Oppgavesett kap. 4 (1 av 2)
GEF2200

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Exercise 1: Wavelengths and wavenumbers (We will NOT go through this in the group session)

a)

What's the relation between wavelength and wavenumber?

$$\nu = \frac{1}{\lambda}$$

They're inversely proportional.

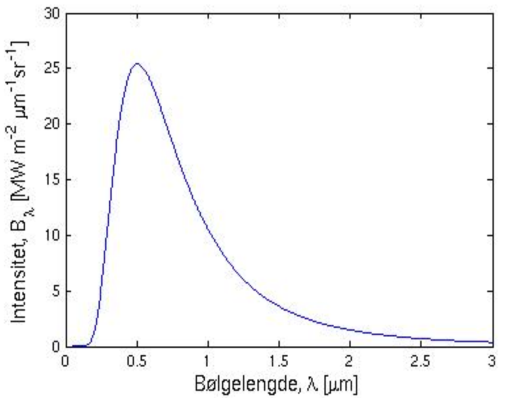
Figure 1 shows emssion spectra from two different bodies. Spectrum A in Figure 1a shows emitted intensity as a function of wavelength, λ, while spectrym B in Figure 1b shows emitted intensity as a function of wavenumber, ν.

b)

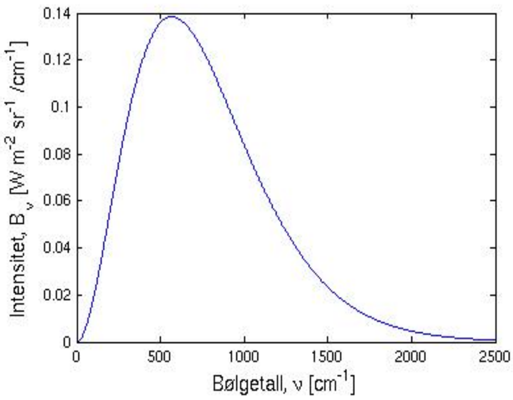
Look at the x-axis in Figure 1a. Is more energetic, short wave radiation towards the left or right of the spectrum?
Wavelength along the x-axis: smaller wavelength means more energetic radiation, which means towards the left.

c)

Look at the x-axis in Figure 1b. Is more energetic, short wave radiation towards the left or right of the spectrum?
Wavenumber along the x axis: larger wavenumber means more energetic radiation, which means towards the rigth.



(a) *Spectrum A*



(b) *Spectrum B*

Figure 1: *Emission spectra from two different bodies*

d)

Which types of radiation do the Sun and Earth emit most intensely?
The sun emits most intensely in visible light, while the earth emits most intensely in infrared radiation. There is very little overlap between the solar and terrestrial spectra

Exercise 2: Radiance og irradiance

a)

What is a solid angle?
Solid angle = The area of the projection of a body onto the unit sphere centered

on the observer. The total area os the unit sphere is $A = 4\pi r^2 = 4\pi 1^2 = 4\pi$. A full sphere thus comprises a solid angle of 4π, while a hemisphere corresponds to 2π. The unit is steradian (sr). From Figure 2 we can see that two objects with different shapes and sizes can give the same solid angle it they're at different distances from the observer and the unit sphere.

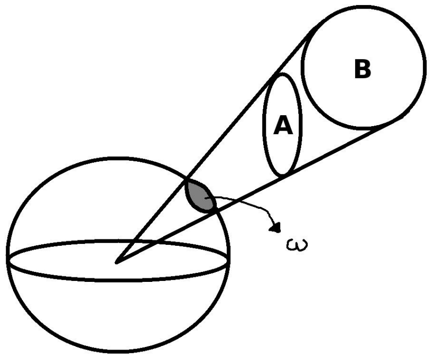


Figure 2: *Solid angle ω (grey area). Both objects subtend the same solid angle, dexpite different shapes and sizes.*

b)

Define the following quantities and specify their corresponding units:

- Monochromatic intensity (monochromatic *radiance*): $I_\lambda = \frac{d^4E}{dA\cos\theta dt d\lambda d\omega}$ with unit $\mathrm{Wm^{-2}\mu m^{-1}\ sr^{-1}}$..
- Intensity (*radiance*).
- Monochromatic flux density (monochromatic *irradiance*).
- Flux density (*irradiance*).

The others are all defined on pages 114-115 in W&H

c)

Consider Figure 4.3 in W&H and show that a small change in solid angle ω is given as

$$d\omega = \sin\theta d\theta d\phi$$

Change in solid angle, $d\omega$, can be caused by changes zenith angle, $d\theta$, or chagnged azimuth angle, $d\phi$. Figure 3 illustrates this. $d\omega$ can be approximated as a rectangle with sides a and b, which are found using the formulas for arc length

$$angle = \frac{arclength}{radius}$$

Giving

$$d\theta = \frac{b}{1}$$

and

$$d\phi = \frac{a}{x}$$

Solve for a and b and multiply to find $d\omega$. To find x you use $\sin\theta = \frac{x}{1}$. We get

$$\begin{aligned} d\omega &= a \cdot b \\ &= \sin\theta d\phi \cdot d\theta \\ \Rightarrow d\omega &= \sin\theta d\theta d\phi \end{aligned}$$

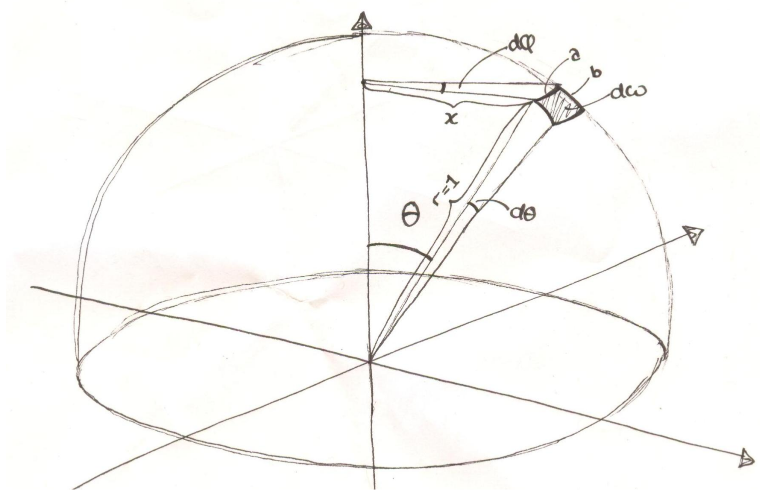


Figure 3: *endring i romvinkelen ω som konsekvens av endret senitvinkel, θ, eller endret azimutvinkel, ϕ.*

d)

Exercise 4.14 in W&H

We find the irradiance by integrating monochromatic irradiance over all wavelengths. In this case we can express the integral as a sum:

$$
\begin{aligned}
F &= \int_\lambda F_\lambda d\lambda \\
&= \sum_i F_{\lambda,i} \cdot \Delta\lambda_i \\
&= (1,0 \cdot (0,50 - 0,35) + 0,5 \cdot (0,70 - 0,50) + 0,2 \cdot (1,00 - 0,70))\,\mathrm{Wm}^{-2} \\
&= 0,31\mathrm{Wm}^{-2}
\end{aligned}
$$

e)

What does *isotropic* radiation mean?

The intensity of isotropic radiation has no dependency on solid angle. It is the same no matter which direction we consider.

f)

A measuring station which measures monochromatic flux density, F_λ is located in an area with mountains on all sides so the horizon is not located at zenith angle 90°, but at 80° (see Figure 4). This means the sky does not span a solid angle of 2π as shown in example 4.1 in W&H, but a little less. The station is exposed to isotropic, monochromatic intensity, I_λ. Calculate the monochromatic flux density F_λ recieved at the station.

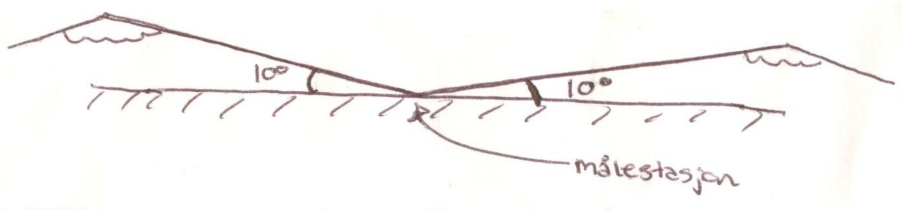


Figure 4: *Measuring station surrounded by mountains on all sides.*

The angle of 10° gives us the horizon with zenith angle 80°, which is $4\pi/9$. *We get*

$$
\begin{aligned}
F_\lambda &= \int_0^{2\pi} \int_0^{4\pi/9} I_\lambda \cos\theta \sin\theta d\theta d\phi \\
&= I_\lambda 2\pi \int_0^{4\pi/9} \cos\theta \sin\theta d\theta
\end{aligned}
$$

Do a substitution to make integration simpler. $u = \cos\theta \Rightarrow du = -\sin\theta d\theta$*. This gives us a change of limits:* $\cos\left(\frac{4\pi}{9}\right) \approx 0,174$*,* $\cos 0 = 1$

$$
\begin{aligned}
F_\lambda &= -I_\lambda 2\pi \int_1^{0,174} u du \\
&= -I_\lambda 2\pi \frac{1}{2}\left((0,174)^2 - 1^2)\right) \\
&= 0,97\pi I_\lambda
\end{aligned}
$$

g)

Radiation is emitted by a body. How has the intesity and flux density changed when we are at a distance d away from the body? Use *the inverse square law* in your explanation.

Unless the beam of radiation has been attenuated by absorption or scattering, the intensity will remain the same. We can see this from 2. Even though the area of the sphere will increase with distance, the area of the solid angle will increase correspondingly. The flux density on the other hand, which says how much energy is transported through the sphere per unit area will not stay the same. Since the area of the sphere increases as ($A_{sphere} = 4\pi r^2$*) with distance from the source, the energy per area will decrease like the square of the distance from the source. We call this the inverse square law,*

$$F \propto d^{-2}$$

Oppgave 3: Sortlegemestråling

a) (fra eksamen 2007)

What is a black body? Which of these can be approximated as a black body:

- The Earth

- The Sun
- The atmosphere

A black body absorbs all radiation incident on it. Nothing is reflected or transmitted. None of them are black bodies, but the sun and the earth can be approximated as black bodies in some cases. The atmosphere transmits almost all incoming short wave solar radiation and can not be approximated as a black body

b)

Write down the following laws/functions and explain briefly what thei describe and the mathematic relations between them:

- **Planck function** *Eq. 4.10 in W&H. Can be used to calculate the monochromatic intensity* B_λ *emitted by a black body (*B_λ *is the same as* I_λ *for black bodies) If you plot this against wavelength you get the spectrum of a black body which is only dependent on temperature and is illustrated in Figure 4.6*
- **Wien's law** *Eq. 4.11 in W&H. Cen be used to find the wavelength of the peak monochromatic intensity emitted by a black body. Can be found by differentiating the Planck function with respect to* λ*, euating the resulting expression with zero and solving for* λ*.*
- **Boltzmanns law** *Eq. 4.12 in W&H. Gives us the flux density emitted from a black body. Found by integrating the Planck function over all solid angles and all wavelengths.*

c)

The fux density of solar radiation incident on the top of the Earth's atmosphere is given by the solar constant $F_s = 1368$ Wm^{-2}. The earth's radius is $R_E = 6,371 \cdot 10^6$ m.

1. How much energy does the Earth recieve from the Sun every second? Neglect the Earth's albedo.
2. How large must the irradiance of the Earth be to achieve radiative equilibrium with the recieved energy you calculated abovet? (Again neglecting the Earth's albedo)
3. The black body temperature of the Earth is -18°C. Show that the planetary albedo is 0.3.
4. How can the mean temperature at the Earth's surface be 15°C, when the black body temperature is -18°C?

1)

$$
\begin{aligned}
E &= \int_A F dA \\
&= F_s \cdot A_{tverrsnitt\ jord} \\
&= 1368 \text{Wm}^{-2} \cdot \pi \cdot \left(6,371 \cdot 10^6 \text{ m}\right)^2 \\
&= 1,74 \cdot 10^{17} \text{W}
\end{aligned}
$$

2)

$$
\begin{aligned}
F_E \cdot 4\pi R_E^2 &= F_s \cdot \pi R_E^2 \\
F_E &= \frac{F_s}{4} \\
F_E &= 342 \text{ Wm}^{-2}
\end{aligned}
$$

3)

$$
\begin{aligned}
(1 - A) \cdot F_s \cdot \pi R_E^2 &= \sigma T^4 \cdot 4\pi R_E^2 \\
A &= 1 - \frac{4\sigma T^4}{F_s} \\
A &= 0,3
\end{aligned}
$$

Exercise 4

a)

Define the terms monochomatic...

- emissivity
- absorptivity
- reflectivity
- transmissivity

These are all defined on page 120 in W&H

b)

Exercise 4.15

Since the surface is opaque it will not transmit radiation and everything not absorbed will be reflectes. Thus the Reflectances are $R_\lambda = 1$ for radiation with $\lambda < 0,70\ \mu m$ and $R_\lambda = 0$ for radiation with $\lambda > 0,70\ \mu m$. We get that the reflected flux densitt is only coming from wavelengths with $\lambda < 0,70\ \mu m$. The refelcted flux density is

$$F_{reflektert} = 1 \cdot (1,0 \cdot (0,50 - 0,35) + 0,5 \cdot (0,70 - 0,50))\,\mathrm{Wm}^{-2}$$
$$F_{reflektert} = 0,25\ \mathrm{Wm}^{-2}$$

c)

A non-black body (A) is assumed to emits radiation with the same emissivity for every wavelength. We assume the body emits the same irradiance/flux density, F, as a black body (B). Which of the two bodies (A or B) has the highets temperature? Explain your answer.

Vi har at

$$F_A = \epsilon_A \sigma T_A^4$$
$$F_B = \sigma T_B^4$$

Since the irradiances are equal, we get

$$F_A = F_B$$
$$\epsilon_A \sigma T_A^4 = \sigma T_B^4$$
$$T_A = T_B \frac{1}{\sqrt[4]{\epsilon_A}}$$

We have that $\epsilon_A < 1 \Rightarrow \sqrt[4]{\epsilon_A} < 1 \Rightarrow \frac{1}{\sqrt[4]{\epsilon_A}} > 1$, Which gives us that $T_A > T_B$.

d)

The incident radiation at the top of a non-reflective, absorbing atmospheric layer is

$I_{\lambda,1} = 5\ \mathrm{Wm}^{-2}\mu\mathrm{m}^{-1}\mathrm{sr}^{-1}$. $I_{\lambda,2} = 3\ \mathrm{Wm}^{-2}\mu\mathrm{m}^{-1}\mathrm{sr}^{-1}$ is transmitted through the layer. What is the transmissivity (T_λ) og absorptivity (α_λ) of the layer?

$$T_\lambda = \frac{3 \text{ Wm}^{-2}\mu\text{m}^{-1}\text{sr}^{-1}}{5 \text{ Wm}^{-2}\mu\text{m}^{-1}\text{sr}^{-1}} = 0,6$$
$$\alpha_\lambda = 1 - 0,6 = 0,4$$

e)

What is Kirchhoff's law, and when does it apply?

Kirchhoff's law says that the monochromatic emissivity is equal to the monochromatic absorptivity, $\epsilon_\lambda = \alpha_\lambda$, for a body in thermodynamic equilibrium. Note that:

1. ϵ_λ and α_λ *do NOT need to be the same for all wavelengths, but for a given wavelength they are equal.*

2. *Kirchhoff's law also applies to black bodies. ϵ_λ and α_λ will be 1 for black bodies and between 0 and 1 for grey bodies.*

3. *Even though gases in the atmosphere do not need to be in thermodynamic equilibrium (since they might emit more than they absorb, or vice versa → not radiative equilibrium → not thermodynamic equilibrium) they can still be in approximate equilibrium if the energy transfers through emission/absorption is much smaller than the energy exchange through collisions. We call this local thermodynamic equilibrium (LTE), and we can apply Kirchhoff's law.*

Exercise 5

a)

Blue light with wavelength $\lambda = 0,5$ μm is scattered by air molecules with radiuss 10^{-4} μm.

1. Which type of scattering is this?

2. Which of the figures in Figure 4.12 in W&H best corresponds to this type of scattering?

3. How does the scattering efficiency K_λ change with wavelength in this scattering regime?

1. Calculate the size parameter

$$
\begin{aligned}
x &= \frac{2\pi r}{\lambda} \\
&= \frac{2\pi 10^{-4}\ \mu\text{m}}{0,5\ \mu\text{m}} \\
&= 1,25 \cdot 10^{-3}
\end{aligned}
$$

   We see that $x << 1$, but not so small that it's negigible. This corresponds to Rayleigh scattering.

2. Figure 4.12a in W&H.

3. For Rayleigh scattering we have $K_\lambda \propto \lambda^{-4}$. Blue light is scattered much more strongly than other colors of light.

b)

Explain the following:

- Blue sky
- Red sunsets

Oppgave 6

a)

What is "broadening", and why is this interesting with regards to climate change?
We know from quantum physics that when atoms/moelcules change energy state they emit/absorb energy corresponding to the energy difference between the states. The energy states a atom/moelcule can be in are quantizes which means that the photons emistted/absorbed during the state transition can only have certain discrette values. If we plot emission/absorption as a function of wavelength we'll get discrete lines at certain wavelengths corresponding to the difference between energy levels and zero everywhere else. But if the atoms/molecules recieve or dissipate energy through collisions at the same time as they emit/absorb or if they have a certain velocity relative to the source they recieve radiatiotion from, the lines get broadened. This means that the atom/molecule can change state even though the energy they emit or absorb doesn't correspond exactly to the energy of the state transition. The resulting emission/aborption lines are broadende into a bell-like shape (see Figure 4.21 in W&H).

This means that for high concentrations of greenhouse gases, absorption and emission on the flanks of the line become important. Even though the centre of a line of CO2 is saturated, emitting more makes absorption/emission on the flanks more efficient. Follow this link to read an article about the greenhouse effect of CO2.

b)

Which types of "broadening" do we have and in which parts of the atmosphere does the different types dominate?

Doppler broadening: *Dominated above 50 km (above the stratosphere). The reciever of radiation moves at such a high velocity relative to the source that the radiation is doppler shifted towards longer (shorter) wavelengths when the reciever is moving away from (towards) the source.*

Pressure broadening: *Dominates below 20 km (in the troposphere). The reciever of radiation collides at the same time as recieving radiation. If the radiation has the slightly wrong energy exitation can still happen if the reciever is able to get or dissipate the energy difference through a collision interaction with another particle*