Ocean Circulation

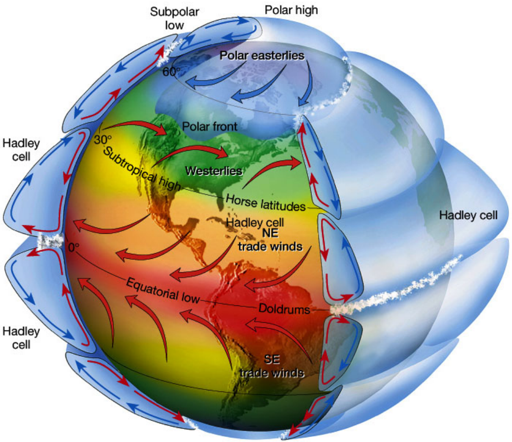
Joe LaCasce
Section for Meteorology and Oceanography

October 21, 2015

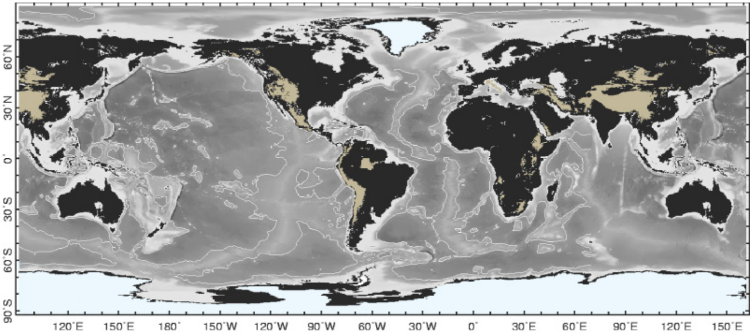
Outline

- Physical characteristics
- Observed circulation
- Geostrophic and hydrostatic balance
- Wind-driven circulation
- Buoyancy-driven circulation

Atmospheric geometry



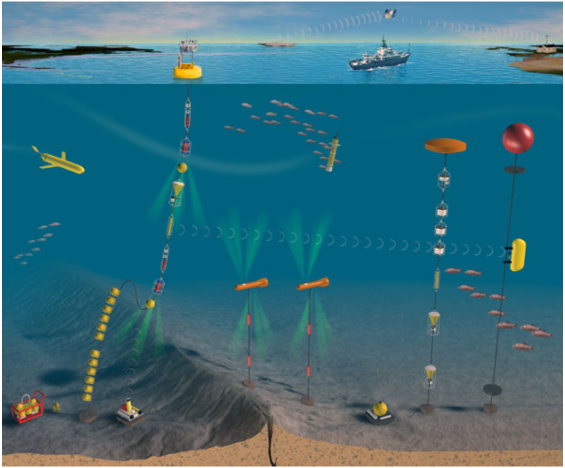
Oceanic geometry



Ocean facts

- Covers 71 % of the earth's surface
- Average depth is 3.7 km
- Volume is 3.2 x 10^{17} $m^3 = 1.3$ x 10^{21} kg
- Heat capacity is 1000 time greater than atmosphere's
- Substantial fraction in ice sheets (Greenland, Antarctica)

Ocean observations



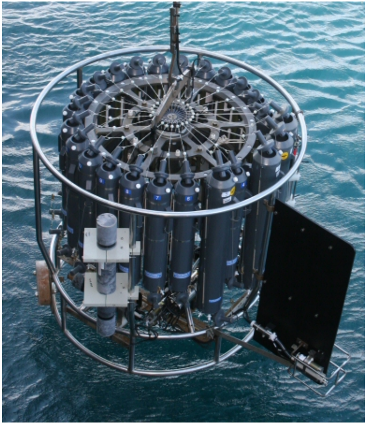
Woods Hole Oceanographic Inst.

Ships

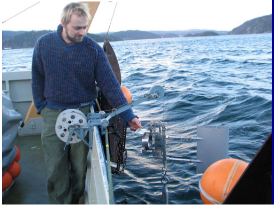
University of Washington, Scripps Inst. of Oceanography

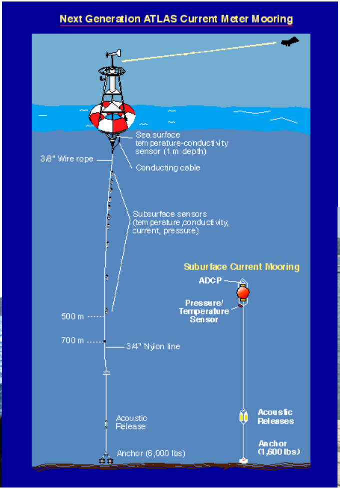
CTD

Conductivity, temperature and depth sensor



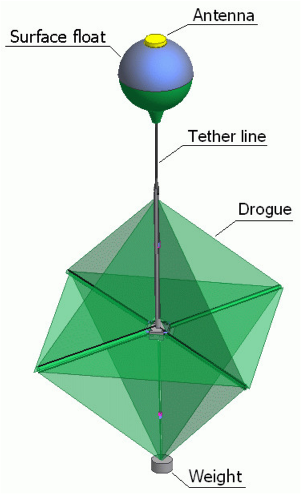
National Oceanographic Center, Southampton

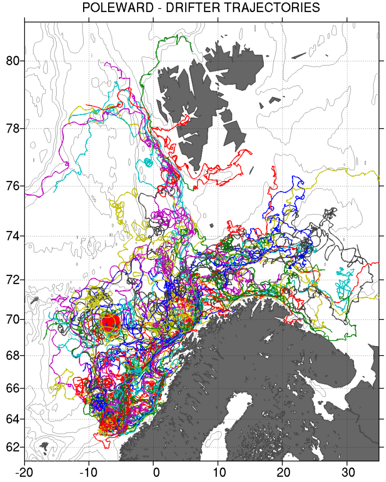




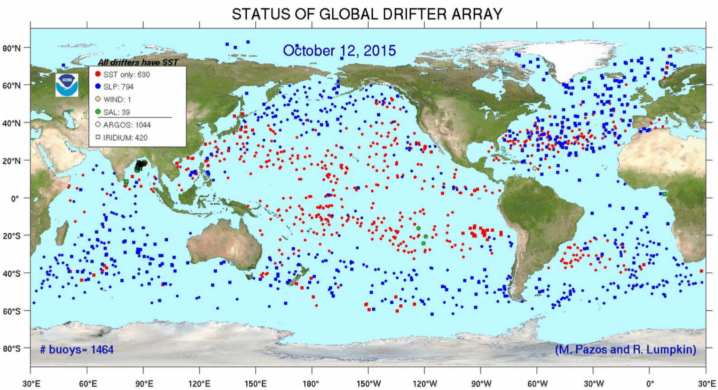
UiO and Falmouth Scientific

The surface drifter



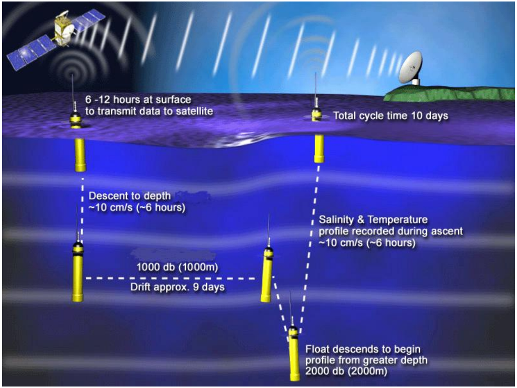


Surface drifter locations: Oct. 12, 2015



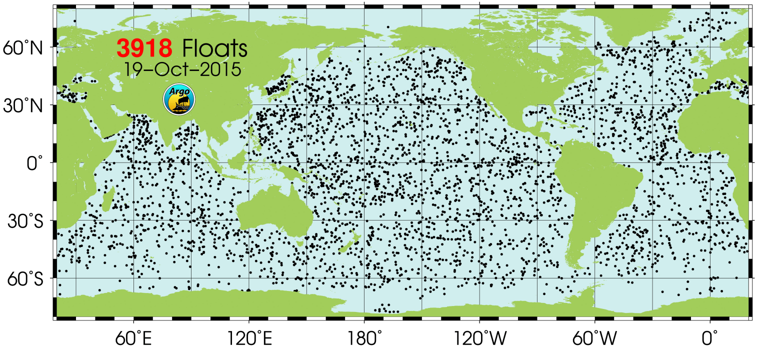
NOAA/AOML

ARGO Floats



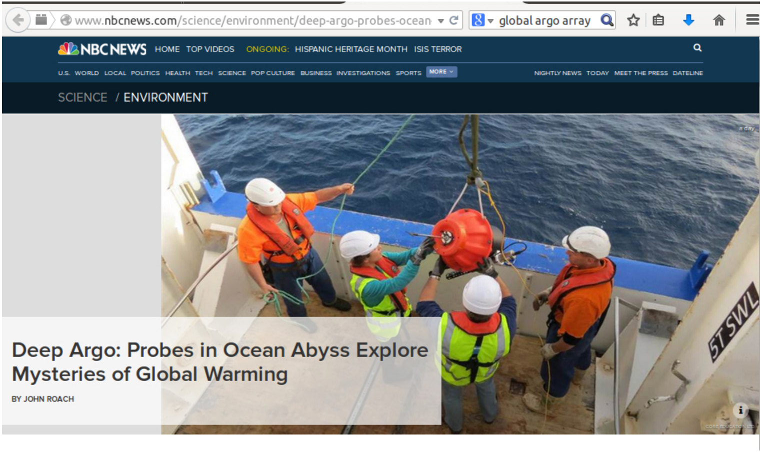
Scripps Inst. Oceanography

ARGO positions, 19 Oct. 2015



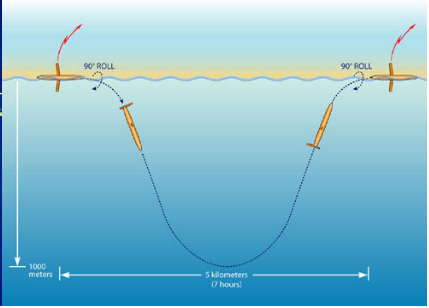
Scripps Inst. Oceanography

Deep ARGO floats



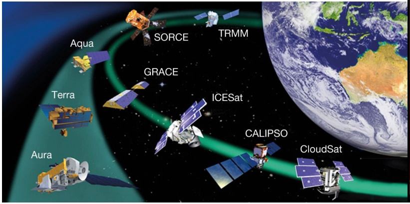
NBC News

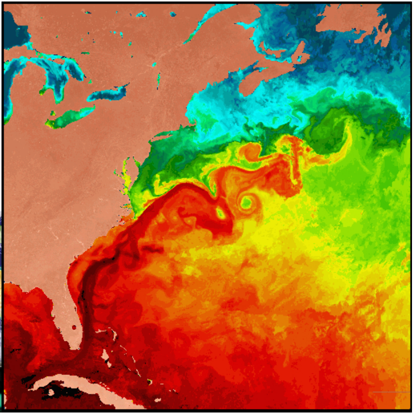
Gliders



Webb Research Inc., Woods Hole Oceanographic Inst.

Satellites





NASA

Observations: measurements

- **Surface temperature** and **salinity** (satellite)
- **Surface height** (satellite)
- **Subsurface temperature** and **salinity** (ships, floats, gliders)
- **Subsurface velocities** (current meters, floats)

Interpreting the observations

- Temperature, salinity and pressure → density
- Sea surface height → surface velocities
- Density profiles → velocity profiles

Calculating density

Density is determined from temperature, salinity and pressure:

$$\rho = \rho(T, S, p) = \rho_c[1 - \alpha_T(T - T_{ref}) + \alpha_S(S - S_{ref}) + ...]$$

where: $\rho_c = 1000 \text{ kg m}^{-3}$

- Warm water is lighter than cold water
- Salty water is heavier than fresh water

Seawater characteristics

Typical temperatures: 0 – 30C

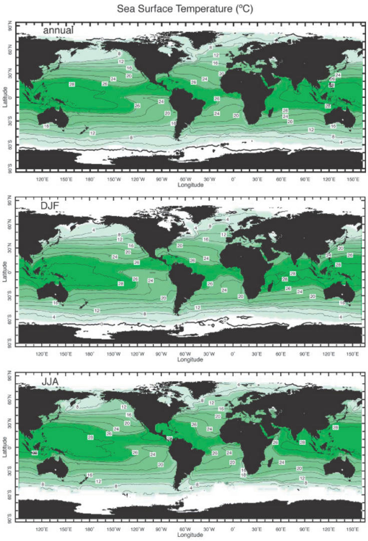
Typical salinities: 33 – 36 psu (practical salinity units)

Compare:

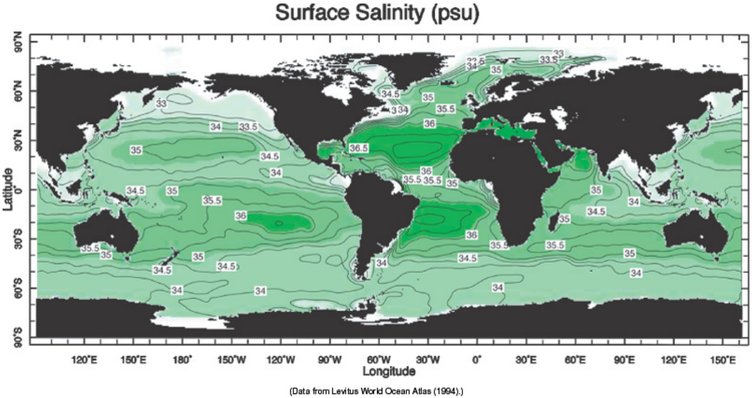
- Fresh water: 0 psu
- Dead Sea: 337 psu

$\rightarrow$ Usually temperature dominates changes in density

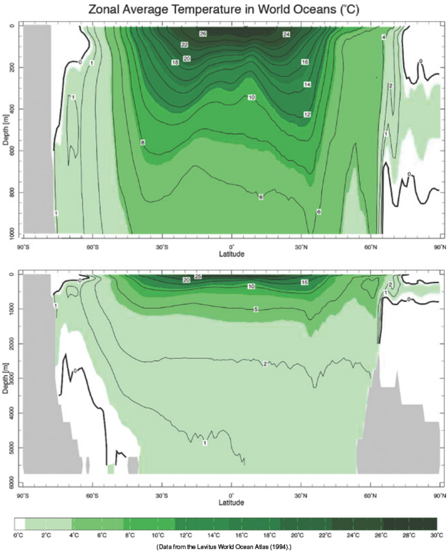
Sea surface temperature (SST)



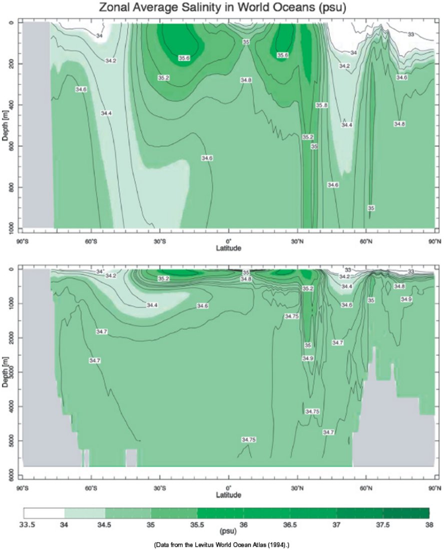
Sea surface salinity



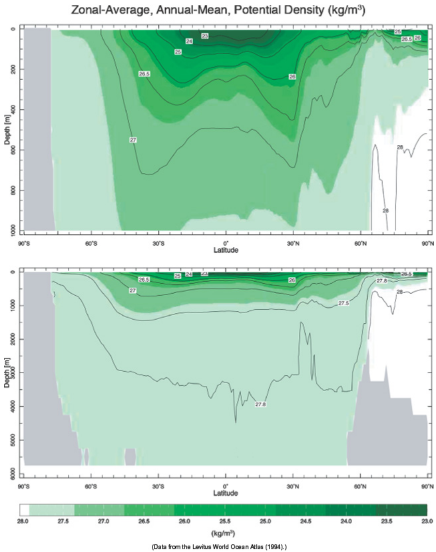
Zonally-averaged temperature vs. depth



Zonally-averaged salinity vs. depth



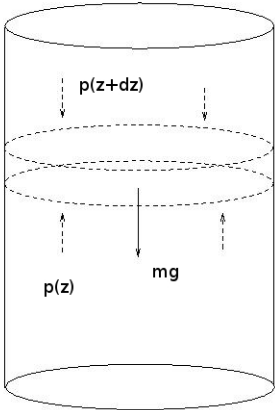
Zonally-averaged density vs. depth



Interpreting the observations

- ~~Temperature, salinity and pressure $\rightarrow$ density~~
- Sea surface height $\rightarrow$ <u>surface velocities</u>
- Density profiles $\rightarrow$ <u>velocity profiles</u>

Basic balances: hydrostatic



Downward force: $-mg = -\rho V g = \rho (A\, dz) g$

Upward force: $(p(z+dz) - p(z))A$

The fluid is static if these are equal:

$$[p(z+dz) - p(z)]A = -\rho g A\, dz$$

or:

$$\frac{dp}{dz} = -\rho g$$

Basic balances: geostrophy

The momentum equations can be scaled:

$$\frac{d}{dt}\vec{u} + f\hat{k} \times \vec{u} = -\frac{1}{\rho_c}\nabla p$$

$$\frac{U}{T} \qquad fU \qquad \frac{\triangle p}{\rho_c L}$$

Divide by fU:

$$\frac{1}{fT} \qquad 1 \qquad \frac{\triangle p}{\rho_c fUL}$$

The temporal Rossby number is given by:

$$\epsilon \equiv \frac{1}{fT}$$

Basic balances: geostrophy

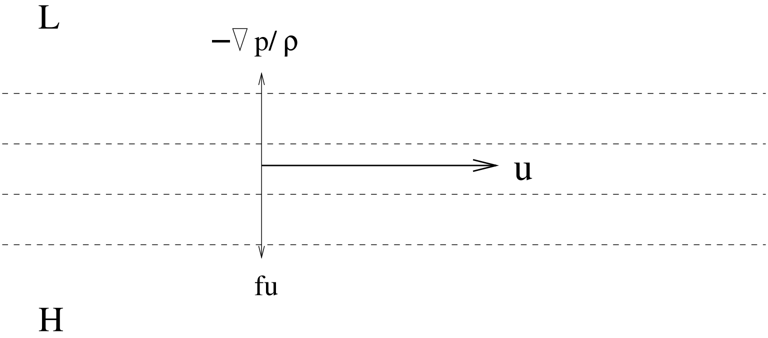
Large scale currents and eddies have a typical time scale of a week:

$$\epsilon = \frac{1}{10^{-4}(10^6)} = 0.01 \ll 1$$

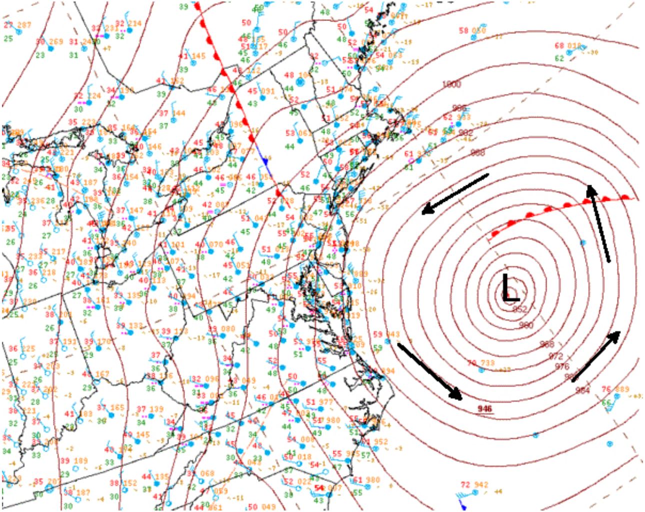
The horizontal velocities are approximately in geostrophic balance:

$$fu = -\frac{1}{\rho_c}\frac{\partial}{\partial y}p, \qquad fv = \frac{1}{\rho_c}\frac{\partial}{\partial x}p$$

Basic balances: geostrophy



Hurricane Sandy



Combine geostrophic and hydrostatic balances

Integrate hydrostatic relation from z to the surface ($z = \eta$):

$$\int_{-z}^{\eta} \frac{\partial p}{\partial z}\, dz = p(\eta) - p(-z) = -g \int_{-z}^{\eta} \rho\, dz$$

The density is roughly constant and $p(\eta) = p_{atmos}$, so:

$$p(-z) = p_{atmos} + \rho_c g(\eta + z)$$

The atmospheric pressure has little effect:

$$\nabla p(-z) = \nabla p_{atmos} + \rho_c g \nabla \eta \approx \rho_c g \nabla \eta$$

Surface velocities

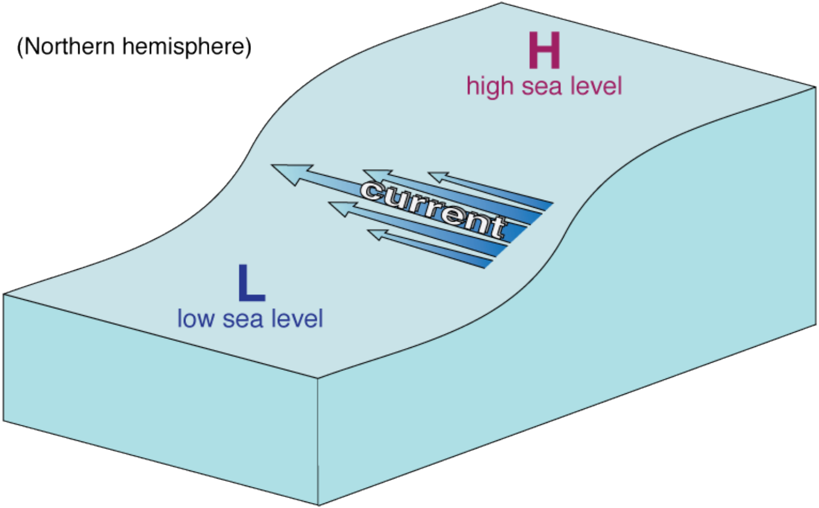
Thus the geostrophic surface velocities are:

$$u_g = -\frac{g}{f}\frac{\partial}{\partial y}\eta, \qquad v_g = \frac{g}{f}\frac{\partial}{\partial x}\eta$$

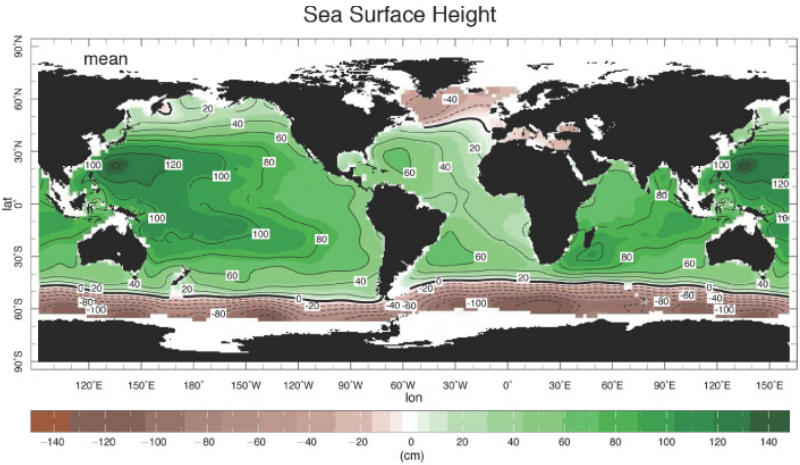
This is how satellite measurements of sea surface height can be used to estimate surface velocities

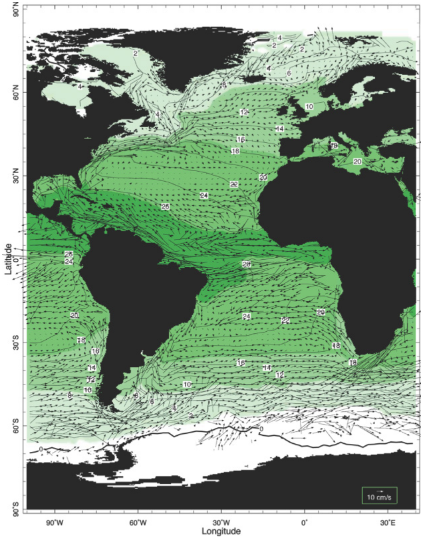
Can obtain global pictures of the surface velocity, with the same resolution as the satellite data (roughly 100 km)

Surface geostrophic flow



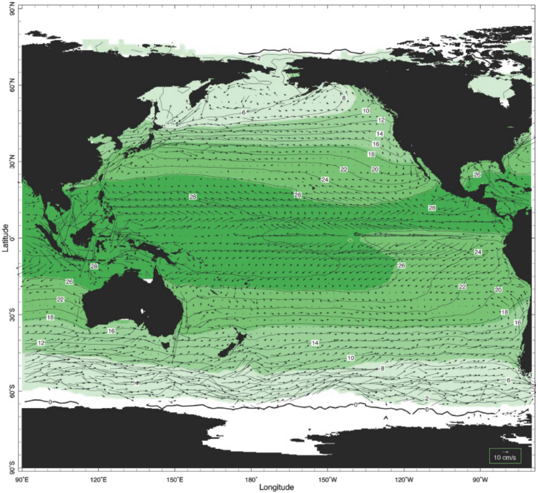
Mean sea surface height, global





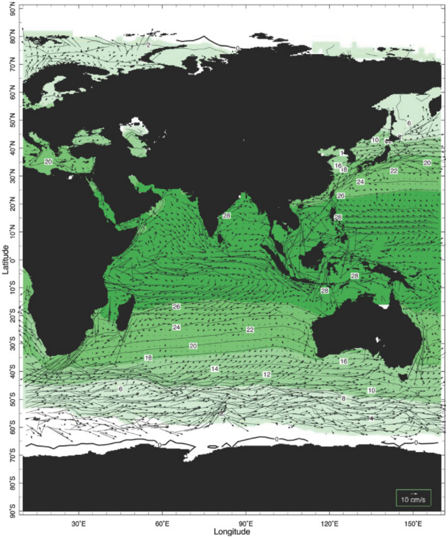
(Data courtesy of Maximenko and Niiler (personal communication, 2003).)

Surface velocities, Pacific



(Data courtesy of Maximenko and Niiler (personal communication, 2003).)

Surface velocities, Indian



(Data courtesy of Maximenko and Niiler (personal communication, 2003).)

Interpreting the observations

- ~~Temperature, salinity and pressure $\rightarrow$ density~~
- ~~Sea surface height $\rightarrow$ surface velocities~~
- Density profiles $\rightarrow$ velocity profiles

Basic balances: thermal wind

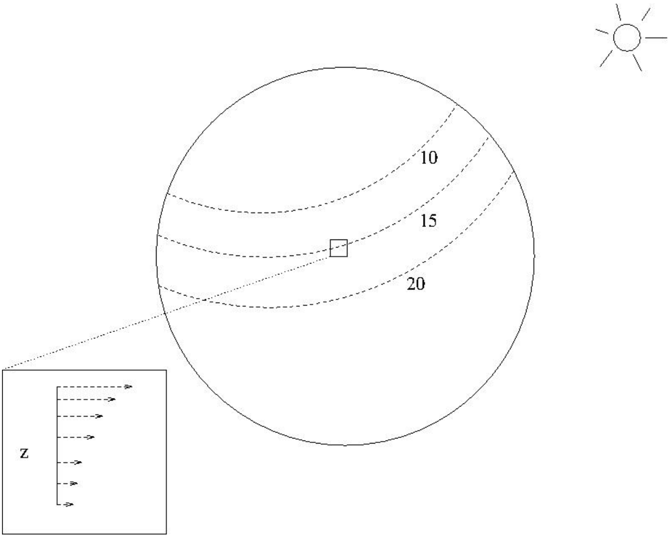
Combine the geostrophic and hydrostatic balances:

$$\begin{aligned}
\frac{\partial}{\partial z} u_g &= \frac{\partial}{\partial z}\left(-\frac{1}{\rho_c f}\frac{\partial p}{\partial y}\right) \\
&= -\frac{1}{\rho_c f}\frac{\partial}{\partial y}\left(\frac{\partial p}{\partial z}\right) \\
&= -\frac{1}{\rho_c f}\frac{\partial}{\partial y}(-\rho g) \\
&= \frac{g}{\rho_c f}\frac{\partial}{\partial y}\rho
\end{aligned}$$

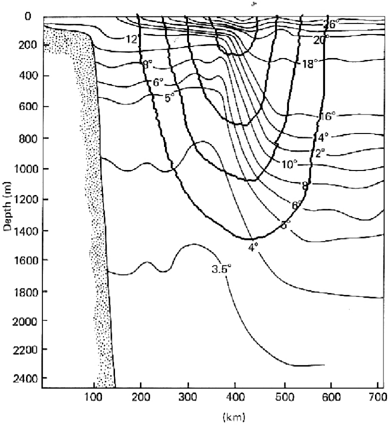
Likewise:

$$\frac{\partial}{\partial z} v_g = -\frac{g}{\rho_c f}\frac{\partial}{\partial x}\rho$$

Basic balances: thermal wind

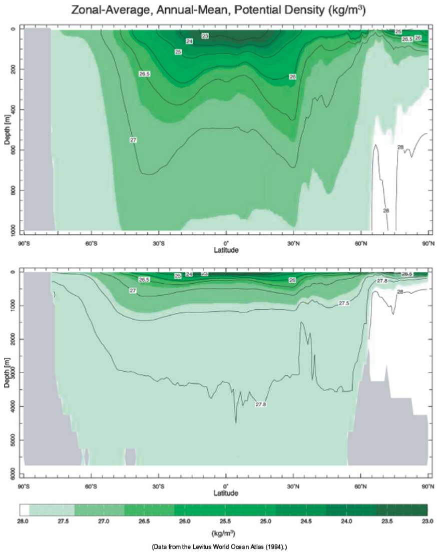


Basic balances: thermal wind



Gulf Stream velocity (dark) and temperature (thin) contours

Deducing the velocity



Deducing the velocity

Integrate the thermal wind relation vertically:

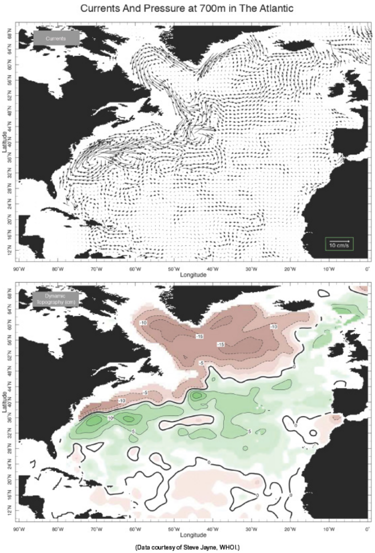
$$\int_{-zref}^{-z} \frac{\partial}{\partial z} u_g \, dz = \int_{-zref}^{-z} \frac{g}{\rho_c f} \frac{\partial}{\partial y} \rho \, dz$$

Or:

$$u_g(-z) = u_g(-z_{ref}) + \frac{g}{\rho_c f} \frac{\partial}{\partial y} \int_{-zref}^{-z} \rho \, dz$$

- Must specify the reference level velocity, $u_g(-z_{ref})$

Subsurface velocity



Basic balances: incompressibility

The full continuity equation is:

$$\frac{\partial}{\partial t}\rho + \vec{u} \cdot \nabla \rho + \rho(\nabla \cdot \vec{u}) = 0$$

In the ocean, $\rho \approx \rho_c =$ const. So:

$$\nabla \cdot \vec{u} = \frac{\partial}{\partial x}u + \frac{\partial}{\partial y}v + \frac{\partial}{\partial z}w = 0$$

Ocean velocities are approximately incompressible

Basic balances: incompressibility

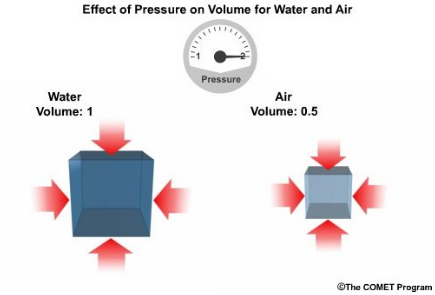
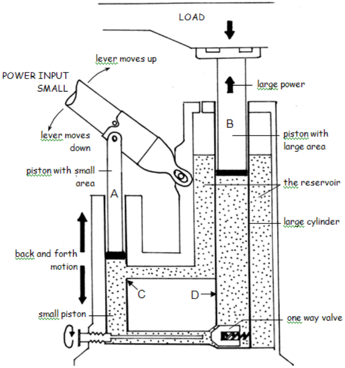


Figure 2: This is a cross-section of a simple hydraulic jack.



Basic balances: summary

- Hydrostatic balance

$$\frac{\partial p}{\partial z} = -\rho g$$

- Geostrophic balance

$$fu = -\frac{1}{\rho}\frac{\partial p}{\partial y}, \quad fv = \frac{1}{\rho}\frac{\partial p}{\partial x}$$

- Incompressibility

$$\nabla \cdot \vec{u} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$