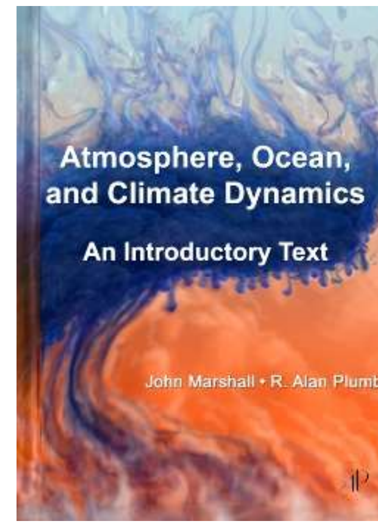
GEF 1100 – Klimasystemet

Chapter 7: Balanced flow



Prof. Dr. Kirstin Krüger (MetOs, UiO)

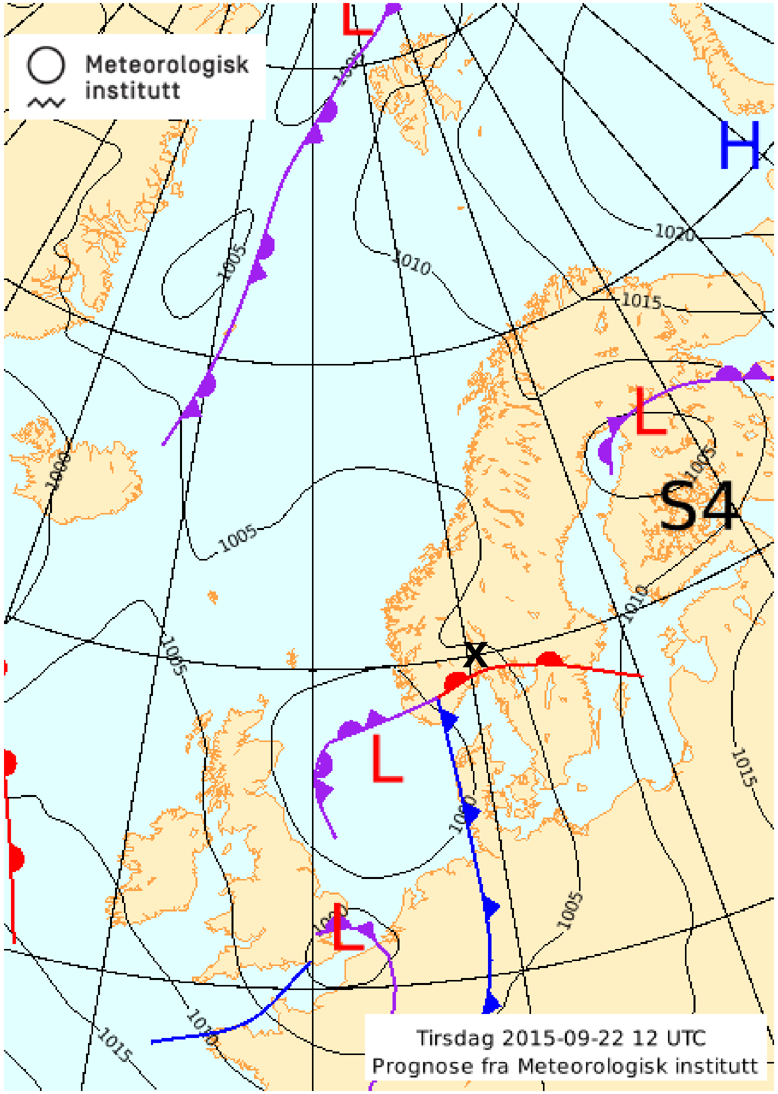
UiO : University of Oslo

Lecture Outline – Ch. 7

Ch. 7 – Balanced flow

**With add ons.*

Motivation



www.yr.no

x Oslo wind forecast for today 12–18 LT:

weak wind, 3 m/s, Northeast

S4 = lufttrykket har steget 4 hPa siste 3 timer

Scale analysis – geostrophic balance -1-

First consider magnitudes of the first 2 terms in momentum eq. for a fluid on a rotating sphere (Eq. 6-43 $\frac{D\mathbf{u}}{Dt} + f\,\hat{\mathbf{z}} \times \boldsymbol{u} + \frac{1}{\rho}\nabla p + \nabla\Phi = \mathcal{F}$) for horizontal components in a free atmosphere ($\mathcal{F}$=0,$\nabla\Phi$=0):

- $$\frac{D\mathbf{u}}{Dt} + f\hat{\mathbf{z}} \times \boldsymbol{u} = \underbrace{\frac{\partial u}{\partial t}}_{\frac{U}{T}\;\;10^{-4}} + \underbrace{\boldsymbol{u}\cdot\nabla\boldsymbol{u}}_{\frac{U^2}{L}\;\;10^{-4}} + \underbrace{f\hat{\mathbf{z}} \times \boldsymbol{u}}_{fU\;\;10^{-3}}$$

  Large-scale flow magnitudes in atmosphere:
  u and $v \sim \mathcal{U}$, $\mathcal{U}$~10 m/s, length scale: L ~10^6 m,
  time scale: $\mathcal{T}$ ~10^5 s, $\mathcal{U}$ / $\mathcal{T}$≈ $\mathcal{U}^2$/L ~10^{-4} ms^{-2},
  $f_{45°}$ ~ 10^{-4} s^{-1}

- Rossby number R_0: - Ratio of acceleration terms (U^2/L) to Coriolis term (f U),
  - R_0=U/f L
  - R_0≃0.1 for large-scale flows in atmosphere
  (R_0≃10^{-3} in ocean, Chapter 9)

Scale analysis – geostrophic balance -2-

- Coriolis term is left (≙ smallness of R_0) together with the pressure gradient term (~10^{-3}):

$$f\hat{\mathbf{z}} \times \boldsymbol{u} + \frac{1}{\rho}\nabla p = 0$$ Eq. 7-2 ***geostrophic balance***

- It can be rearranged to ($\hat{\mathbf{z}} \times \hat{\mathbf{z}} \times \boldsymbol{u} = -\boldsymbol{u}$):

$$\boldsymbol{u}_{\mathrm{g}} = \underbrace{\frac{1}{f\rho}\hat{\mathbf{z}} \times \nabla p}$$ Eq. 7-3 ***geostrophic flow***

$$\hat{\mathbf{z}} \times \nabla p = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & 1 \\ \frac{\partial p}{\partial x} & \frac{\partial p}{\partial y} & \frac{\partial p}{\partial z} \end{vmatrix} = \hat{x}\left(-\frac{\partial p}{\partial y}\right) + \hat{y}\left(\frac{\partial p}{\partial x}\right)$$

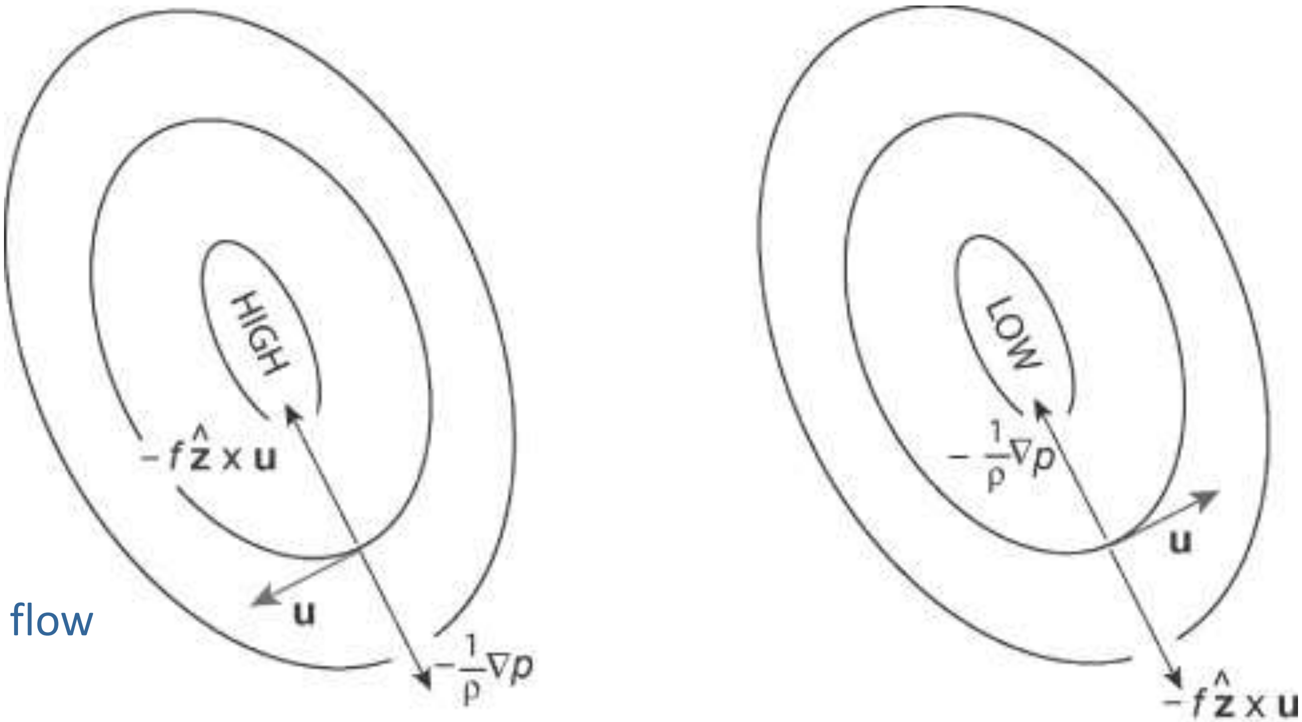
- In vector component form:

$$(u_g, v_g) = \left(-\frac{1}{f\rho}\frac{\partial p}{\partial y}, \frac{1}{f\rho}\frac{\partial p}{\partial x}\right)$$ Eq. 7-4

Note: For *geostrophic flow* the pressure gradient is balanced by the Coriolis term; to be approximately satisfied for flows of small R_0.

Geostrophic flow (NH: f>0)

High:
clockwise flow



Low:
anticlockwise flow

Figure 7.1: Geostrophic flow around a high pressure center (left) and a low pressure center (right). (Northern hemisphere case, $f > 0$.) The effect of Coriolis deflecting flow "to the right" (see Fig. 6.10) is balanced by the horizontal component of the pressure gradient force, $-1/\rho \nabla p$, directed from high to low pressure.

Marshall and Plumb (2008)

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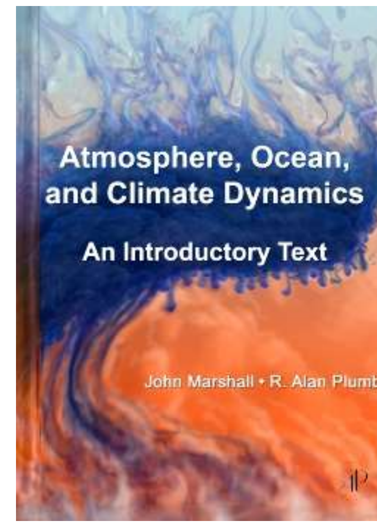
Chapter 7: Balanced flow



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Lecture Outline – Ch. 7

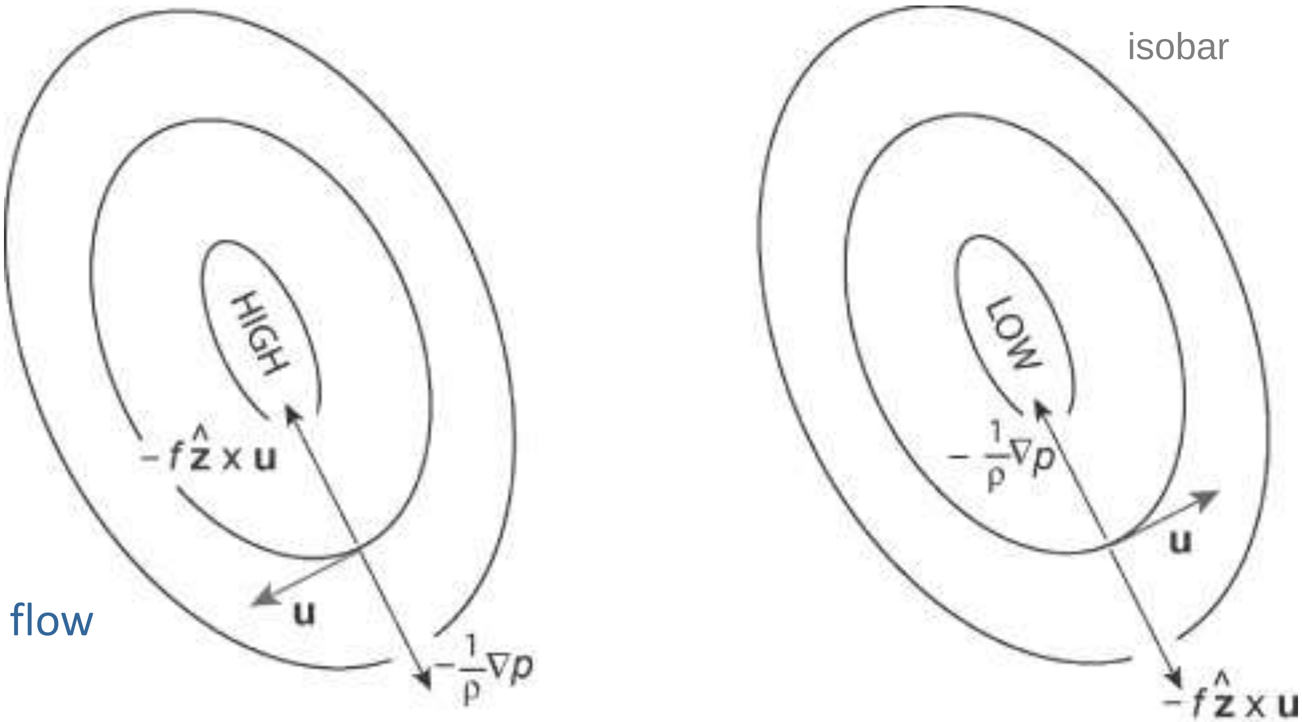
Ch. 7 – Balanced flow

1. Motivation
2. Geostrophic motion
    - 2.1 Geostrophic wind
    - 2.2 Synoptic charts
    - 2.3 Balanced flows
3. Thermal wind equation*
4. Subgeostrophic flow: The Ekman layer
    - 5.1 The Ekman layer*
    - 5.2 Surface (friction) wind
    - 5.3 Ageostrophic flow
5. Summary
6. Take home message

**With add ons.*

Geostrophic flow (NH: *f*>0)

High:
clockwise flow

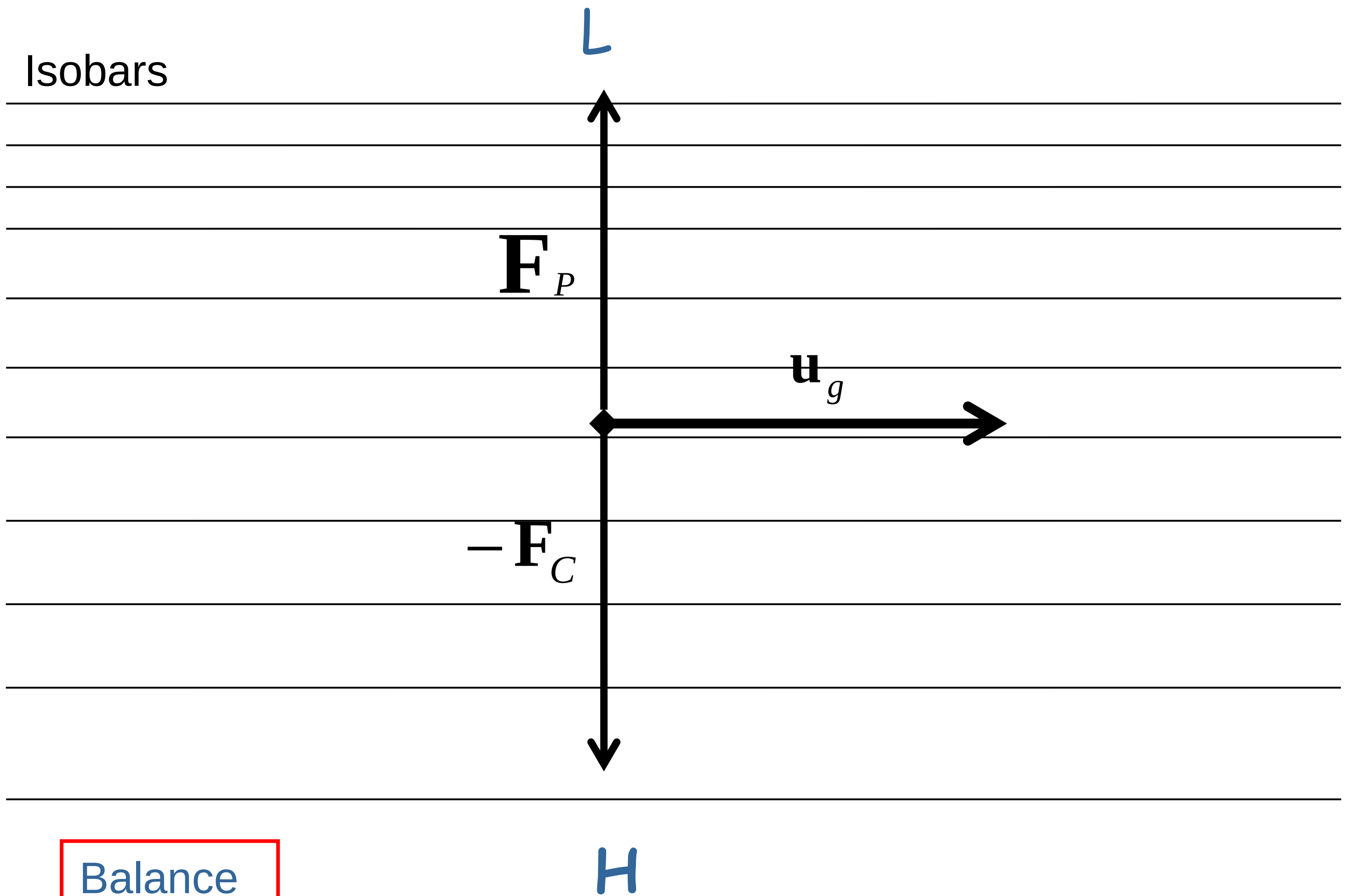


Low:
anticlockwise flow

Figure 7.1: Geostrophic flow around a high pressure center (left) and a low pressure center (right). (Northern hemisphere case, $f > 0$.) The effect of Coriolis deflecting flow "to the right" (see Fig. 6.10) is balanced by the horizontal component of the pressure gradient force, $-1/\rho\,\nabla p$, directed from high to low pressure.

Marshall and Plumb (2008)

Geostrophic wind



Balance between:

$\vec{F}_P = -\vec{F}_C$

$$\boldsymbol{u}_g = \frac{1}{f\rho}\hat{\boldsymbol{z}} \times \nabla p$$

Geostrophic flow

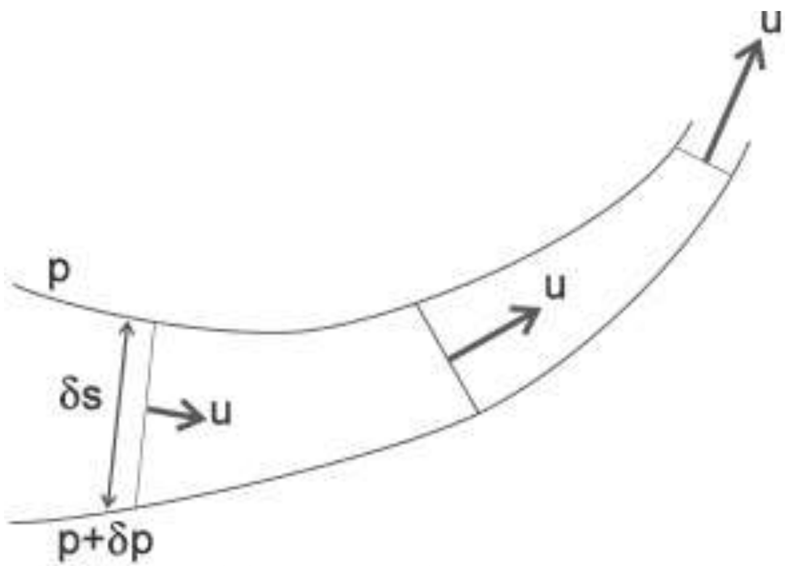


Figure 7.2: Schematic of two pressure contours (isobars) on a horizontal surface. The geostrophic flow, defined by Eq. 7-3, is directed along the isobars; its magnitude increases as the isobars become closer together.

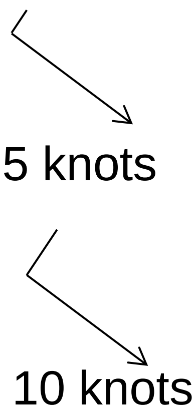
$$|\boldsymbol{u}_g| = \frac{1}{f\rho}|\nabla p| = \frac{1}{f\rho}\frac{\delta p}{\delta s}$$

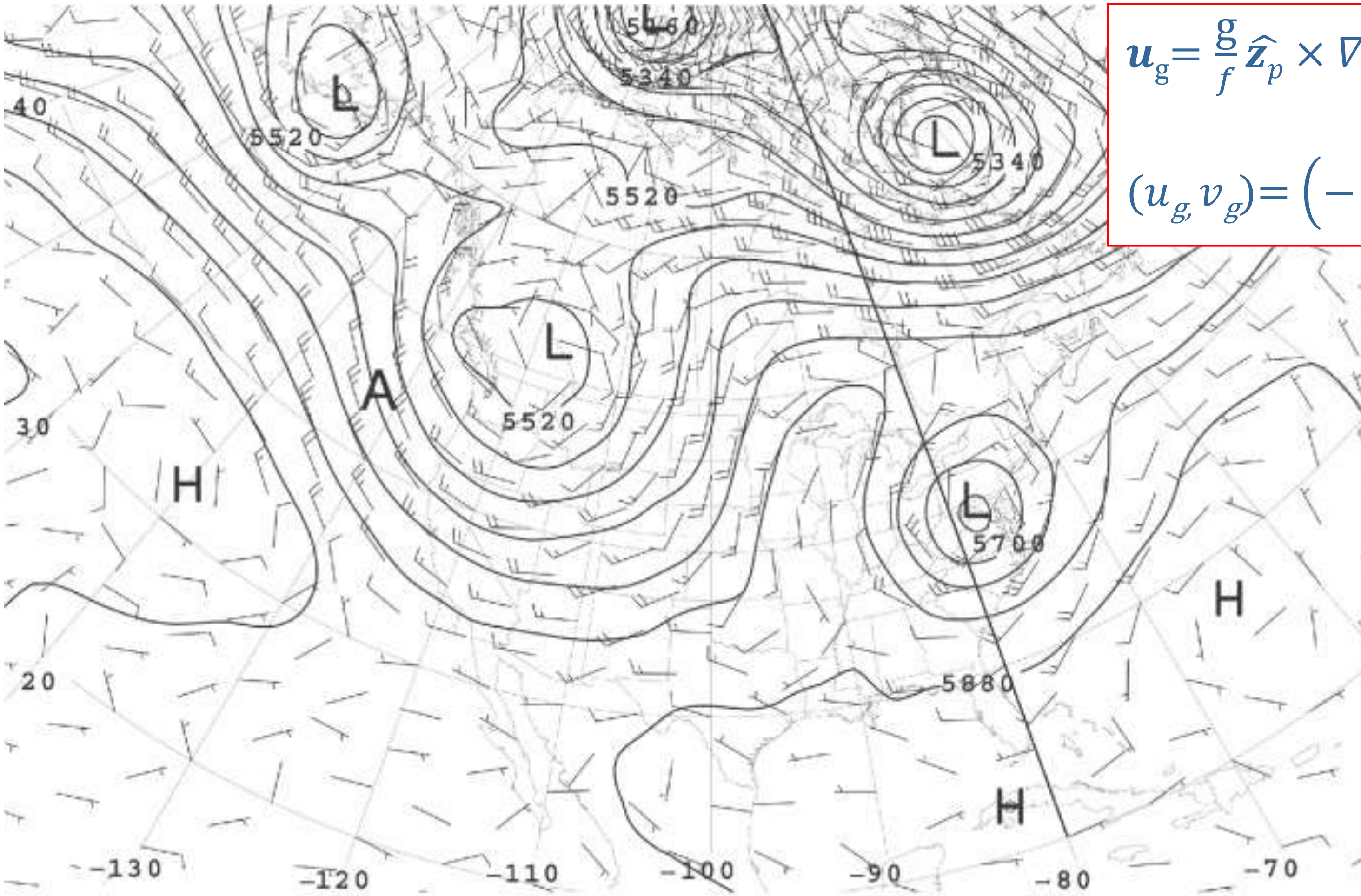
s: distance

Marshall and Plumb (2008)

Note: The geostrophic wind flows parallel to the isobars and is strongest where the isobars are closest (pressure maps > winds).

500 hPa wind and geopotential height (gpm), 12 GMT June 21, 2003 over USA





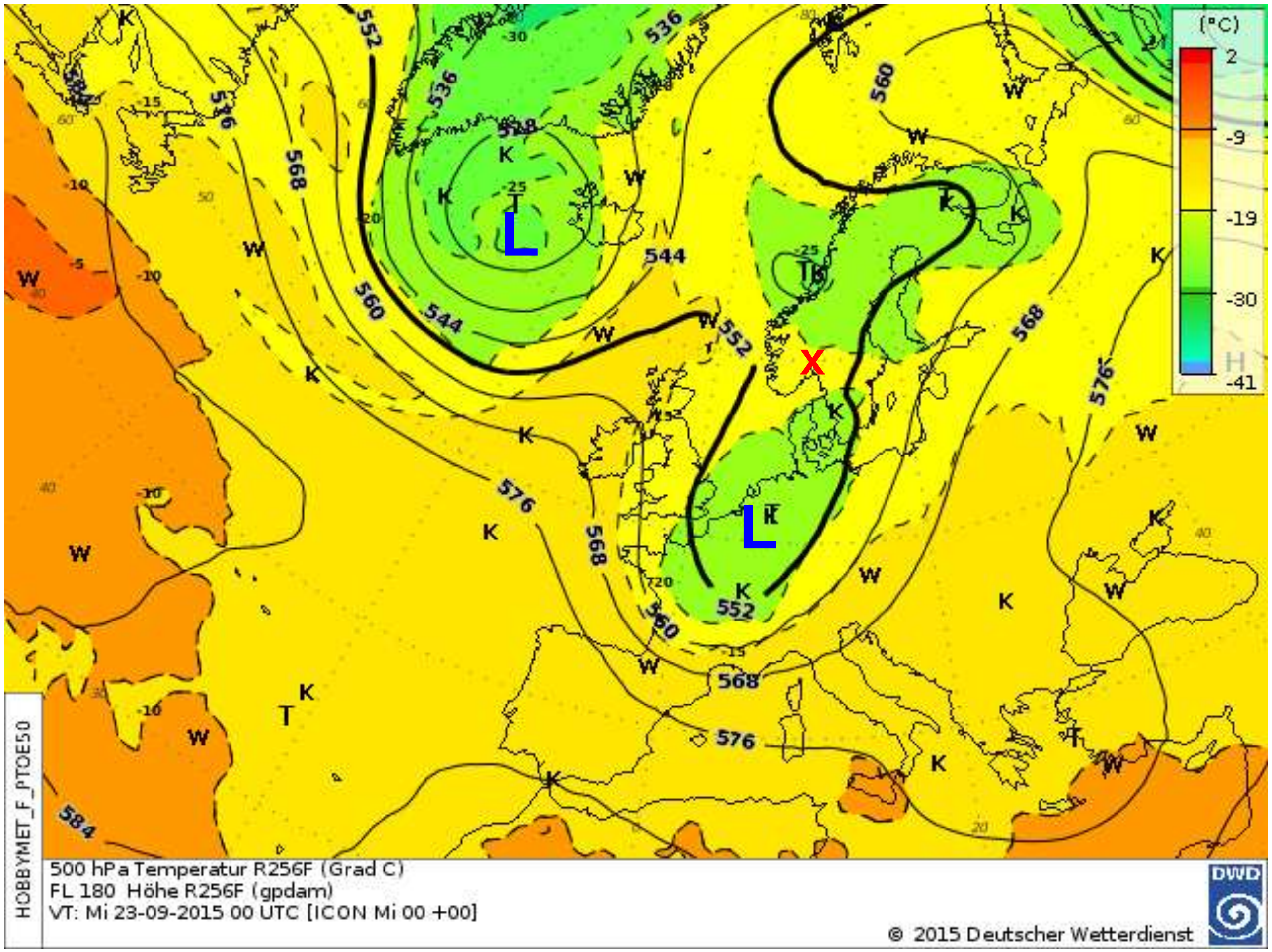
$$\boldsymbol{u}_g = \frac{g}{f}\hat{\boldsymbol{z}}_p \times \nabla_p z$$

$$(u_g, v_g) = \left(-\frac{g}{f}\frac{\partial z}{\partial y}, \frac{g}{f}\frac{\partial z}{\partial x}\right)$$

Note: Geopotential height ($z_{GH} \approx z$; *see Chapter 5 Eq. 5.5*) contours are streamlines of the geostrophic flow on pressure surfaces; geostrophic flow streams along *z* contours (as *p* contours on *height* surfaces) (Fig. 7-4).

Marshall and Plumb (2008)

Today's 500 hPa GH (gpdam) map



x Oslo

www.dwd.de

Summary: The geostrophic wind $\boldsymbol{u}_g$

- Geostrophic wind: “wind on the turning earth“: Geo (Greek) “Earth“, strophe (Greek) “turning”.
- The friction force and centrifugal forces are negligible.
- Balance between pressure gradient and Coriolis force
  → **geostrophic balance**
- The geostrophic wind is a horizontal wind.
- It displays a good approximation of the horizontal wind in the free atmosphere; not defined at the equator:
  - Assumption: $u=u_g$ $v=v_g$

$$\Rightarrow \boldsymbol{u_g} = \frac{1}{f\rho}\hat{\mathbf{z}} \times \nabla p \qquad u_g = -\frac{1}{f\rho}\frac{\partial p}{\partial y} \quad v_g = \frac{1}{f\rho}\frac{\partial p}{\partial x}$$

ρ: density
$f = 2\,\Omega \sin\phi$;
at equator $f = 0$
at Pole f is maximum

The geostrophic wind $\mathbf{u}_g$

- **Geostrophic wind balance (GWB)**:

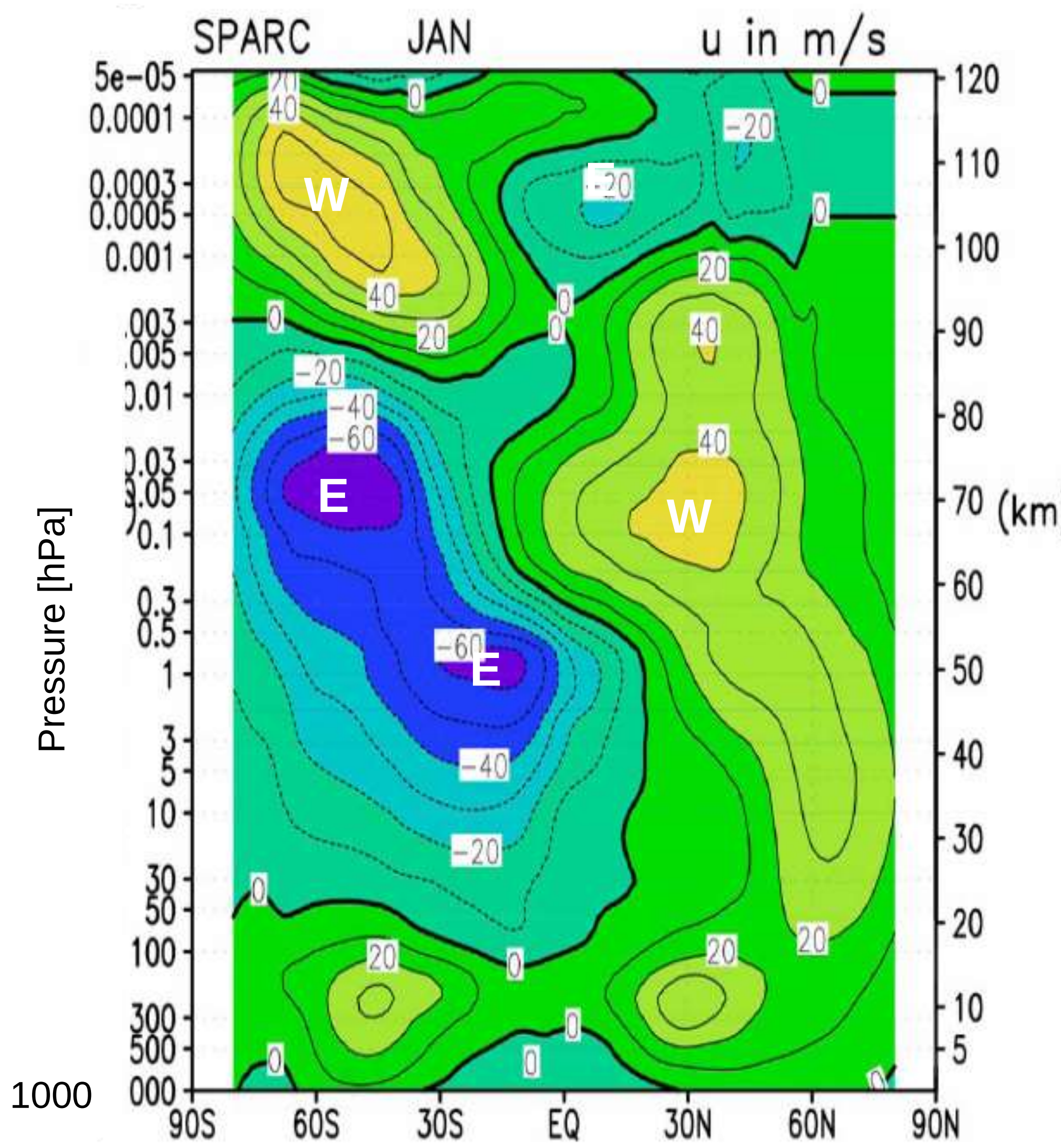
  Simultaneous wind and pressure measurements in the open atmosphere show that mostly the GWB is fully achieved.

  Advantage: The wind field can be determined directly from the pressure or height fields.

  Disadvantage: The horizontal equation of motion becomes purely diagnostic; stationary state.

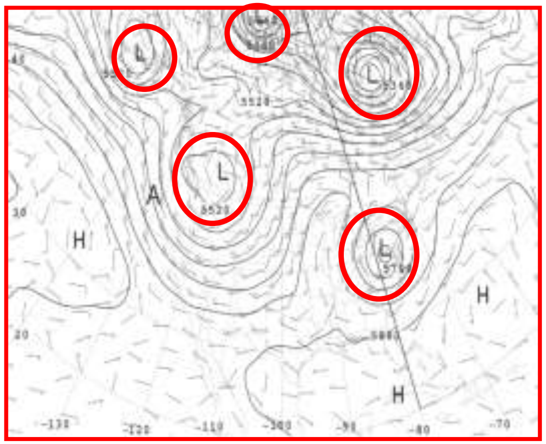
Zonal mean **zonal wind** u (m/s)

January average
SPARC climatology



Randel et al 2004

Rossby number @500hPa, 12 GMT June 21, 2003



$R_0 > 0.1$

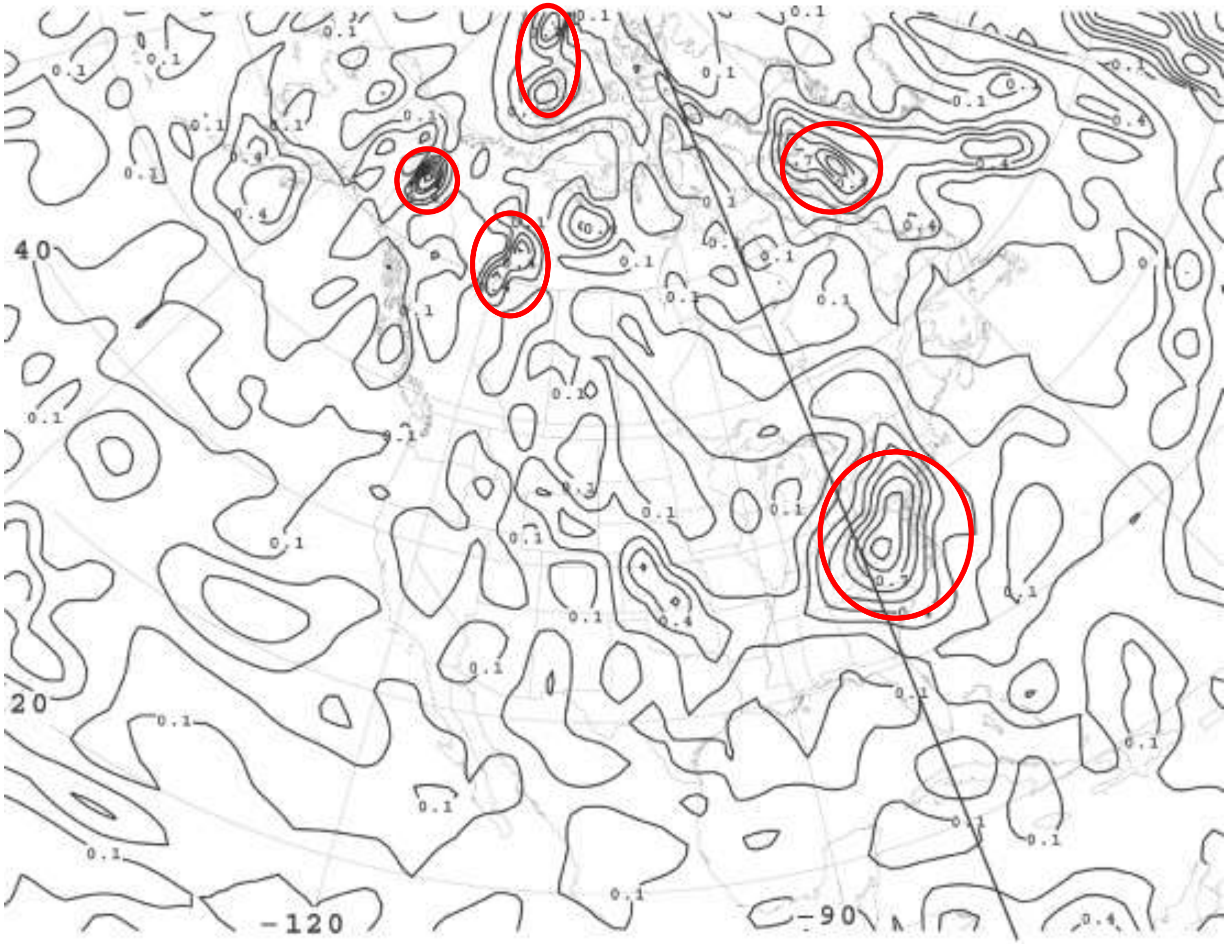


Figure 7.5: The Rossby number for the 500-mbar flow at 12 GMT on June 21, 2003, the same time as Fig. 7.4. The contour interval is 0.1. Note that Ro ~ 0.1 over most of the region but can approach 1 in strong cyclones, such as the low centered over 80° W, 40° N.

Marshall and Plumb (2008)

(Other) Balanced flows

When geostrophic balance does not hold, then $R_0 \geq 1$:

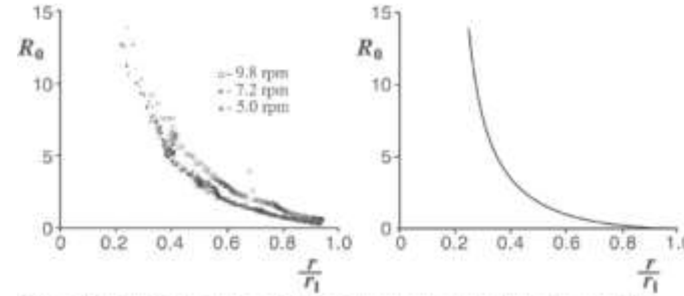


Figure 7.6: Left: The R_o number plotted as a function of nondimensional radius (r/r_1) computed by tracking particles in three radial inflow experiments (each at a different rotation rate, quoted here in revolutions per minute [rpm]). Right: Theoretical prediction based on Eq. 7-12.

- **Gradient wind balance** ($R_0 \sim 1$)

Pressure gradient force, Coriolis force and Centrifugal force play a role.

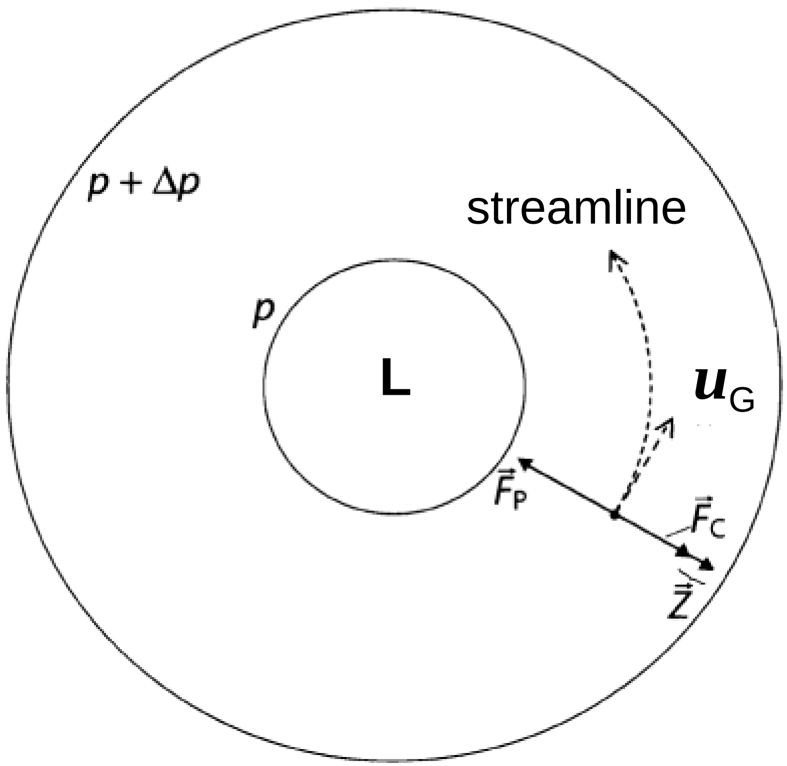
- **Cyclostrophic wind balance** ($R_0 > 1$)

Pressure gradient force and Centrifugal force play a role.

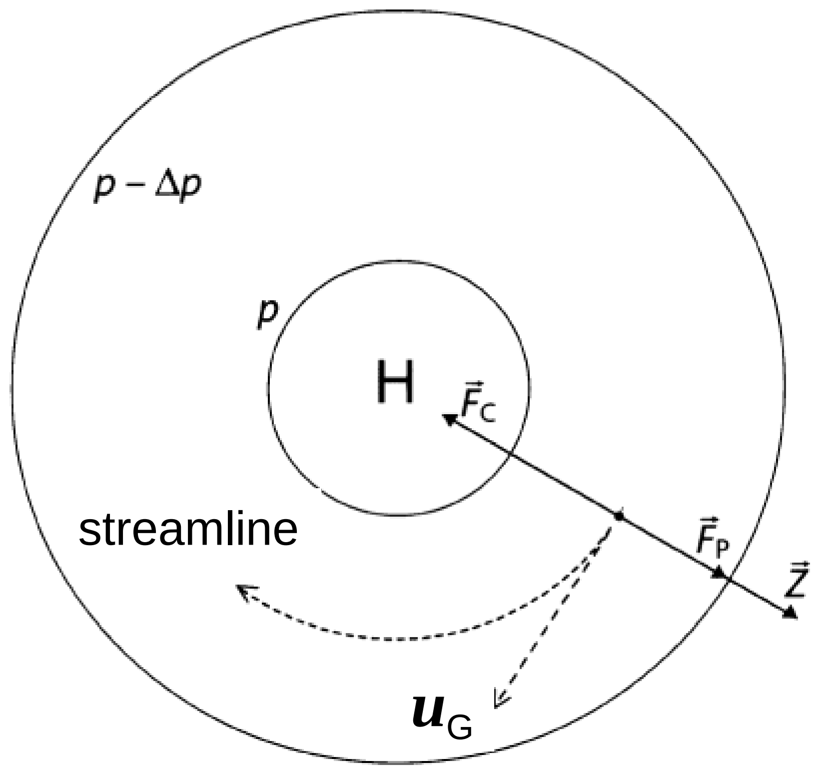
Marshall and Plumb (2008)

Gradient wind $\boldsymbol{u}_G$ – 2 cases with same $\boldsymbol{F}_p$

Case 1: sub-geostrophic



Case 2: super-geostrophic



Balance between the forces:

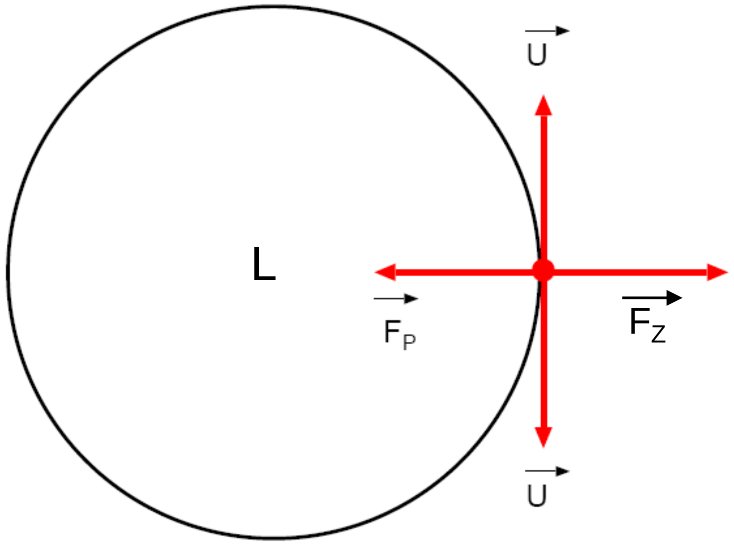
$$\vec{F}_{pressure} = -(\vec{F}_{Coriolis} + \vec{F}_{Z(centrifugal)})$$

Balance between the forces:

$$\vec{\mathbf{F}}_{pressure} + \vec{\mathbf{F}}_{Z(centrifugal)} = -\vec{\mathbf{F}}_{Coriolis}$$

Kraus (2004)

Cyclostrophic wind



Balance between:

$\mathbf{F}_P = -\mathbf{F}_Z$

- Physically appropriate solutions are orbits around a low pressure centre, that can run both cyclonically (counter clockwise) as well as anticyclonically (clockwise).
- The cyclostrophic wind appears in **small-scale whirlwinds** (dust devils, tornados).

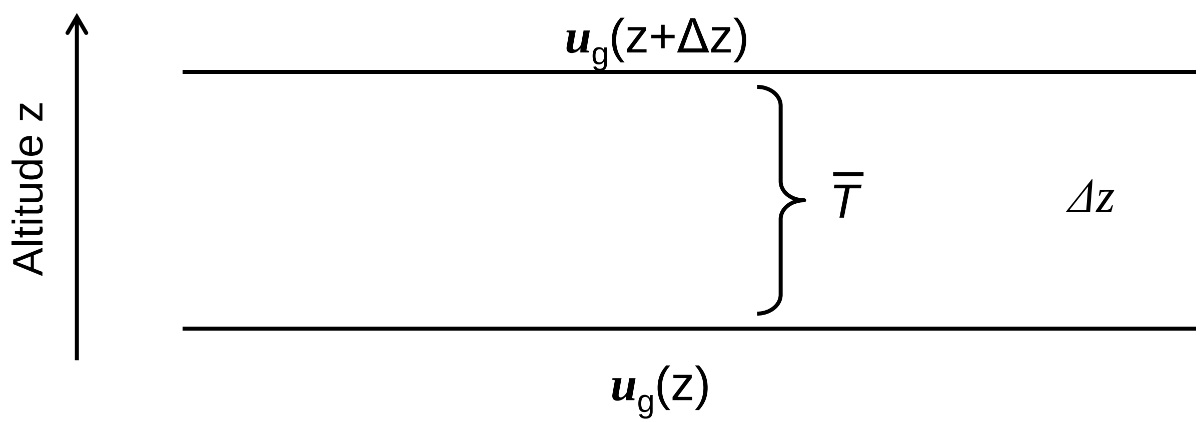
The thermal wind

Geostrophic wind change with altitude z?

→"Thermal wind" ($\Delta_z u_g$)

Applying geostrophic wind equation together with ideal gas law (Eq. 1-1 *p=ρRT*); insert finite vertical layer Δz* with average temperature $\overline{T}$, leads to:
*(*Δ: finite difference)*

$$\Delta_z \boldsymbol{u}_g = \frac{1}{\overline{T}}(\Delta_z T)\boldsymbol{u}_g + \frac{g}{f\overline{T}}\Delta z\, \hat{\mathbf{z}} \times \nabla_h \overline{T}$$



$\boldsymbol{u}_g$: geostrophic wind vector, *T:* temperature ρ: density *p:* pressure; R: gas constant for dry air, g: gravity acceleration, f: *Coriolis parameter, z: geometric height,* $\hat{\mathbf{z}}$: unit vector in z−direction

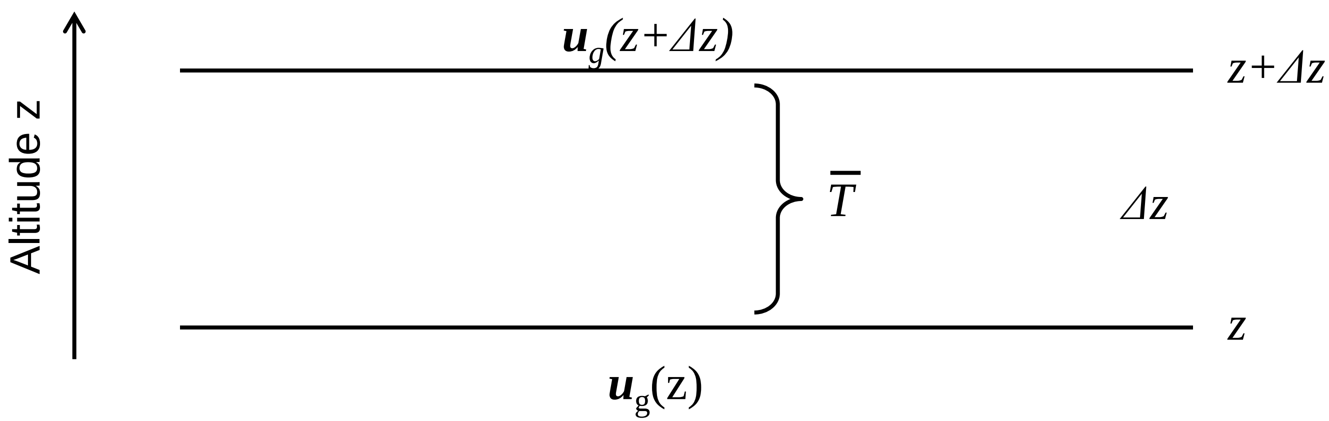
Thermal wind approximation

In many cases: 1) $\boldsymbol{u}_g$<10 m/s; or 2) strong horizontal temperature gradient; then the first term can be neglected.

$$\Delta_z \boldsymbol{u}_g = \cancel{\frac{1}{\overline{T}}(\Delta_z T)\boldsymbol{u}_g} + \frac{g}{f\overline{T}}\Delta z\, \hat{\mathbf{z}} \times \nabla_h \overline{T}$$

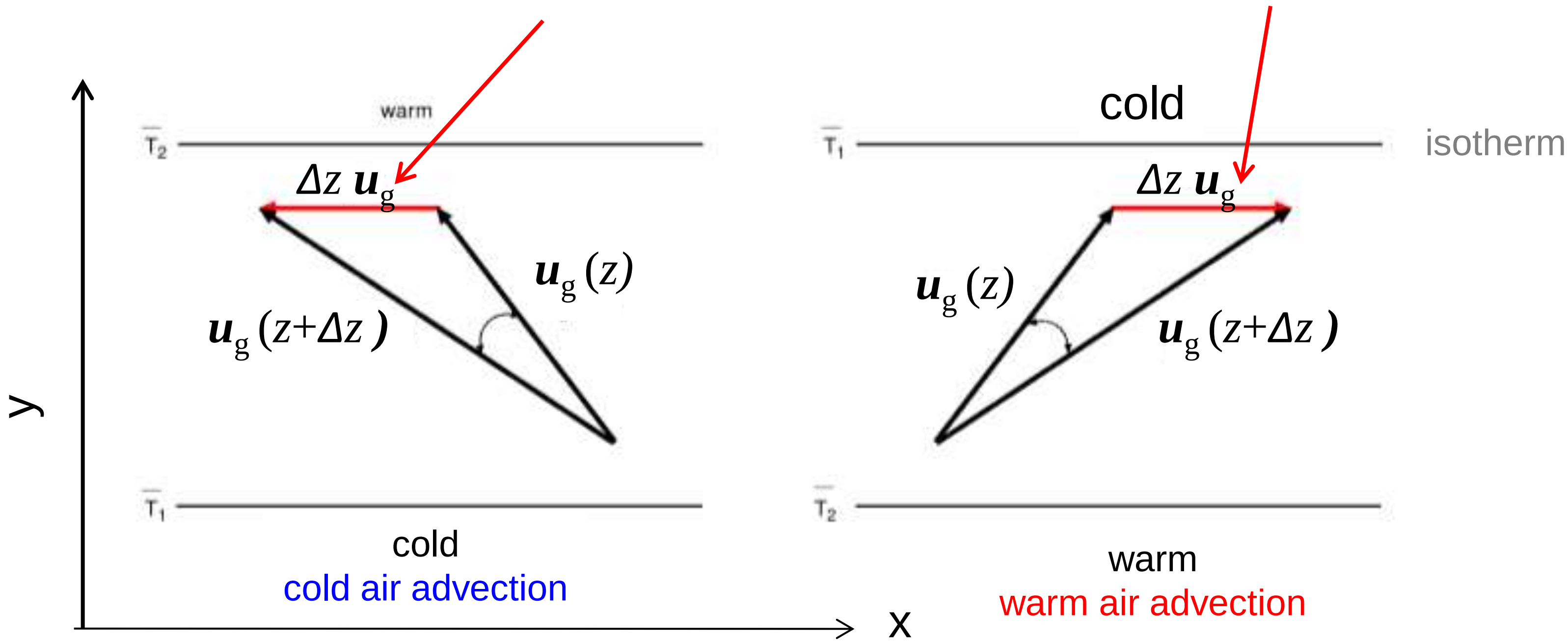
$$\Delta_z \mathbf{u}_g = \mathbf{u}_g(z+\Delta z) - \mathbf{u}_g(z) \approx \frac{g}{f\overline{T}}\Delta z\ \hat{\mathbf{z}} \times \nabla_h \overline{T}$$

$$\boxed{\Rightarrow \Delta_z \mathbf{u}_g \approx \hat{\mathbf{z}} \times \nabla_h \overline{T}}$$



Note: In this approximation, the wind change is always parallel to the average isotherms (lines of const. temperature).

Thermal wind approximation



Which rules can I deduce from the thermal wind approximation, if $u_g < 10\ m/s$?

- The geostr. wind change with height **is parallel to the mean isotherms**,
- whereby the **colder air remains to the left on the NH**,
- **geostrophic left rotation with increasing height for cold air advection**,
- **geostrophic right rotation with increasing height for warm air advection**.

Thermal wind:

$$\frac{\partial \boldsymbol{u}_g}{\partial z} = \frac{\alpha g}{f} \hat{\boldsymbol{z}} \times \nabla T \quad \text{Eq. 7-18}$$

$$\begin{cases} \dfrac{\partial u_g}{\partial z} = u_g(z+\Delta z) - u_g(z) \approx -\dfrac{ag}{f}\dfrac{\partial T}{\partial y} \\ \dfrac{\partial v_g}{\partial z} = v_g(z+\Delta z) - v_g(z) \approx \dfrac{ag}{f}\dfrac{\partial T}{\partial x} \end{cases}$$

α: Thermal expansion coefficient (Ch. 4)
g: Gravity acceleration

Geostrophic wind:

$$\boldsymbol{u}_g = \frac{1}{f\rho} \hat{\boldsymbol{z}} \times \nabla p \quad \text{Eq. 7-3}$$

$$\boldsymbol{u}_g = \frac{g}{f} \hat{\boldsymbol{z}}_p \times \nabla_p z \quad \text{Eq. 7-7}$$

$$\begin{cases} u_g = -\dfrac{1}{f\rho}\dfrac{\partial p}{\partial y} \\ v_g = \dfrac{1}{f\rho}\dfrac{\partial p}{\partial x} \end{cases}$$

Compare:
- **Thermal wind** is determined by the **horizontal temperature gradient**,
- **geostrophic wind** through the **horizontal pressure gradient**.

Thermal wind – examples

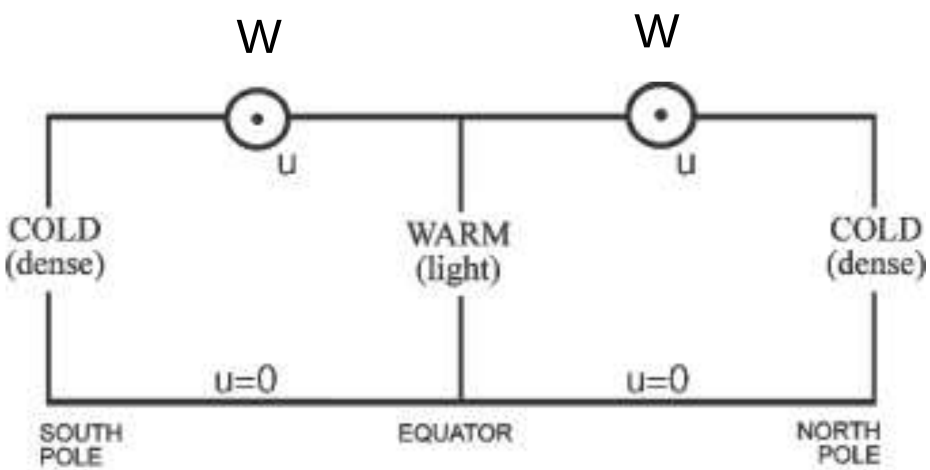
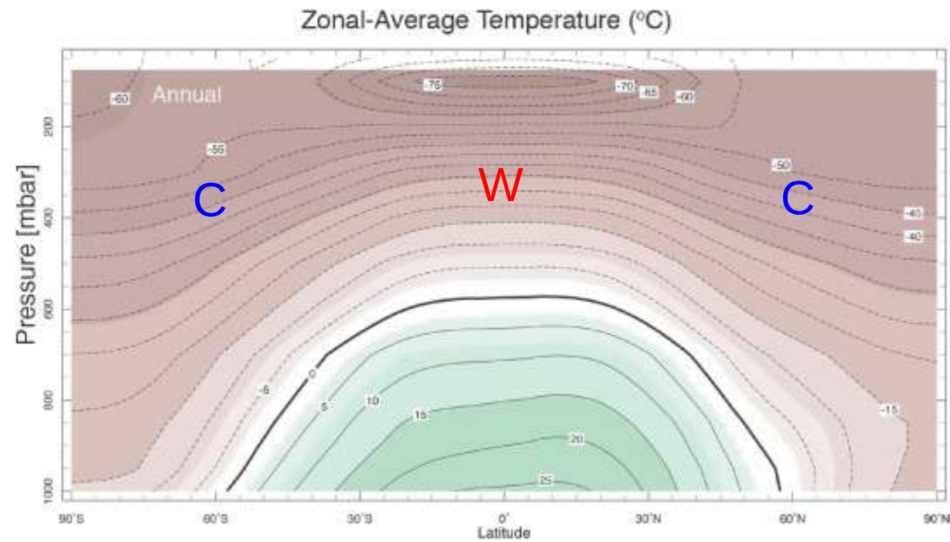
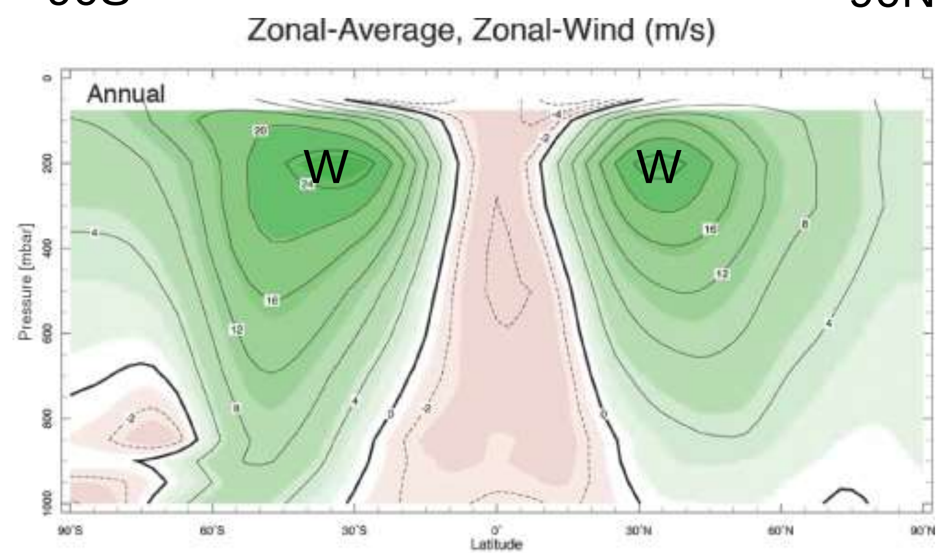


Figure 7.19: A schematic of westerly winds observed in both hemispheres in thermal wind balance with the equator-to-pole temperature gradient. (See Eq. 7-24 and the observations shown in Figs. 5.7 and 5.20.)



90S

90N



Marshall and Plumb (2008)

Temperature (°C) @500hPa, 12 GMT June 21, 2003

cold air
warm air

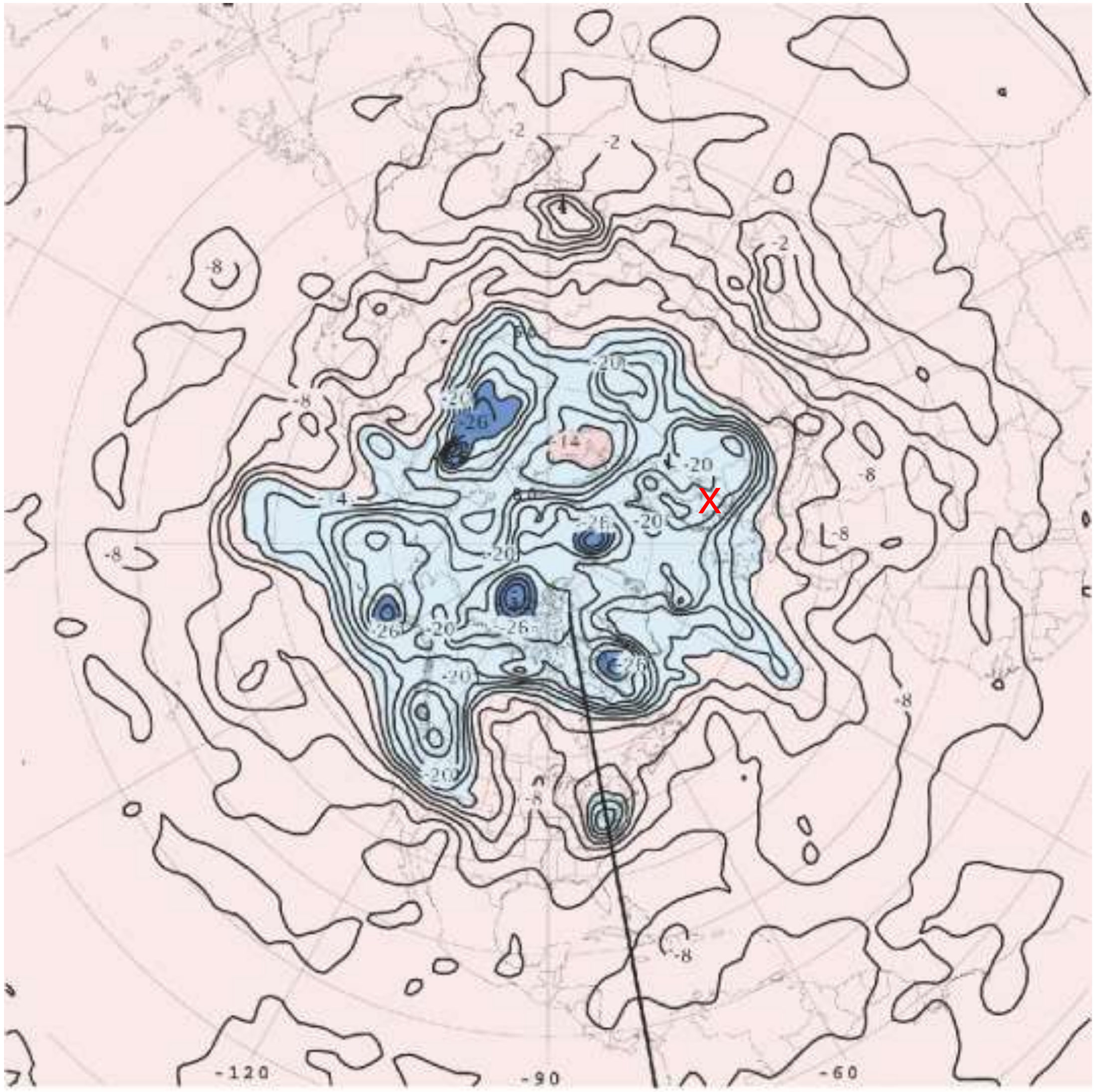


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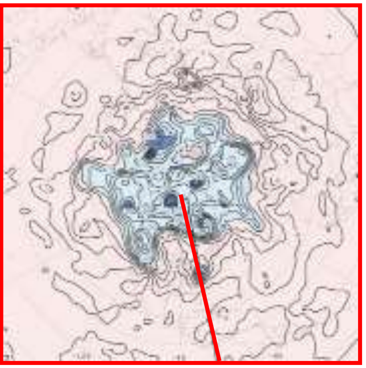
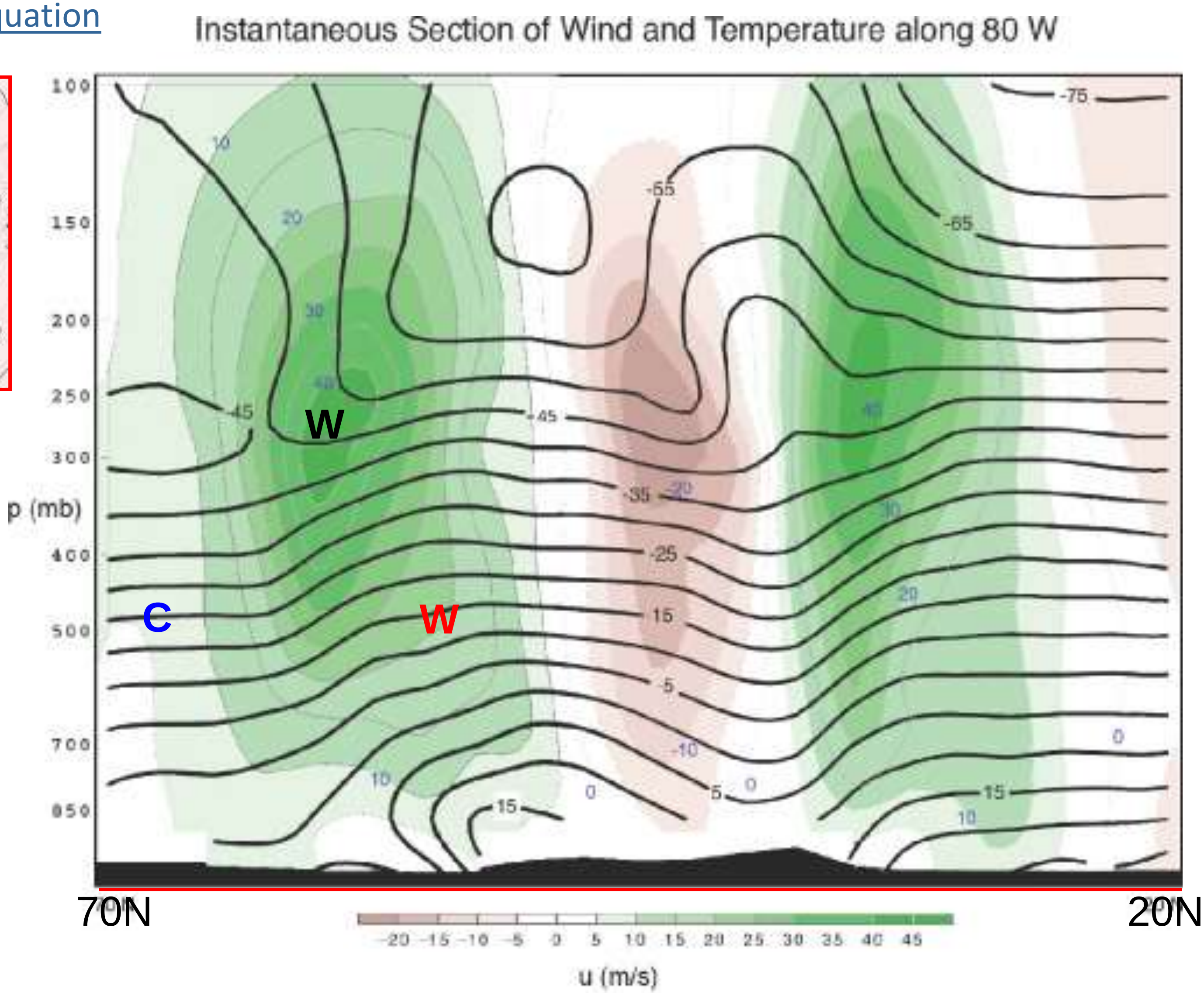
Figure 7.20: The temperature, T, on the 500-mbar surface at 12 GMT on June 21, 2003, the same time as Fig. 7.4. The contour interval is 2°C. The thick black line marks the position of the meridional section shown in Fig. 7.21 at 80°W extending from 20° N to 70° N. A region of pronounced temperature contrast separates warm air (pink) from cold air (blue). The coldest temperatures over the pole get as low as –32°C.

Marshall and Plumb (2008)

4. Thermal wind equation





Westerly wind

Easterly wind

Figure 7.21: A cross section of zonal wind, u (color-scale, green indicating away from us and brown toward us, and thin contours every 5 m s^{-1}), and temperature, T (thick contours every 5°C), through the atmosphere at 80° W, extending from 20° N to 70° N, on June 21, 2003, at 12 GMT, as marked on Figs. 7.20 and 7.4. Note that $\partial u/\partial p < 0$ in regions where $\partial T/\partial y < 0$ and vice versa.

Marshall and Plumb (2008)

Stratospheric polar vortex

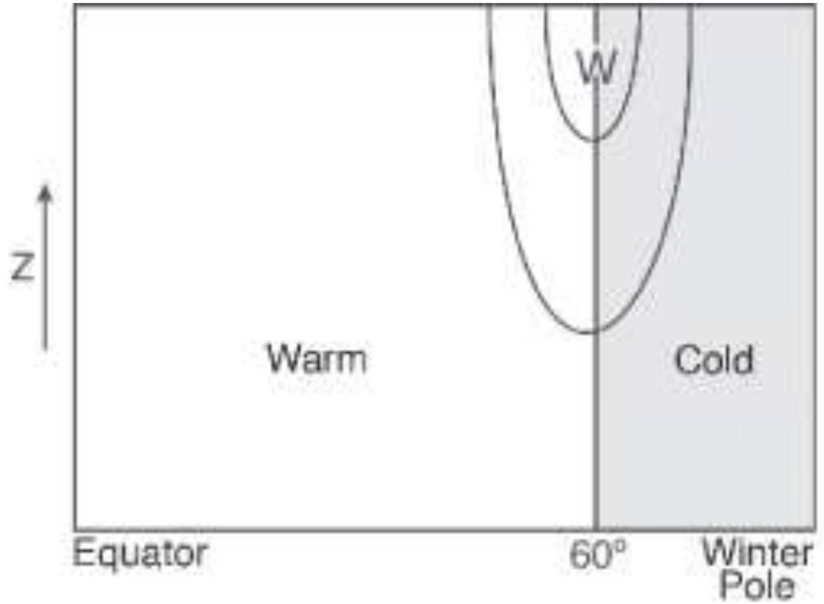


Figure 7.29: A schematic of the winter polar stratosphere dominated by the "polar vortex," a strong westerly circulation at ~60° around the cold pole.

Marshall and Plumb (2008)

Sub-geostrophic flow: The Ekman layer

- **Large scale flow** in **free atmosphere** and **ocean** is close to **geostrophic** and **thermal wind balance**.
- Boundary layers have large departures from geostrophy due to friction forces.

Ekman layer:
- Friction acceleration $\mathcal{F}$ becomes important,
- roughness of surface generates turbulence in the first ~1 km of the atmosphere,
- wind generates turbulence at the ocean surface down to first 100 meters.

Sub-geostrophic flow:
R_0 small, $\mathcal{F}$ exists, then horizontal component of momentum (geostrophic) balance (Eq. 7-2) can be written as:

$$f\hat{\boldsymbol{z}} \times \boldsymbol{u} + \frac{1}{\rho}\nabla p = \mathcal{F} \qquad \text{Eq. 7-25}$$

Surface (friction) wind

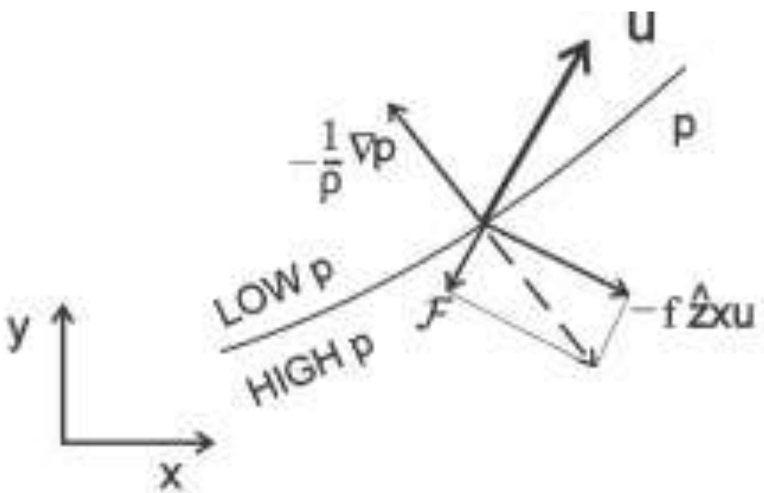


Figure 7.22: The balance of forces in Eq. 7-25: the dotted line is the vector sum $\mathcal{F} - f\hat{\mathbf{z}} \times \boldsymbol{u}$ and is balanced by $-1/\rho\,\nabla p$.

$$f\hat{\mathbf{z}} \times \boldsymbol{u} + \frac{1}{\rho}\nabla p = \mathcal{F} \qquad \text{Eq. 7-25}$$

Marshall and Plumb (2008)

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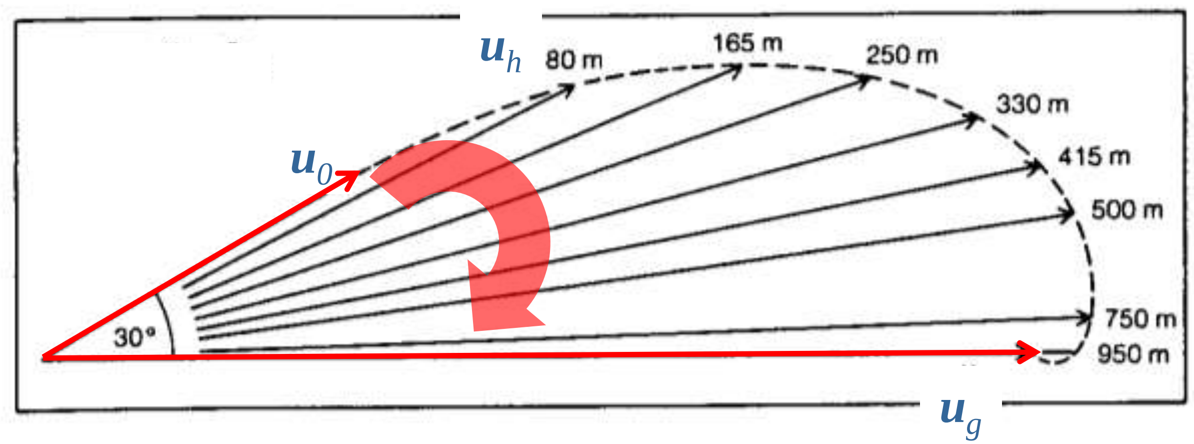
1. Start with $\boldsymbol{u}$
2. Coriolis force per unit mass must be to the right of $\boldsymbol{u}$
3. Frictional force per unit mass acts as a drag and must be opposite to $\boldsymbol{u}$.
4. Sum of the two forces (dashed arrow) must be balanced by the pressure gradient force per unit mass
5. Pressure gradient is not normal to the wind vector or the wind is no longer directed along the isobars, however the low pressure system is still on the left side but with a deflection.

=> The flow is sub-geostrophic (less than geostrophic); ageostrophic component directed from high to low.

The Ekman spiral – theory

Ekman spiral: simplified theoretical calculation



Roedel, 1987

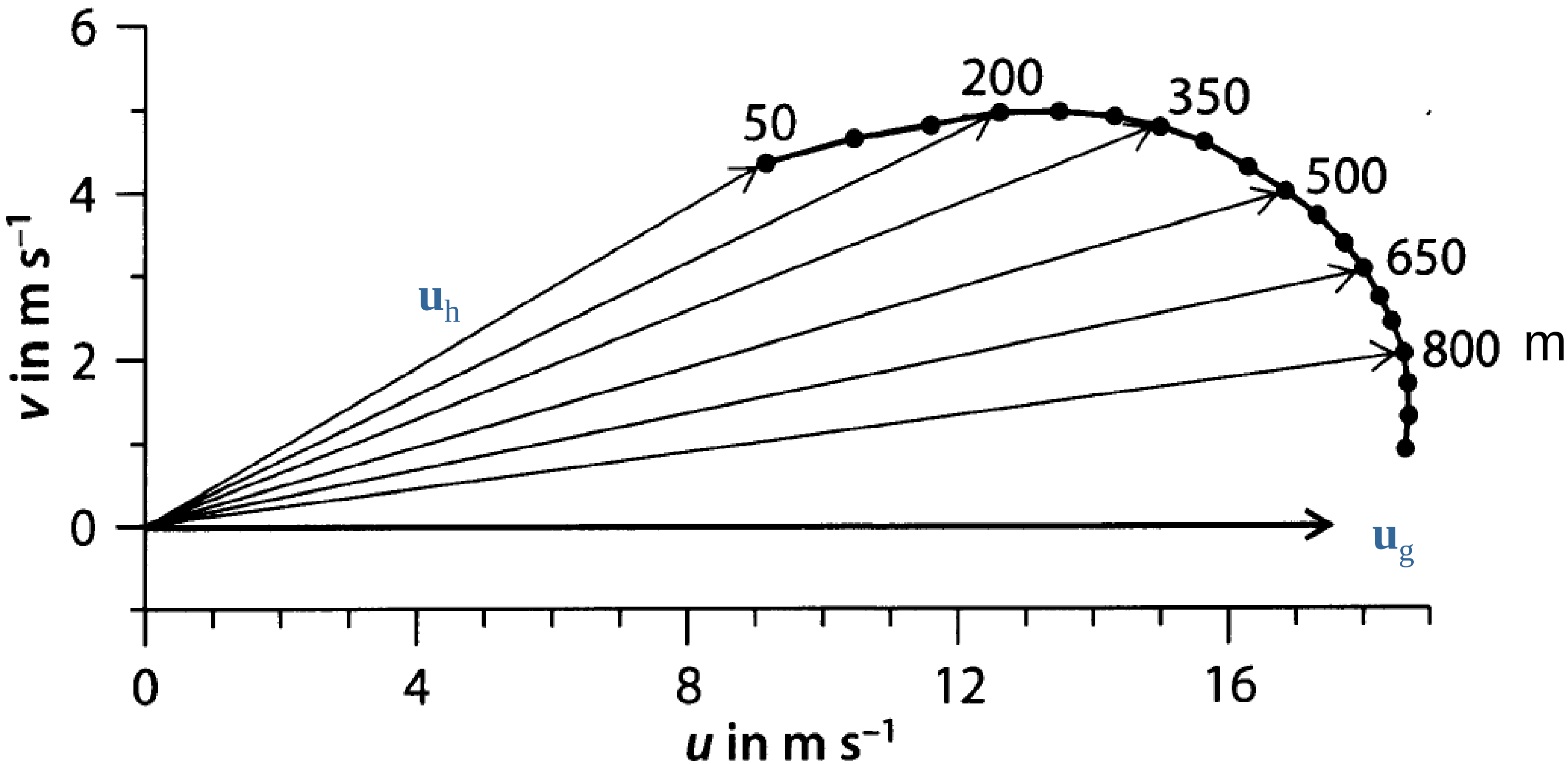
$f\hat{z} \times \boldsymbol{u} + \frac{1}{\rho}\nabla p = \mathcal{F}$ Eq. 7-25

$\mathcal{F} = \mu\, \partial^2\boldsymbol{u}/\partial z^2$

μ: constant eddy viscosity

The Ekman spiral was first calculated for the oceanic friction layer by the Swedish oceanographer in 1905. In 1906, a possible application in Meteorology was developed.

Wind measurement in the boundary layer



The "Leipzig wind profile"

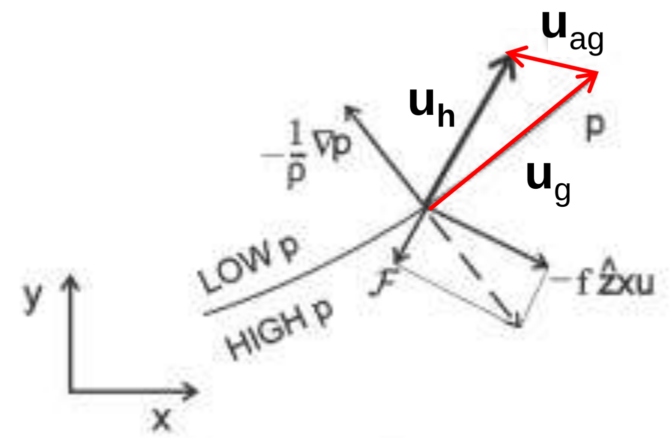
Kraus, 2004

The ageostrophic flow $\mathbf{u}_{ag}$

The *ageostrophic* flow is the difference between the geostrophic flow ($\mathbf{u}_g$) and the horizontal flow ($\mathbf{u}_h$):

$$\mathbf{u}_h = \mathbf{u}_g + \mathbf{u}_{ag} \qquad \text{Eq. 7-26}$$

$$f\hat{\mathbf{z}} \times \boldsymbol{u}_{ag} = \mathcal{F} \qquad \text{Eq. 7-27}$$



The ageostrophic component is always directed to the right of $\mathcal{F}$ in the NH.

Surface (friction) wind

Low pressure (NH):
- anticlockwise flow
- often called "cyclone"

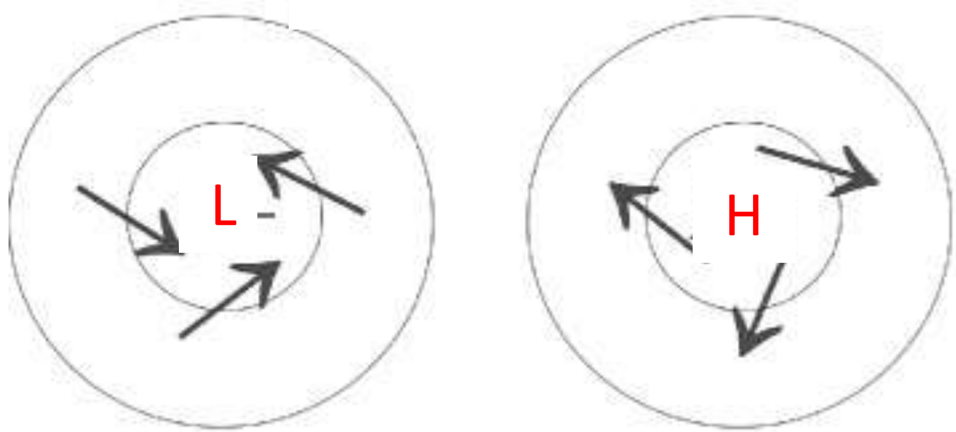


Figure 7.24: Flow spiraling in to a low-pressure region (left) and out of a high-pressure region (right) in a bottom Ekman layer. In both cases the ageostrophic flow is directed from high pressure to low pressure, or down the pressure gradient.

Marshall and Plumb (2008)

High pressure (NH):
- clockwise flow
- often called "anticyclone"

Note: At the surface the wind is blowing from the high to the low pressure system.

Vertical motion induced by Ekman layers

- Ageostrophic flow is horizontally divergent.
- Convergence/divergence drives vertical motions.
- In pressure coordinates, if *f* is const., the continuity equation (Eq. 6-12) becomes:

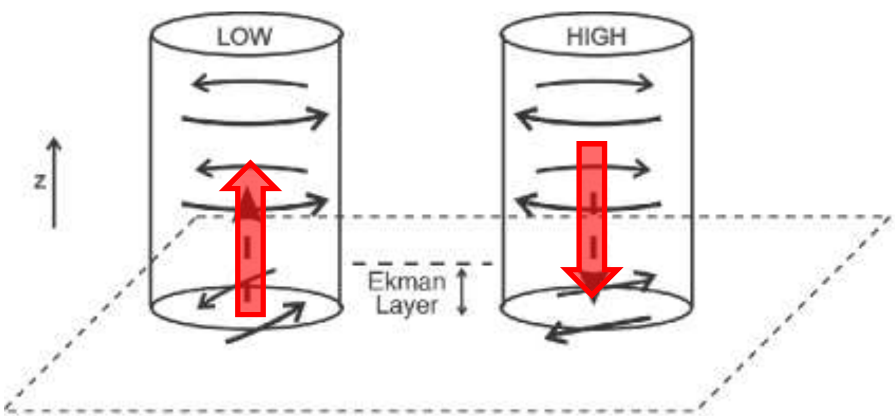


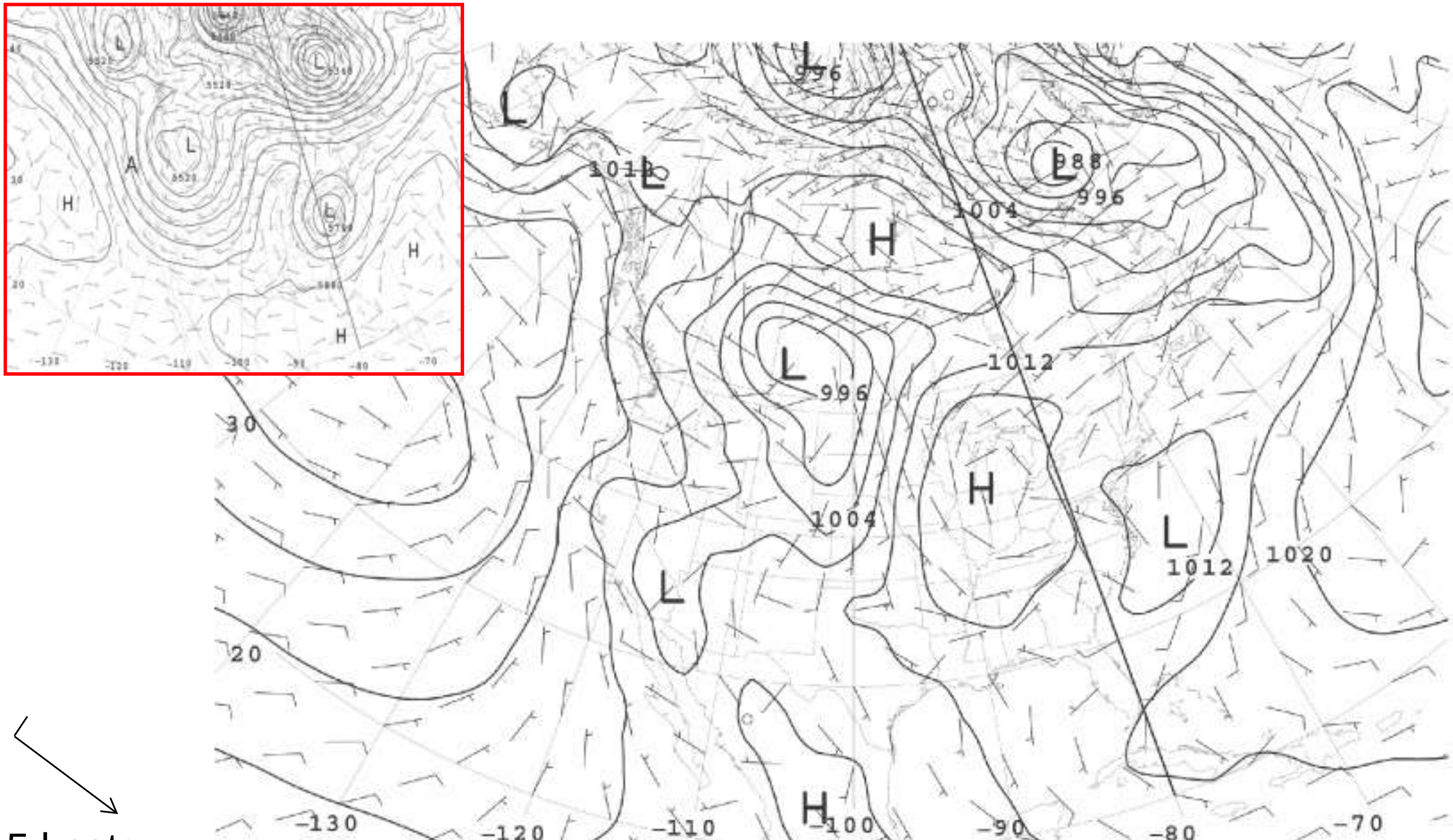
Figure 7.26: Schematic diagram showing the direction of the frictionally induced ageostrophic flow in the Ekman layer induced by low pressure and high pressure systems. There is flow into the low, inducing rising motion (the dotted arrow), and flow out of a high, inducing sinking motion.

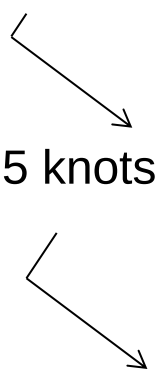
Marshall and Plumb (2008)

$$\nabla_p \cdot \boldsymbol{u}_{ag} + \frac{\partial w}{\partial p} = 0$$

- Low: convergent flow
- High: divergent flow
- Continuity equation requires vertical motions:
  - “Ekman pumping“ > ascent
  - “Ekman suction” > descent

Surface pressure (hPa) and wind (kn), 12 GMT June 21, 2003, USA




5 knots

10 knots

Marshall and Plumb (2008)

Summary

TABLE 7.1. Summary of key equations. Note that (x, y, p) is not a right-handed coordinate system. So although $\hat{\mathbf{z}}$ is a unit vector pointing toward increasing z, and therefore upward, $\hat{\mathbf{z}}_p$ is a unit vector pointing toward decreasing p, and therefore also upward.

(x, y, z) coordinates	(x, y, z) coordinates	(x, y, p) coordinates
$\nabla \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ general	$\nabla \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$ (incompressible—OCEAN)	$\nabla_p \equiv \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial p}\right)$ (comp. perfect gas—ATMOS)
Continuity $\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0$	$\nabla \cdot u = 0$	$\nabla_p \cdot u = 0$
Hydrostatic balance $\frac{\partial p}{\partial z} = -g\rho$	$\frac{\partial p}{\partial z} = -g\rho$	$\frac{\partial z}{\partial p} = -\frac{1}{g\rho}$
Geostrophic balance $fu = \frac{1}{\rho}\hat{z} \times \nabla p$	$fu = \frac{1}{\rho_{ref}}\hat{z} \times \nabla p$	$fu = g\hat{z}_p \times \nabla_p z$
Thermal wind balance	$f\frac{\partial \boldsymbol{u}}{\partial z} = ag\,\hat{\boldsymbol{z}} \times \nabla T$	$f\frac{\partial u}{\partial p} = -\frac{R}{p}\hat{z}_p \times \nabla T$

Take home message

- Balanced flows for horizontal fluids (atmosphere and ocean).
- Balanced horizontal winds:
  - **Geostrophic wind balance** good approximation for the observed wind in the free troposphere.
  - **Gradient wind balance** and **cyclostrophic wind balance** occur with higher Rossby number.
  - **Surface wind** (subgeostrophic flow) **balance** occurs within the Ekman layer.
- **Thermal wind** is good **approximation** for the geostrophic wind change with height *z*.