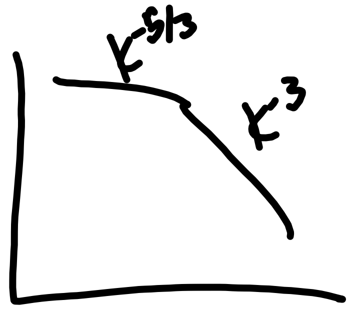
K67 $\frac{\partial}{\partial t}\zeta + \vec{u}\cdot\nabla\zeta = \nabla\times F + \nu\nabla^2\zeta$

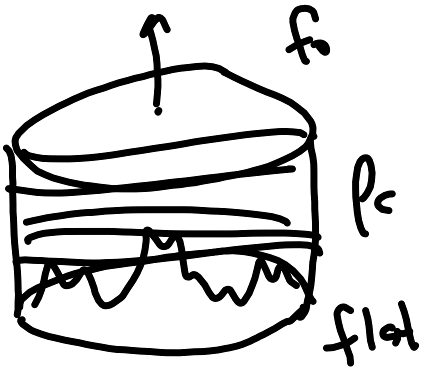
f-plane $\frac{\partial}{\partial t}\zeta + \vec{u}\cdot\nabla(\zeta+f) = \nabla\times F + \nu\nabla^2\zeta$

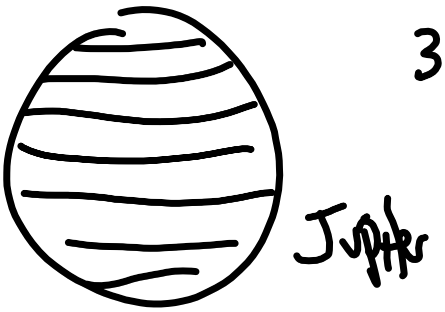
$f = f_0 + \underline{\underline{\beta y}}$ "f plane" $f = f_0$

$\downarrow$

$\vec{u}\cdot\nabla\zeta$







complications

1.) $f \neq$ const.

$f = f_0 + \beta y$ 1975

2.) topography 1975

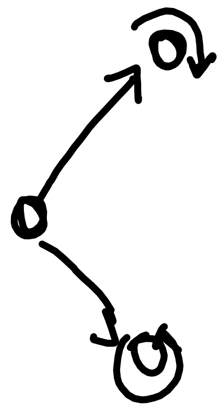
3.) $\rho = \rho(z)$ ξ 1971

$$\frac{\partial}{\partial t}\zeta + \vec{u}\cdot\nabla(\zeta + f_0 + \beta y) = F + D$$

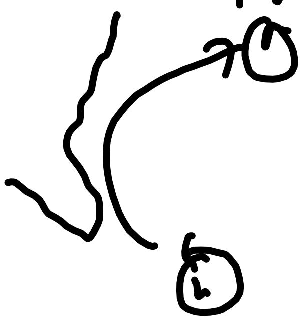
$$\frac{\partial}{\partial t}\zeta + \vec{u}\cdot\nabla\zeta + \beta v = F + D = 0$$

$$\frac{d}{dt}\zeta + \beta v = 0$$

$$\frac{d}{dt}(\zeta + \beta y) = 0$$



Kelvin's $\rightarrow$ $\zeta + \beta y = \text{const.}$



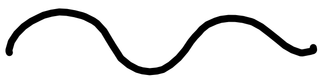
Rhines 1975

linear $\frac{\partial}{\partial t}\zeta + \beta v = 0$

$v = \frac{\partial \psi}{\partial x}$ $\zeta = \nabla^2 \psi$

$\frac{\partial}{\partial t}\nabla^2\psi + \beta \frac{\partial \psi}{\partial x} = 0$ Rossby 1936

wave $\psi \sim \hat{\psi} e^{ikx+ily-i\omega t}$



$\left[-i\omega(-k^2-l^2)\cancel{\hat{\psi}} + ik\beta\cancel{\hat{\psi}}\right]\cancel{e^{ikx+ily-i\omega t}} = 0$

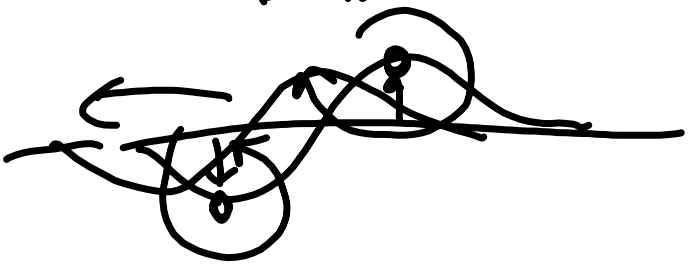
$i\omega(k^2+l^2) + ik\beta = 0$

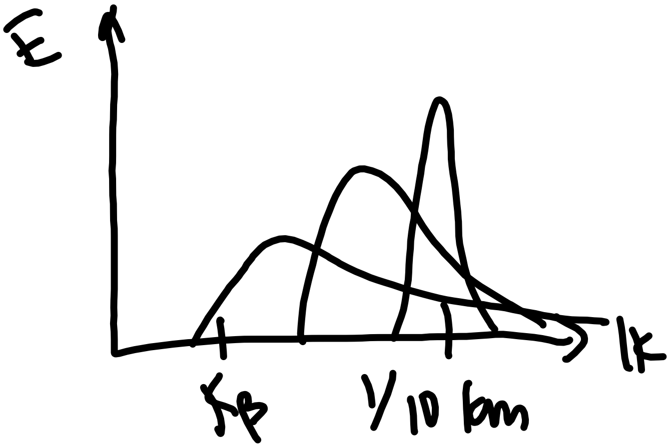
$\omega = \frac{-\beta k}{k^2+l^2}$ Rossby wave dispersion relation

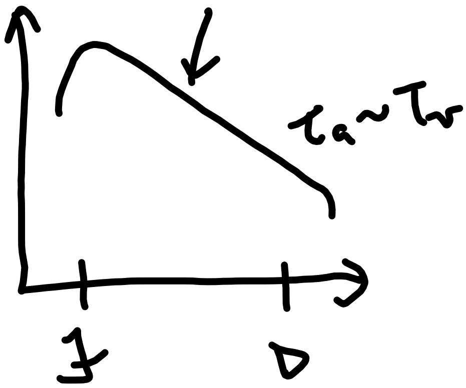
$$\omega = \frac{2\pi}{T} \quad T \sim \frac{k^2 + l^2}{\beta k}$$

frequency higher small scales

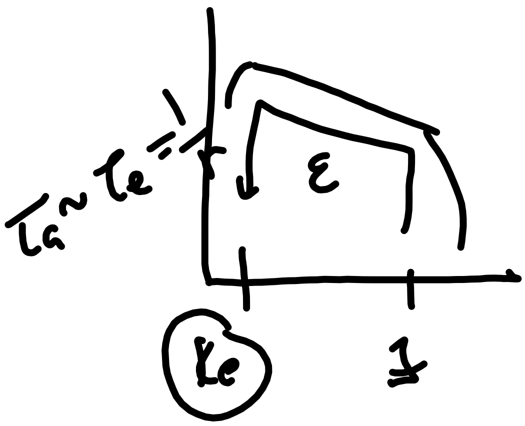
$$c_x = \frac{\omega}{k} = \frac{-\beta}{k^2 + l^2}$$ westward!







$$\tau_q \sim \tau_\beta = \frac{k^2 + l^2}{\beta k}$$



$$\frac{\partial}{\partial t}\zeta + \vec{u}\cdot\nabla\zeta + \beta v = 0$$

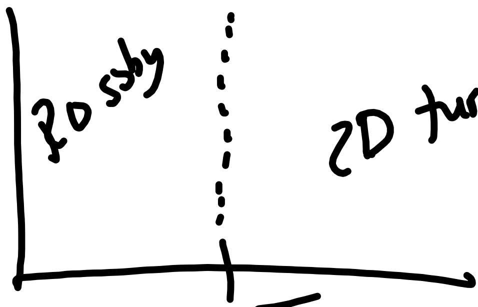
$$\frac{U}{LT} \qquad \frac{UU}{LL} \qquad \beta U$$

$$\frac{1}{\beta L T} \qquad \frac{U}{\beta L^2} \qquad 1$$

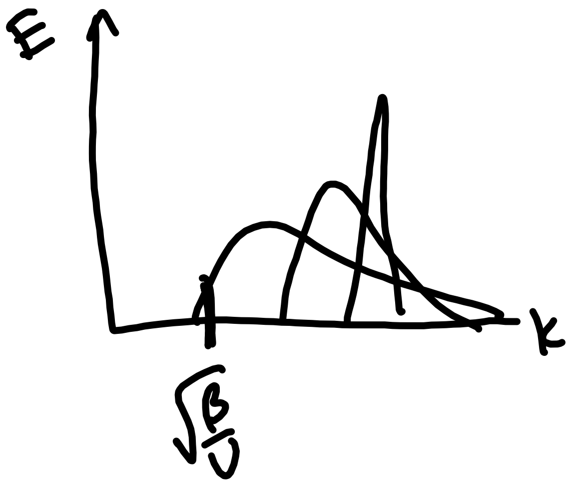
$$Ro = \frac{U}{fL} \rightarrow \frac{U}{\beta L^2}$$

$\frac{U}{\beta L^2} \ll 1$ Rossby waves

$\frac{U}{\beta L^2} \gg 1$ 2D turbulence



$L = \sqrt{\frac{U}{\beta}}$ Rhines Scale



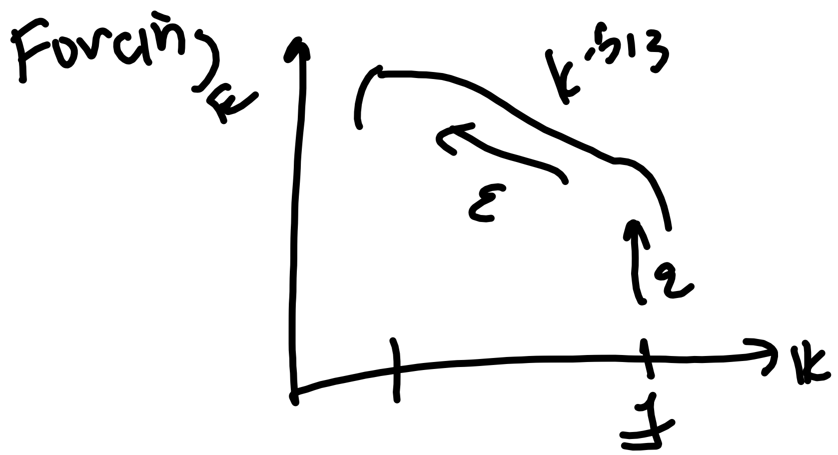
"arrest"

→ Forcing

→ zonal bands ?

anisotropy

Vallis + Maltrud 1993



$$T_q \sim T_\beta = \frac{k^2+l^2}{\beta k}$$

$$T_q \sim \varepsilon^{-1/3} k^{-2/3}$$

$$s \quad \left(\frac{m^2}{s^3}\right)\left(\frac{1}{m}\right)$$

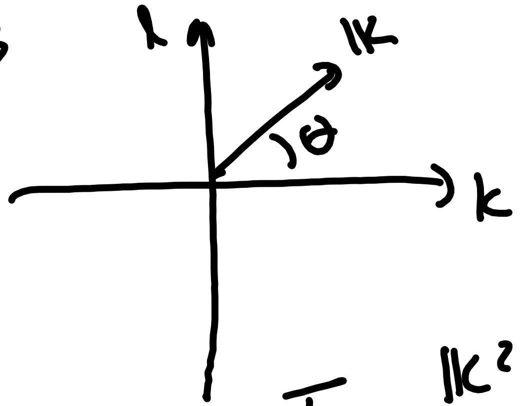
Rhines → isotropic $k = l \to \mathbb{K}$

$$\varepsilon^{-1/3}\,\mathbb{K}^{-2/3} = \frac{\mathbb{K}^2}{\beta\,\mathbb{K}} \sim \frac{\mathbb{K}}{\beta}$$

$$\frac{\beta}{\varepsilon^{1/3}} = \mathbb{K}^{5/3} \qquad \mathbb{K}_\beta = \frac{\beta^{3/5}}{\varepsilon^{1/5}}$$

anistropy?

VA 93



$$k = \mathbb{K}\cos\theta$$
$$l = \mathbb{K}\sin\theta$$

$$T = \frac{\mathbb{K}^2}{\beta\,\mathbb{K}\cos\theta} = \frac{\mathbb{K}}{\beta\cos\theta}$$

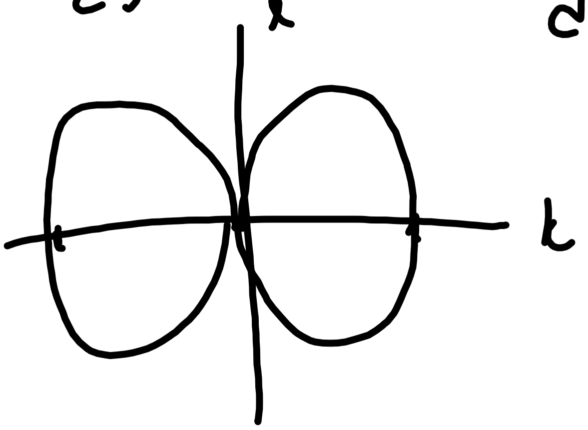
$$\varepsilon^{-1/3}\,\mathbb{K}^{-2/3} = \frac{\mathbb{K}}{\beta\cos\theta}$$

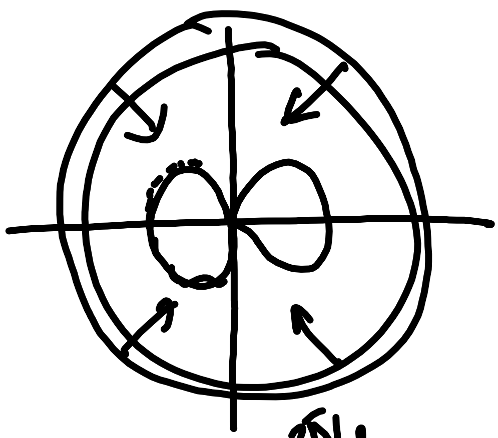
$$\mathbb{K}^{5/3} = \frac{\beta}{\varepsilon^{1/3}}\cos\theta \qquad \mathbb{K} = \frac{\beta^{3/5}}{\varepsilon^{1/5}}\cos\theta^{3/5}$$

$$k = \mathbb{K}\cos\theta = \frac{\beta^{3/5}}{\varepsilon^{1/5}}\cos\theta^{8/5}$$

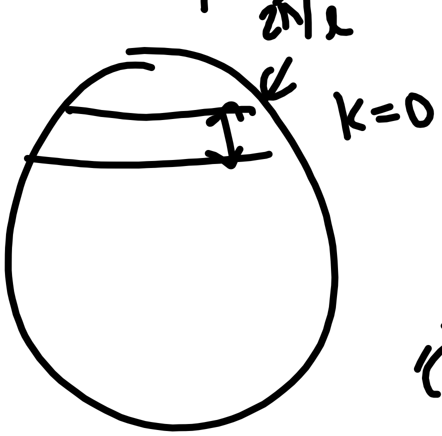
$$l = \mathbb{K}\sin\theta = \frac{\beta^{3/5}}{\varepsilon^{1/5}}\cos\theta^{3/5}\sin\theta$$

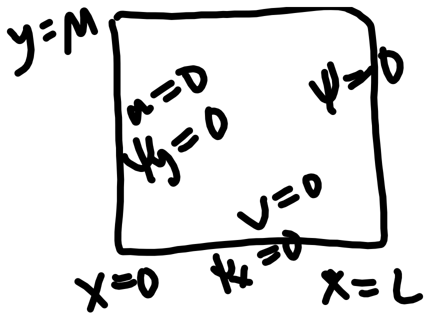
dumbbell






E
isotropic
k


2π/L
k=0



$$f = f_0 + \beta y$$

JHL 2001

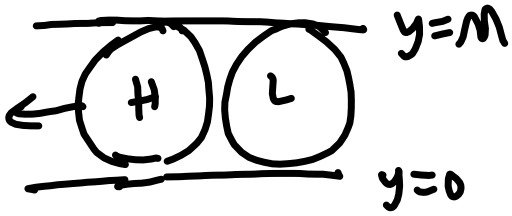
linear $\frac{d}{dt}\zeta + \beta v = 0$

$$\psi = \hat{\psi} e^{ikx + ily - i\omega t}$$ plane wave

$$\psi = \hat{\psi} \sin\left(\frac{n\pi x}{L}\right) \sin\left(\frac{m\pi y}{M}\right) \cos(kx - \omega t)$$

Rossby basin mode

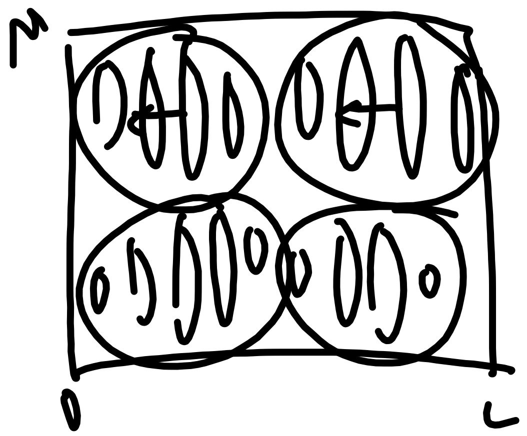
channel



$$\psi = \hat{\psi} \sin\left(\frac{m\pi y}{M}\right) \cos(kx - \omega t)$$

channel wave

Basin



n = 2
m = 2

$$\frac{\partial}{\partial t} \nabla^2 \psi + \beta \frac{\partial \psi}{\partial x} = 0$$

$$\psi = \hat{\psi} \sin(\alpha x) \sin(\gamma y) \cos(kx - \omega t)$$

$$\psi_x = \alpha \hat{\psi} \cos \sin \cos \quad - k \alpha \hat{\psi} \sin \sin \sin$$

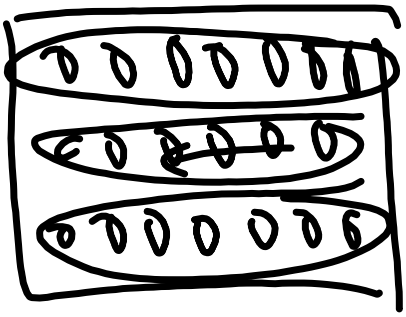
$$\psi_y = \gamma \hat{\psi} \sin \cos \cos$$

$$\psi_{xx} = -\alpha^2 \hat{\psi} \sin \sin \cos + \ldots$$

algebra $\rightarrow$ dispersion relation

$$\omega = \frac{-\beta}{2\pi \sqrt{(n\pi/L)^2 + (m\pi/M)^2}} = \frac{-\beta}{2\pi K_{nm}}$$

$$K_{nm} = \sqrt{\left(\frac{n\pi}{L}\right)^2 + \left(\frac{m\pi}{M}\right)^2}$$ isotropic



$m = 3$

$n = 1$

$$\tau_c = \tau_b$$

$$\varepsilon^{-1/3} k_{nm}^{-2/3} = \frac{k_{nm}}{\beta}$$

$$k_{nm} = \frac{\beta^{3/5}}{\varepsilon^{1/5}}$$ isotropic!

$$\frac{\partial}{\partial t}\zeta + \vec{u}\cdot\nabla\zeta + \beta v = \mathcal{F} - r\nabla^2\psi$$

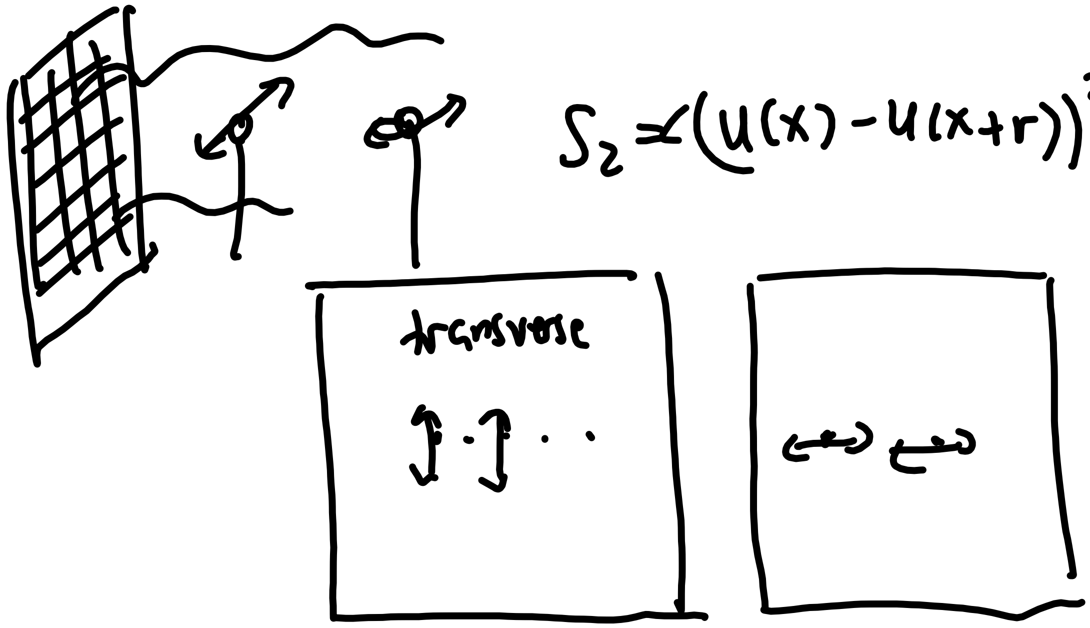
$$\frac{d}{dt}E = \iint \psi \mathcal{F}\, dA - r\iint u^2+v^2\, dA$$

$$\varepsilon \qquad - 2rE$$

steady $\frac{dE}{dt} \approx 0$ $\quad \varepsilon = 2rE$

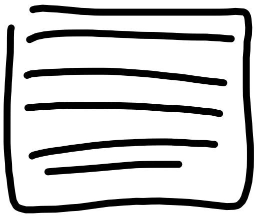
$$L = \sqrt{\frac{U}{\beta}} = \frac{\varepsilon^{1/5}}{\beta^{3/5}} = \frac{(2rE)^{1/5}}{\beta^{3/5}}$$

correlation ellipses → structure function

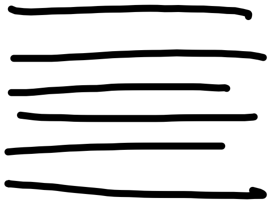


$$S_2 = \langle (u(x) - u(x+r))^2 \rangle$$

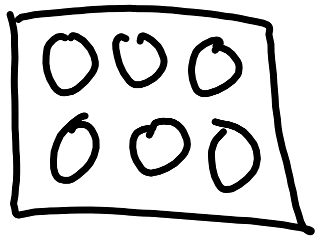
Rhines (1975)	VM (1993)	JHC (2001)
isotropic	anisotropic	isotropic
periodic	periodic	basin
	bands	modes



periodic?



channel



basin