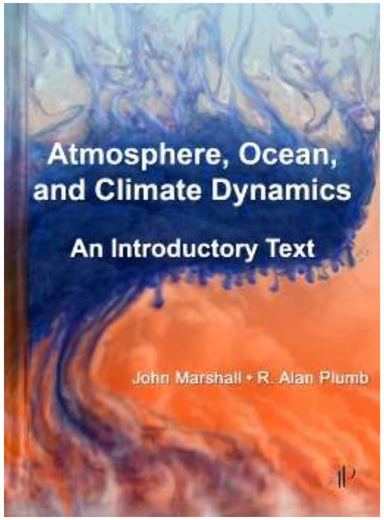
GEF 1100 – Klimasystemet

Chapter 6-8: Recap



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Lecture Outline – Ch. 6

Ch. 6 - The equations of fluid motions

1. Motivation
2. Basics
    - 2.1 Lagrange - Euler
    - 2.2* Mathematical add on
3. Equation of motion for a nonrotating fluid
    - 3.1 Forces
    - 3.2 Equations of motion
    - 3.3 Hydrostatic balance
4. Conversation of mass
5. Thermodynamic equation
6. Equation of motion for a rotating fluid
    - 6.1 Forces
    - 6.2 Equations of motion
7. Take home messages

**Addition, not in book.*

Ch. 6 - Take home message

- 5 equations for the temporal evolution of the fluid with 5 unknowns (u,v,w,p,T): *equations of motion (3), conservation of mass (1), thermodynamic equation (1)*

- Equations of motion on a *non-rotating fluid*: *Pressure gradient force, gravitational force and friction force act.* $\frac{D\mathbf{u}}{Dt} + \frac{1}{\rho}\nabla p + g\hat{\mathbf{z}} = \mathcal{F}$

- Equations of motion on a *rotating fluid*: *Pressure gradient force, modified gravitational potential (gravitational and centrifugal force), Coriolis force and friction force act.* $\frac{D\mathbf{u}}{Dt} + \frac{1}{\rho}\nabla p + \nabla\Phi + f\hat{\mathbf{z}} \times \boldsymbol{u} = \mathcal{F}$

$\left(\frac{D}{Dt}\right)_{\text{fixed particle}} \equiv \frac{\partial}{\partial t} + \boldsymbol{u}\cdot\boldsymbol{\nabla}$ (Eq. 6-1) : Lagrangian or total derivative

→5 equations for the evolution of the fluid (5 unknowns)

Ia.) $\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial x} = \mathcal{F}_{\mathrm{x}}$ (6-7a)

Ib.) $\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial y} = \mathcal{F}_{y}$ (6–7b)

Ic.) $\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} + \mathrm{g} = \mathcal{F}_{\mathrm{z}}$ (6-7c)

II.) $\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\boldsymbol{u}) = 0$ (6-9 or 6-11/6-12)

III.) $\frac{DQ}{Dt} = c_p\frac{DT}{Dt} - \frac{1}{\rho}\frac{Dp}{Dt}$ (6-13 or 6-14)

Restrictions:
- application to average motion is often incorrect i.e. turbulence,
- fixed coordinate system.

u: zonal velocity
v:meridional velocity
w: vertical velocity
ρ: density

$\mathcal{F}$: frictional force *per unit mass*
p: pressure
g: gravity acceleration

Q: heat
$\frac{DQ}{Dt}$: diabatic heating rate per unit mass
c_p : specific heat at constant pressure
T: temperature

Hydrostatic balance

If friction $\mathcal{F}_z$ and vertical acceleration $\frac{Dw}{Dt}$ are negligible, we derive from the vertical eq. of motion (6-7c) the **hydrostatic balance** *(Ch.3, Eq.3-3):*

$$\frac{\partial p}{\partial z} = -\rho g \quad \text{(Eq. 6-8)}$$

Balance between vertical pressure gradient and gravitational force!

Note: This approximation holds for large-scale atmospheric and oceanic circulation with weak vertical motions.

Forces on rotating sphere

Fictitious forces:

- occur in revolving/accelerating system
  (e.g. car in curve)
- occur on the earth's surface
  (Fixed system on earth's surface forms an accelerated system, because a circular motion is performed once a day. Movement of earth around the sun compared to earth's rotation is insignificant.)

→ **Coriolis force**

→ **Centrifugal force**

Equations of motion on a *rotating fluid*

$$\underbrace{\frac{D\mathbf{u}}{Dt}}_{\text{Lagrangian (or total) derivative of vector } \mathbf{u}} + \underbrace{\frac{1}{\rho}\nabla p}_{\text{Pressure gradient accel.}} + \underbrace{\nabla \Phi}_{\text{Gravitational +Centrifugal accelerations}} + \underbrace{f\hat{\mathbf{z}} \times \boldsymbol{u}}_{\text{Coriolis acceleration}} = \underbrace{\mathcal{F}}_{\text{Friction acceleration}} \qquad \text{Eq. 6–43}$$

$$\Phi = \mathrm{g}z - \frac{\Omega^2 \mathrm{r}^2}{2} \qquad \text{Eq. 6–30}$$

Φ (Greek “Phi“) is modified gravitational potential on Earth.

Coriolis parameter f: $f = 2\Omega \sin \varphi$

Angular velocity: Ω= 7.27 x 10^{-5} s^{-1}

Lecture Outline – Ch. 7

Ch. 7 – Balanced flow

1. Motivation

2. Geostrophic motion
   - 2.1 Geostrophic wind
   - 2.2 Synoptic charts
   - 2.3 Balanced flows

3. Thermal wind equation*

4. Subgeostrophic flow: The Ekman layer
   - 5.1 The Ekman layer*
   - 5.2 Surface (friction) wind
   - 5.3 Ageostrophic flow

5. Summary

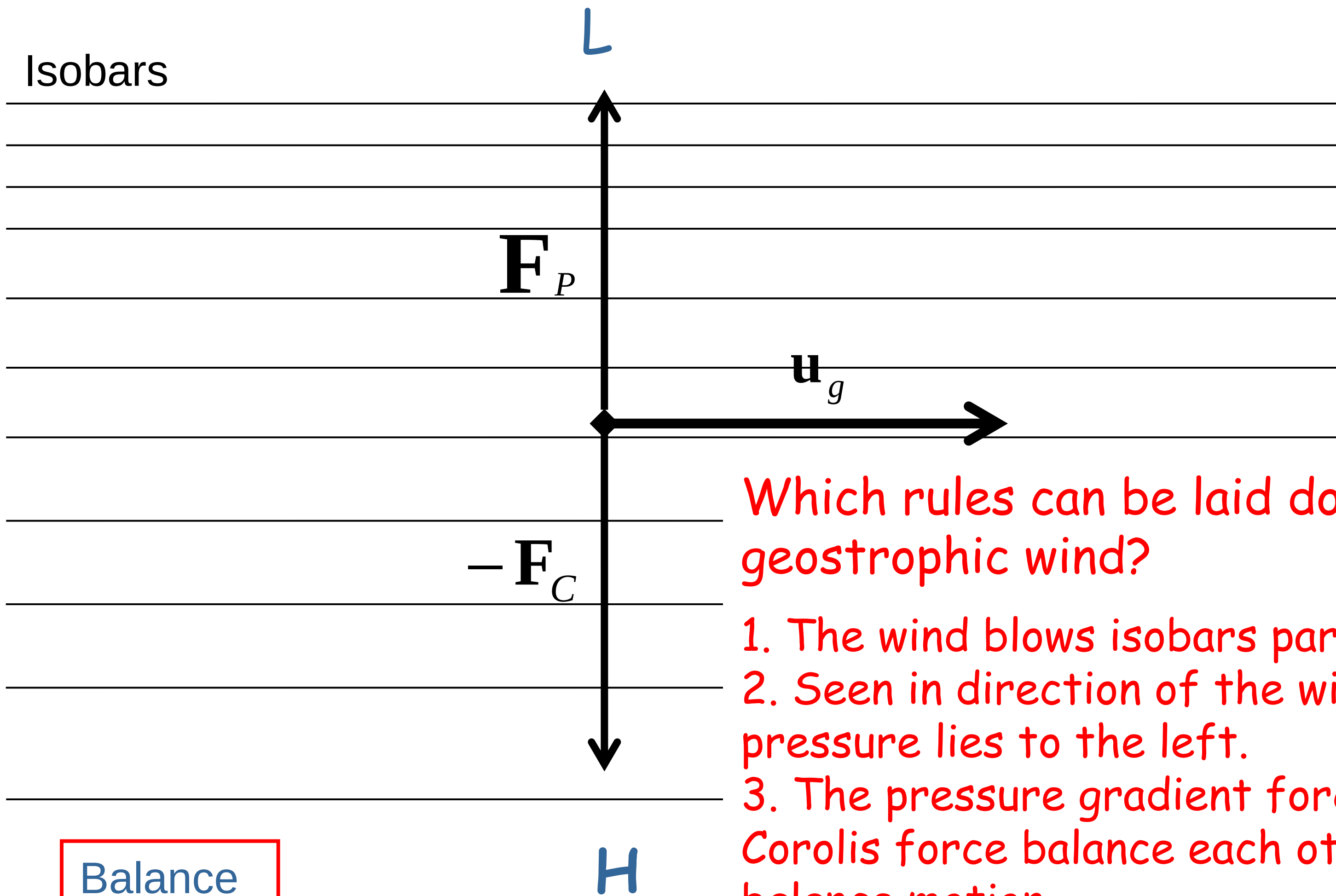
6. Take home message

**With add ons.*

Ch. 7- Take home message

- Balanced flows for horizontal fluids (atmosphere and ocean).
- Balanced horizontal winds:
  - Geostrophic wind balance good approximation for the observed wind in the free troposphere.
  - Gradient wind balance and cyclostrophic wind balance occur with higher Rossby number.
  - Surface wind (subgeostrophic flow) balance occurs within the Ekman layer.
- Thermal wind is good approximation for the geostrophic wind change with height *z*.

Geostrophic wind



Balance between:

$\vec{F}_P = -\vec{F}_C$

Which rules can be laid down for the geostrophic wind?

1. The wind blows isobars parallel.
2. Seen in direction of the wind, the low pressure lies to the left.
3. The pressure gradient force and the Corolis force balance each other $\Rightarrow$ balance motion.

$\Rightarrow$ The geostrophic wind closely describes the horizontal wind above the boundary layer!

(Other) Balanced flows

When geostrophic balance does
not hold, then the Rossby number $R_0 \geq 1$:

- **Gradient wind balance** ($R_0 \sim 1$)

Pressure gradient force, Coriolis force and Centrifugal force play a role.

- **Cyclostrophic wind balance** ($R_0 > 1$)

Pressure gradient force and Centrifugal force play a role.

Thermal wind:

$$\frac{\partial \boldsymbol{u}_g}{\partial z} = \frac{\alpha g}{f} \hat{\boldsymbol{z}} \times \nabla T \qquad \text{Eq. 7-18}$$

$$\begin{cases} \dfrac{\partial u_g}{\partial z} = u_g(z+\Delta z) - u_g(z) \approx -\dfrac{ag}{f}\dfrac{\partial T}{\partial y} \\ \dfrac{\partial v_g}{\partial z} = v_g(z+\Delta z) - v_g(z) \approx \dfrac{ag}{f}\dfrac{\partial T}{\partial x} \end{cases}$$

α: Thermal expansion coefficient (Ch. 4)
g: Gravity acceleration

Geostrophic wind:

$$\boldsymbol{u}_g = \frac{1}{f\rho} \hat{\boldsymbol{z}} \times \nabla p \qquad \text{Eq. 7-3}$$

$$\boldsymbol{u}_g = \frac{g}{f} \hat{\boldsymbol{z}}_p \times \nabla_p z \qquad \text{Eq. 7-7}$$

$$\begin{cases} u_g = -\dfrac{1}{f\rho}\dfrac{\partial p}{\partial y} \\ v_g = \dfrac{1}{f\rho}\dfrac{\partial p}{\partial x} \end{cases}$$

Compare:
- **Thermal wind** is determined by the **horizontal temperature gradient**,
- **geostrophic wind** through the **horizontal pressure gradient**.

Surface (friction) wind

Low pressure (NH):
- anticlockwise flow
- often called "cyclone"

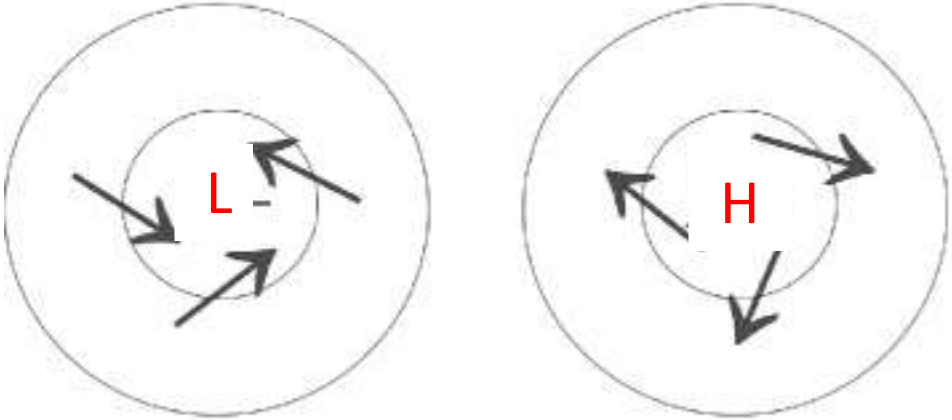


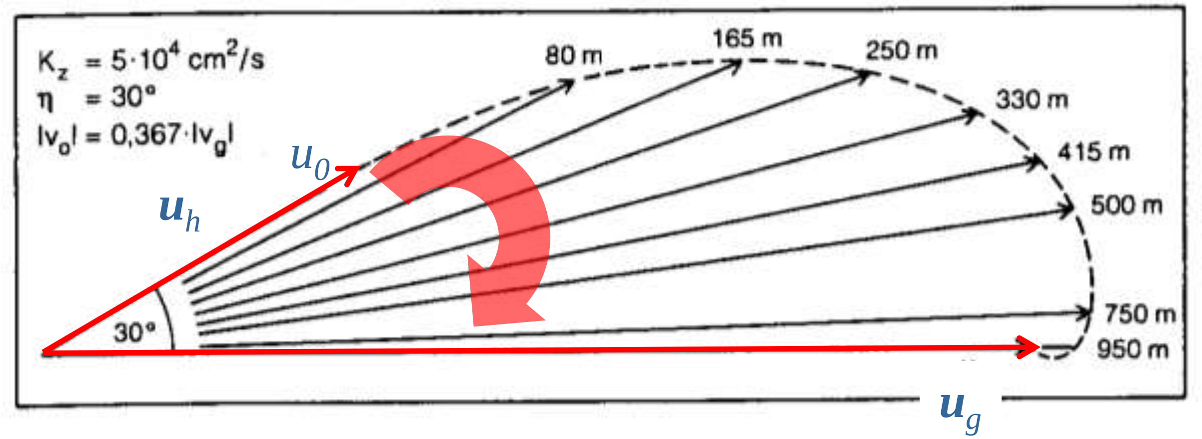
Figure 7.24: Flow spiraling in to a low-pressure region (left) and out of a high-pressure region (right) in a bottom Ekman layer. In both cases the ageostrophic flow is directed from high pressure to low pressure, or down the pressure gradient.

Marshall and Plumb (2008)

High pressure (NH):
- clockwise flow
- often called "anticyclone"

The Ekman spiral – theory

Ekman spiral: simplified theoretical calculation

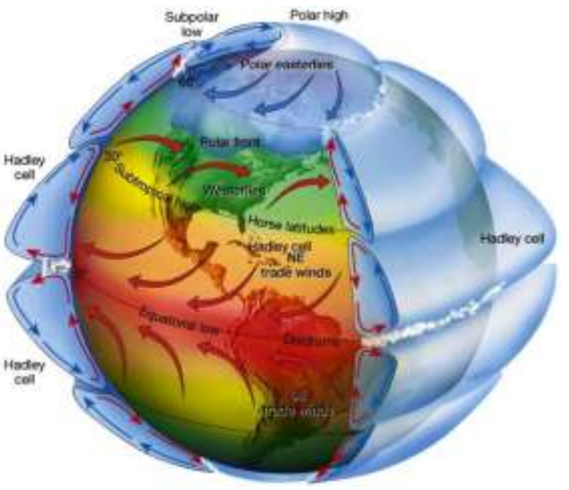


Roedel, 1987

The Ekman spiral was calculated for the oceanic friction layer by the Swedish oceanographer in 1905. In 1906, a possible application in Meteorology was developed.

Lecture Outline – Ch. 8

Ch. 8 – The general circulation of the atmosphere



1. Motivation

2. Observed circulation*
   - 2.1 The tropical Hadley circulation
   - 2.2 The Intertropical Convergence Zone (ITCZ)*

3. Mechanistic view of the circulation
   - 3.1 The tropical Hadley circulation
   - 3.2 The extratropical circulation

4. Large-scale atmospheric energy and momentum budget

5. Summary

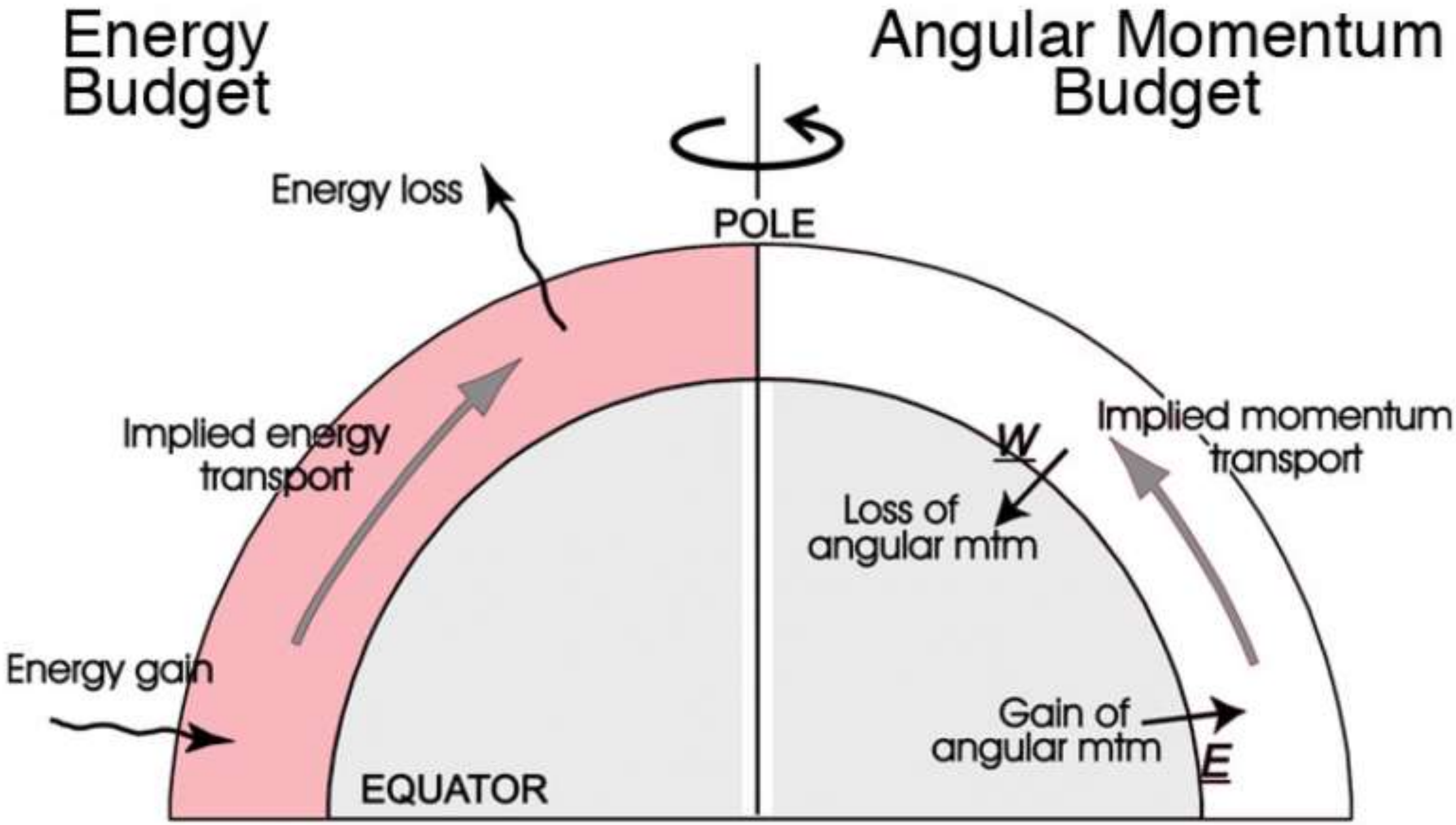
6. Take home message

*Add-ons, not in book.

Ch. 8 - Take home message

- Energy and momentum budgets demands on the GCA.
- Observed atmospheric winds and major climate zones can be explained by dynamic atmosphere on a longitudinal uniform, rotating Earth with latitudinal gradient of solar heating.
- However, distinct deviations on temporal and longitudinally variations exist.
- Geostrophic, hydrostatic and thermal wind balances together with conservation of angular momentum explain most of the observed wind patterns.

Energy and angular momentum budget



Marshall and Plumb (2008)

Can we explain the observed atmospheric circulation?

- Based on fluid dynamics,
- *simple* representation of the atmosphere,
- driven by latitudinal gradients in solar forcing?

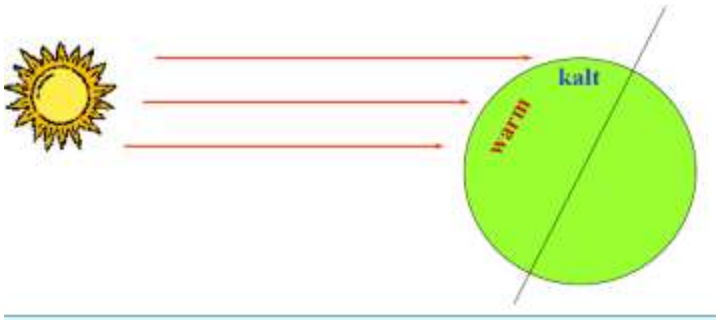
Neglect:

Temporal (seasons and diurnal variations) and surface (oceans, continents, mountains) variations.

Assume:

Atmosphere response to a **longitudinal uniform**, **rotating planet** (Earth) and to **latitudinal gradient of heating** (max at equator).

1.



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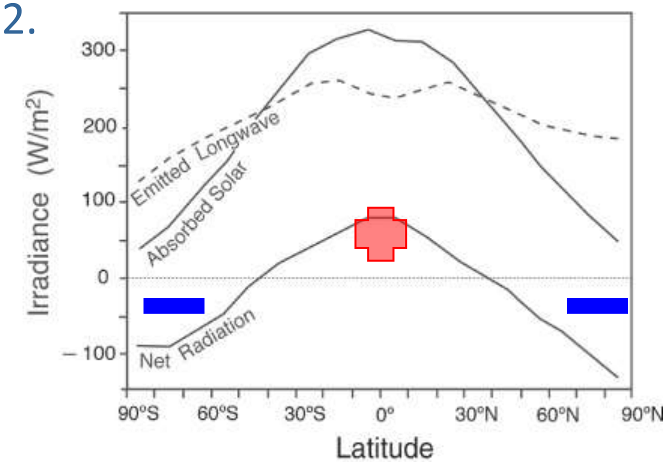
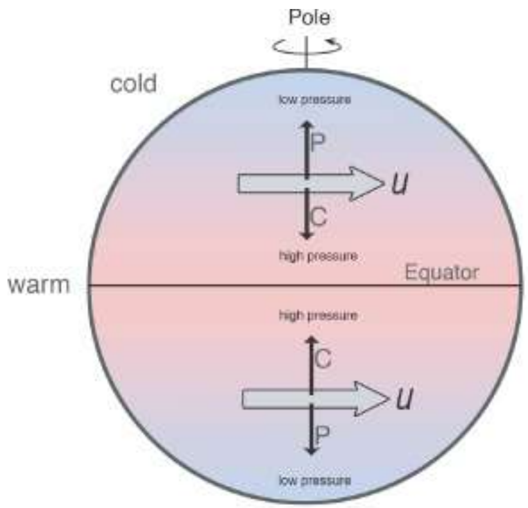


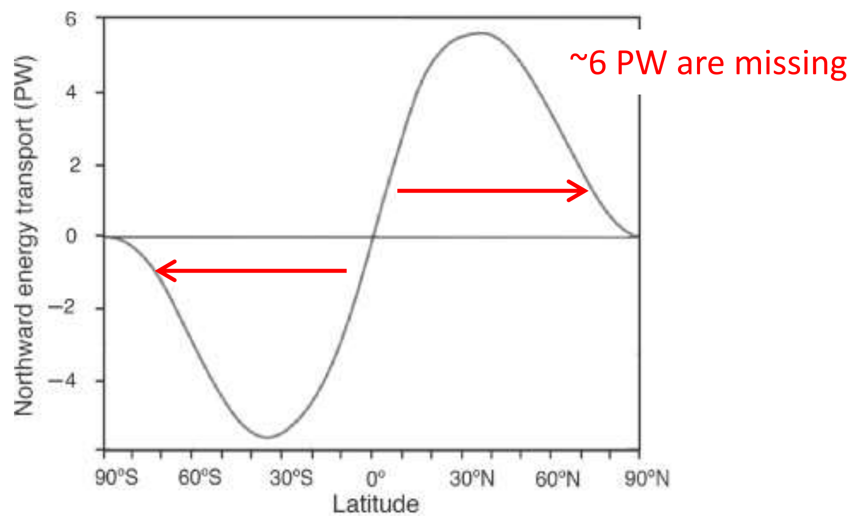
Surplus of radiation balance in the tropics and deficit in the polar regions!

3.



Horizontal temperature gradient > by hydrostatic balance > horizontal pressure gradient "P" balanced by Coriolis force "C" > geostrophic balance> westerly wind (U>0)

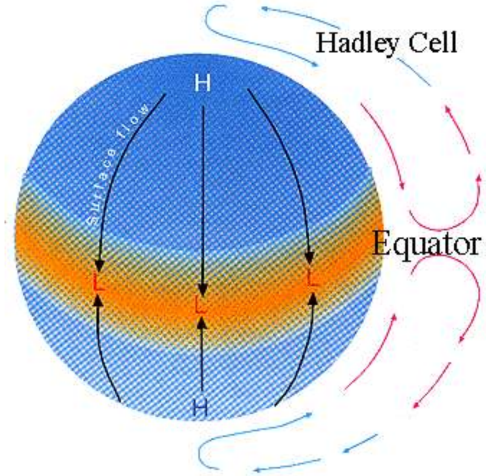
4.



Poleward energy transport

Marshall and Plumb (2008)

Effect of the rotating earth



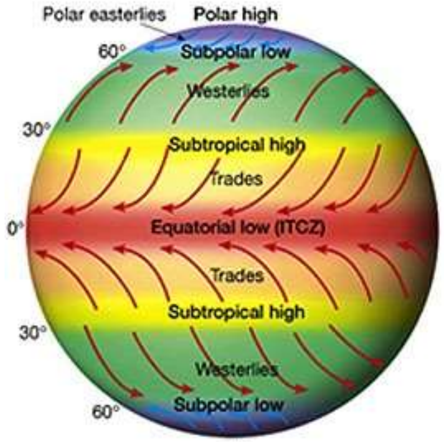
- **If the earth didn't rotate, we would have a single-cell circulation** in each hemisphere.

- But because in reality earth follows a movement on a rotating sphere, it develops a three cell circulation in each hemisphere:

  - Polar cell

  - Ferrel cell

  - Hadley cell



(a)

General circulation of the atmosphere (GCA)

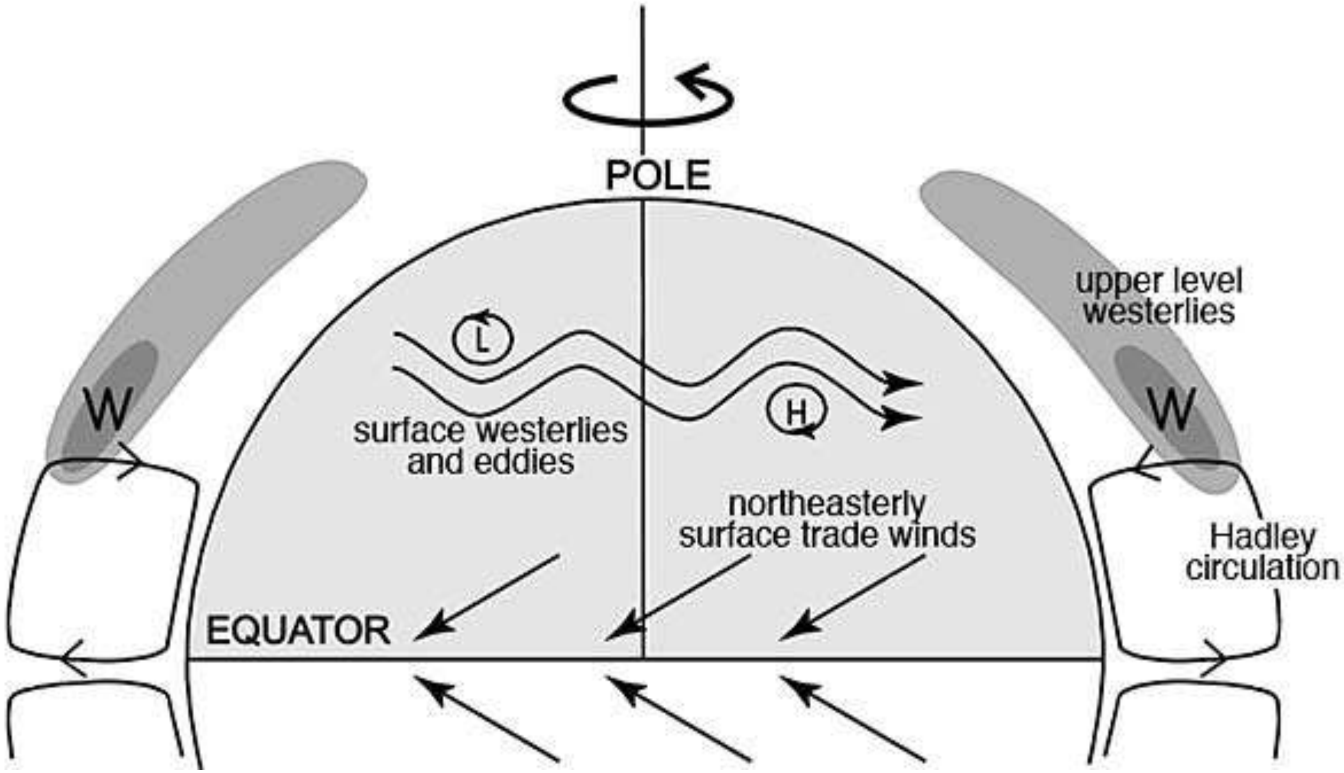
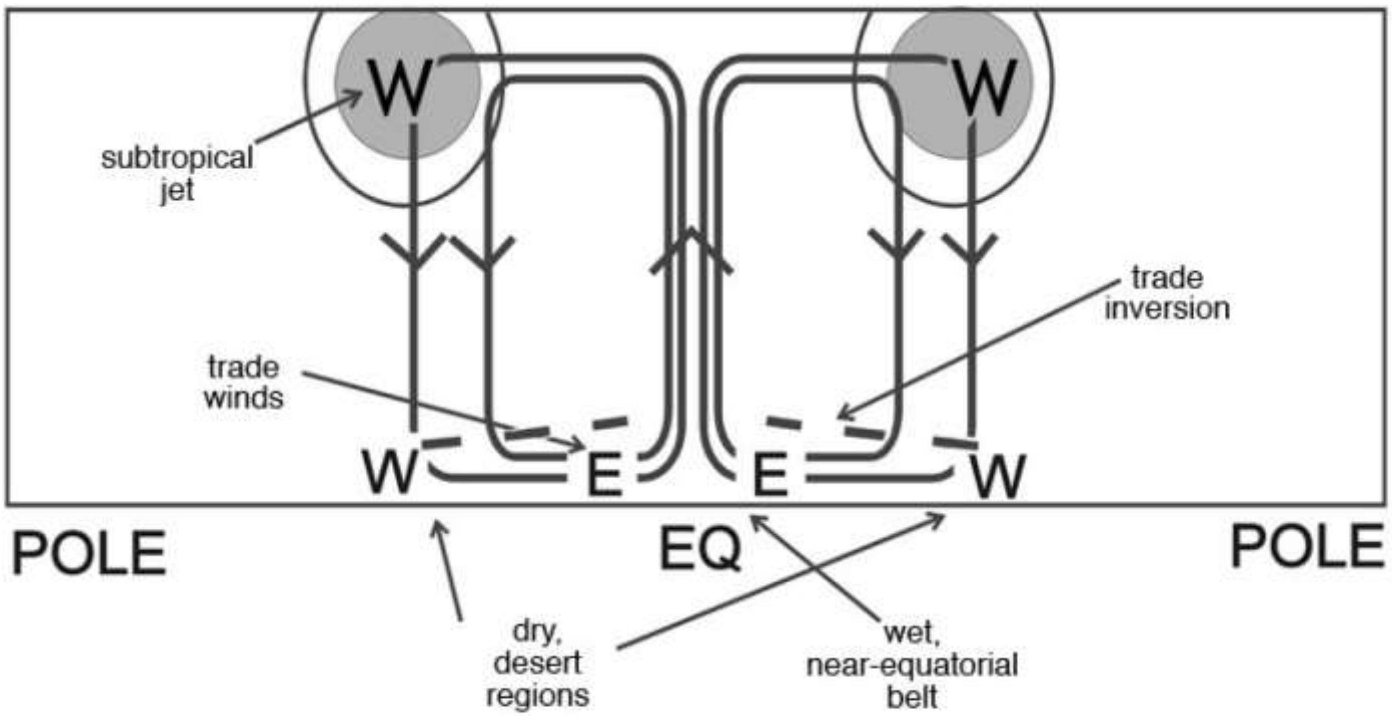


Figure 8.2: Schematic of the observed atmospheric general circulation for annual-averaged conditions. The upper level westerlies are shaded to reveal the core of the subtropical jet stream on the poleward flank of the Hadley circulation. The surface westerlies and surface trade winds are also marked, as are the highs and lows of middle latitudes. Only the northern hemisphere is shown. The vertical scale is greatly exaggerated.

Marshall and Plumb (2008)

Hadley circulation schematic



Marshall and Plumb, 2007

Extratropical circulation – Baroclinic instability

- Strong horizontal temperature gradient in mid-latitudes implies:

  - westerly wind increase with height (thermal wind balance Eq. 7-24)

  - pressure horizontal gradients and by geostrophic balance > **weak meridional circulation.**

But:

- Poleward heat transport required to balance energy budget, but how if Hadley Cell transports heat only up to subtropics?
- Daily observations tell us strong zonal asymmetries (low and high pressure systems)

  ⟶Thus the axisymmetric model can only partly be correct.

  ⟶Mid-latitudes is full of eddies (weather systems).

GCA – more complex

Polar High
circumpolar easterlies

subpolar low pressure channel

Polarfront
Westwinddrift

subtropical high pressure zone

northeastern trades

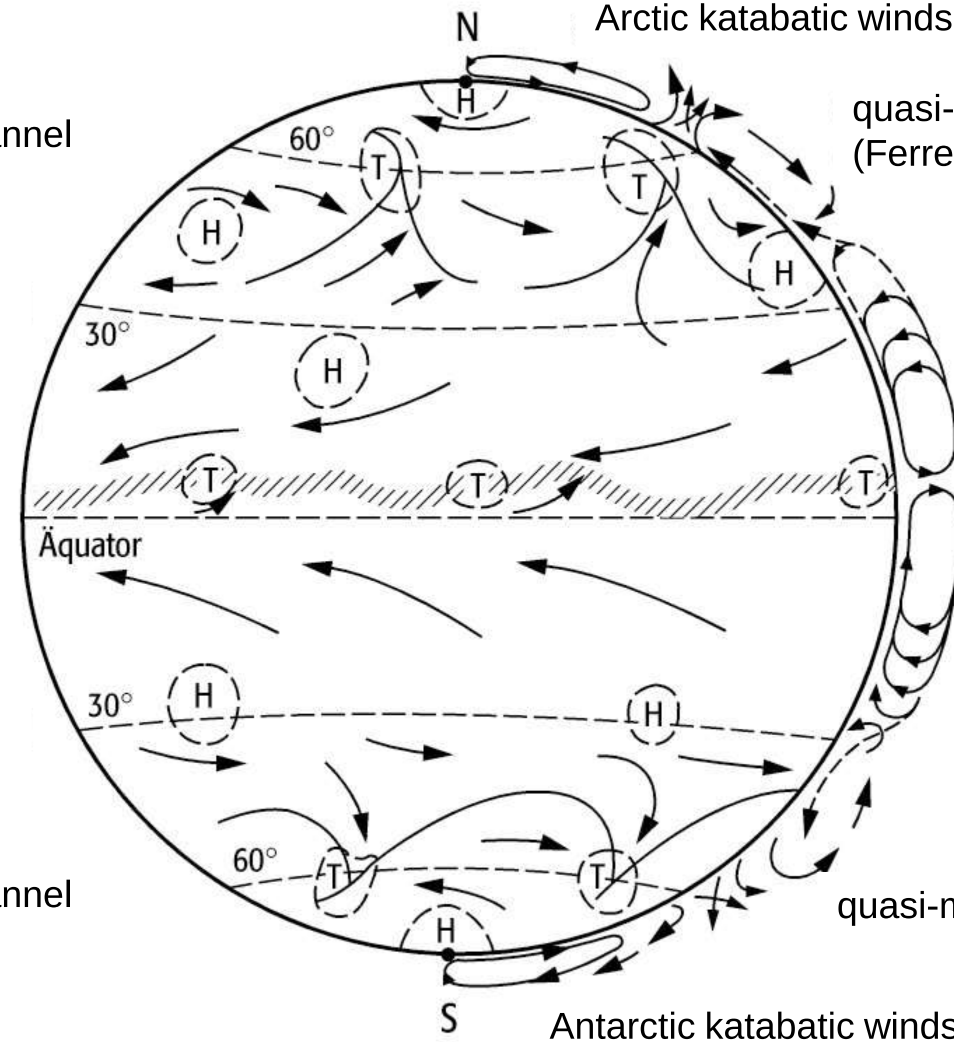
intertropical convergence
equatorial counter flow

southeastern trades

subtropical high pressure zone
Polarfront
Westwinddrift

subpolar low pressure channel

Polar High
circumpolar easterlies



Arctic katabatic winds

quasi-meridional circulation (Ferrel cell)

northern Hadley cell

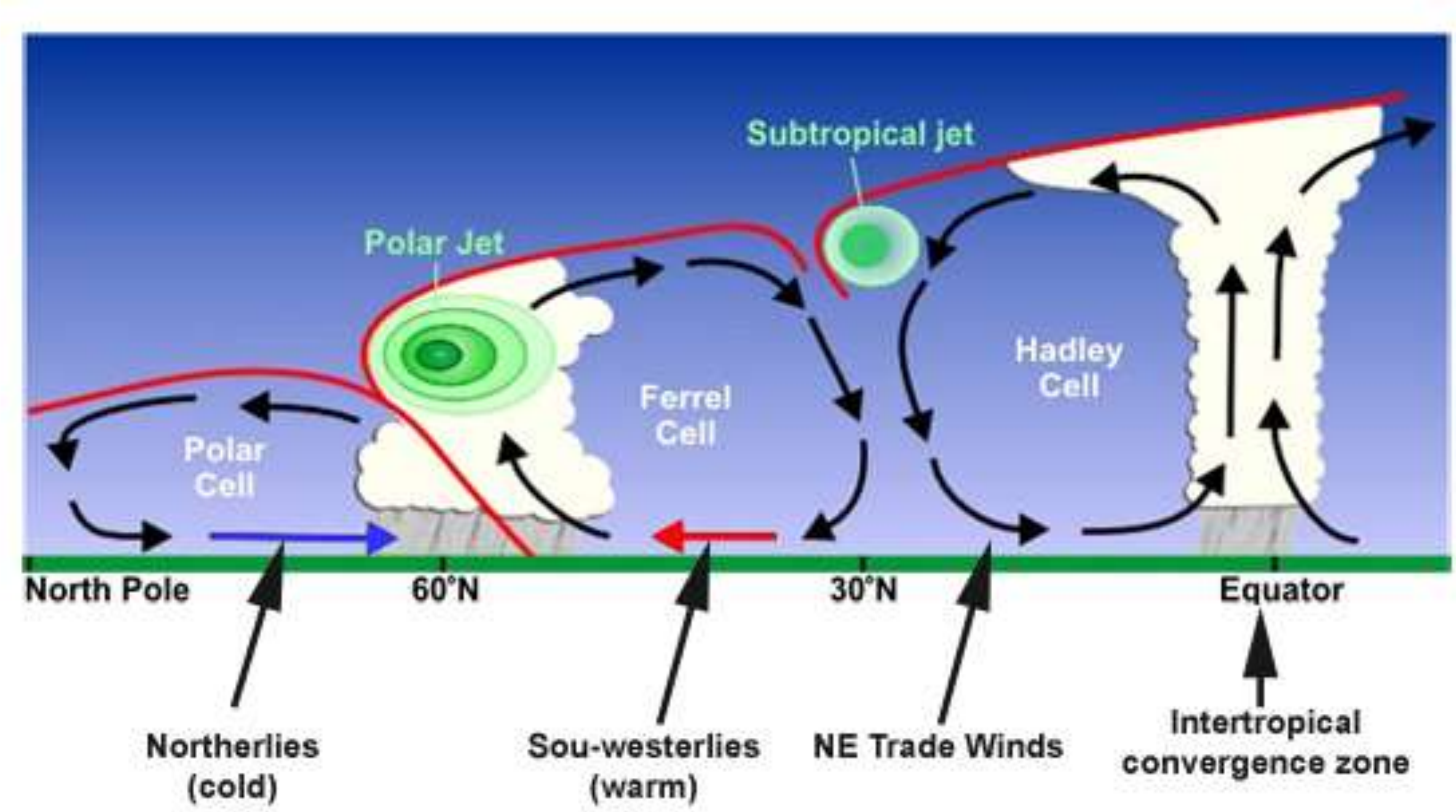
southern Hadley cell

quasi-meridional circulation

Antarctic katabatic winds

www.wissenschaft-online.de

GCA: vertical-meridional flow



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