GEF2200 Atmospheric Physics

Exam 2006

Tuesday 13th of June

Solution

Problem 1

a. Potential temperature is the temperature an air parcel will have when compressed or expanded adiabatically to the reference pressure $p_0 = 1000\text{hPa}$.

The equation for it is derived by using the First law of thermodynamics, where the heat added to the system dq is zero:

$$dq = c_p dT - \alpha dp = 0 \tag{1}$$

and then insert α from the equation of state:

$$\frac{dT}{T} - \frac{R}{c_p}\frac{dp}{p} = 0 \tag{2}$$

Integrating from T, p to the reference level θ, p_0 we have

$$\int_T^\theta \frac{dT}{T} = \frac{R}{c_p}\int_p^{p_0}\frac{dp}{p} \tag{3}$$

$$\ln\frac{\theta}{T} = \frac{R}{c_p}\ln\frac{p_0}{p} = \ln\left(\frac{p_0}{p}\right)^{\frac{R}{c_p}} \tag{4}$$

$$\theta = T\left(\frac{p_0}{p}\right)^{\frac{R}{c_p}} \tag{5}$$

which is the equation for potential temperature.

θ is an important parameter in atmospheric thermodynamics because most of the thermodynamic processes in the atmosphere are adiabatic, meaning that θ is conserved.

b. Lifting condensation level (LCL) is the level where an air parcel reach saturation when lifted adiabatiaclly.

To find the pressure at LCL when $p_1 = 900\text{hPa}$ and $T_1 = 20°\text{C}$, and $T_2 = 13°\text{C}$, we use the equation for potential temperature with reference level p_1,

T_1 instead of p_0, θ and solve for p_2:

$$
\begin{aligned}
T_1 &= T_2 \left(\frac{p_1}{p_2}\right)^{\frac{R}{c_p}} && (6)\\
\frac{p_1}{p_2} &= \left(\frac{T_1}{T_2}\right)^{\frac{c_p}{R}} && (7)\\
p_2 &= p_1 \left(\frac{T_1}{T_2}\right)^{-\frac{c_p}{R}} && (8)\\
p_1 &= 900\text{hPa}\left(\frac{286.15\text{K}}{293.15\text{K}}\right)^{-\frac{1004\text{JK}^{-1}\text{kg}^{-1}}{287\text{JK}^{-1}\text{kg}^{-1}}} && (9)\\
&= 827\text{hPa} && (10)
\end{aligned}
$$

c. The relative humidity before lifting is given by

$$RH = \frac{w(T_1)}{w_s(T_1)} 100\% \tag{11}$$

and using the relationship between w and e

$$RH = \frac{e(T_1)/p_1}{e_s(T_1)/p_1} 100\% = \frac{e(T_1)}{e_s(T_1)} 100\% \tag{12}$$

$e_s(T_1)$ can then be calculated directly from the equation given, but for $e(T_1)$ we need to see that

$$
\begin{aligned}
w(p_1) &= w_s(p_2) && (13)\\
\varepsilon \frac{e(T_1)}{p_1} &= \varepsilon \frac{e_s(T_2)}{p_2} && (14)\\
e(T_1) &= e_s(T_2) \frac{p_1}{p_2} && (15)
\end{aligned}
$$

then we insert the equation for e_s and insert for T_1 and T_2:

$$
\begin{aligned}
RH &= \frac{e(T_1)}{e_s(T_1)} 100\% = \frac{e_s(T_2)\frac{p_1}{p_2}}{e_s(T_1)} 100\% && (16)\\
&= \frac{e_s(T_2)}{e_s(T_1)} \frac{p_1}{p_2} 100\% && (17)\\
&= \frac{\exp\left[\frac{L}{R_v}\left(\frac{1}{273} - \frac{1}{T_1}\right)\right]}{\exp\left[\frac{L}{R_v}\left(\frac{1}{273} - \frac{1}{T_2}\right)\right]} \frac{p_1}{p_2} 100\% && (18)\\
&= \frac{\exp\left[\frac{L}{R_v}\left(\frac{1}{273} - \frac{1}{293.25}\right)\right]}{\exp\left[\frac{L}{R_v}\left(\frac{1}{273} - \frac{1}{286.15}\right)\right]} \frac{900\text{hPa}}{827\text{hPa}} 100\% && (19)\\
&= 69\% && (20)
\end{aligned}
$$

Even more elegant:

$$
\begin{aligned}
RH &= \frac{\exp\left[\frac{L}{R_v}\left(\frac{1}{273}-\frac{1}{T_1}\right)\right]}{\exp\left[\frac{L}{R_v}\left(\frac{1}{273}-\frac{1}{T_2}\right)\right]}\frac{p_1}{p_2}100\% && (21)\\
&= \exp\left[\frac{L}{R_v}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)\right]\frac{p_1}{p_2}100\% && (22)\\
&= \exp\left[\frac{L}{R_v}\left(\frac{1}{293.25}-\frac{1}{286.15}\right)\right]\frac{900\text{hPa}}{827\text{hPa}}100\% && (23)\\
&= 69\% && (24)
\end{aligned}
$$

d. The average turbulent vertical transports of θ, q and u below 400m are

$$
\begin{aligned}
\overline{w'\theta'} &= 0 \quad \text{No transport of heat} && (25)\\
\overline{w'u'} &> 0 \quad \text{Downward transport of momentum} && (26)\\
\overline{w'q'} &< 0 \quad \text{Upward transport of moisture} && (27)\\
& && (28)
\end{aligned}
$$

There is no transport of heat, because $\partial\theta/\partial z = 0$, so the air is neutral between 0 and 400m. Above 400m we have $\partial\theta/\partial z > 0$, so there the air is stable.

Problem 2

a. The mechanisms we have for ice particle growth in cold clouds are 1. deposition from vapor phase, 2. riming of supercooled droplets on the crystal and 3. aggregation.

b. The equation for growth by collision

$$\frac{dr}{dt} = \frac{v_s w_l E_c}{4\varrho_l} \tag{29}$$

can be used for ice particle growth mechanisms which undergo collision, which are riming (collision of ice particle and supercooled droplets) and aggregation (collision of ice particles).

c. To find the radius of a growing ice particle that collects (collides) with supercooled droplets, we use Equation (29) and insert the given values.

$$
\begin{aligned}
v_s &= \frac{r^2\varrho_i g_0}{72\eta} && (30)\\
w_l &= 0.5\text{g/m}^3 = 5\cdot 10^{-4}\text{kg/m}^3 && (31)\\
\eta &= 1.7\cdot 10^{-5} && (32)\\
g_0 &= 9.81\text{ms}^{-1} && (33)\\
r_0 &= 100\mu\text{m} = 10^{-4}\text{m} && (34)\\
E_c &= 0.6 && (35)
\end{aligned}
$$

And then we get

$$\frac{dr}{dt} = \frac{r^2\varrho_i g_0}{72\eta}\frac{w_l E_c}{4\varrho_i} \tag{36}$$

$$\int_{r_0}^{r}\frac{dr}{r^2} = \int_0^t \frac{g_0 w_l E_c}{4\cdot 72\eta} \tag{37}$$

$$-\left[\frac{1}{r}-\frac{1}{r_0}\right] = \frac{g_0 w_l E_c}{288\eta}t \tag{38}$$

And for $t = 15\text{min} = 15\cdot 60 = 900\text{s}$, we solve for r

$$\frac{1}{r} = \frac{1}{r_0} - \frac{g_0 w_l E_c}{288\eta}900\text{s} \tag{39}$$

$$= \frac{1}{10^{-4}\text{m}} - \frac{9.81\text{ms}^{-2}\cdot 5\cdot 10^{-4}\text{kg/m}^3\cdot 0.6}{288\cdot 1.7\cdot 10^{-5}}900\text{s} \tag{40}$$

$$= 9459\text{m}^{-1} \tag{41}$$

$$r = \frac{1}{9459}m = 1.06\cdot 10^{-4}\text{m} = 106\mu\text{m} \tag{42}$$

A particle growing by riming is a graupel.

d. Water vapor pressure over a droplet relative to a plane surface of water (and therefore the surroundings) is given by

$$\frac{e'}{e_s} = 1 + \frac{a}{r} - \frac{b}{r^3} \tag{43}$$

where $a = \frac{2\sigma}{nkT}$ and $b = \frac{3imM_w}{4M_s\pi\varrho}$.

e'/e_s Relative humidity over the droplet.

a/r Constant depending on the surface stress of the droplet divided by droplet radius. Called the curvature effect, which is large for small droplets.

b/r^3 Constant depending on the amount of salt solved in the droplet divided by the r^3. This term is called the solute effect, and is important for small droplets, where the concentration of salt iones is large. For the smallest droplets this effect is more important than the curvature effect, allowing droplets to reach activation by reducing the necessary supersaturation to form droplets.

This means that the solute effect is most important for activating a droplet.

Problem 3

a. The sketch ...

E_λ Incoming irradiance
dE_λ Absorbed irradiance
z Height
dz Thickness of layer
k_λ Absorbtion coefficient
ϱ Density of air
ϕ The zenith angle

This equation shows how much of incoming monochromatic irradiance is absorbed when passing through a layer of thickness dz. When integrated from $z = \infty$ to z, we find the relationship between transmissivity and the optical depth

$$\ln\left(\frac{E_\lambda}{E_{\lambda,\infty}}\right) = \ln\tau = -\sigma_\lambda \tag{44}$$

where

$$\sigma_\lambda = \int_z^\infty k_\lambda \varrho \sec(\phi) dz \tag{45}$$

b. Inserting the density profile

$$\varrho(z) = \varrho(0)\exp\left(\frac{-z}{H}\right) \tag{46}$$

where H =const is the scale height, into the equation of the optical thickness

$$\begin{aligned}
\sigma_\lambda &= \int_z^\infty k_\lambda \varrho \sec(\phi) dz && (47)\\
&= \varrho(0) k_\lambda \sec(\phi) \int_z^\infty \exp\left(\frac{-z}{H}\right) dz && (48)\\
&= k_\lambda \varrho(0) \sec(\phi) H \exp\left(\frac{-z}{H}\right) && (49)\\
&= H k_\lambda \varrho(0) \sec(\phi) \exp\left(\frac{-z}{H}\right) && (50)
\end{aligned}$$

c. With $\phi = 0$, $\sec(\phi) = 1$, and we have

$$\begin{aligned}
\varrho(0) &= 1\text{kgm}^{-3}\\
H &= 10\text{km}\\
k_\lambda &= 1\text{kgm}^{-2}
\end{aligned}$$

so that the optical thickness given by Equation (50) can be written

$$\begin{aligned}\sigma_\lambda &= Hk_\lambda\varrho(0)\sec(\phi)\exp\left(\frac{-z}{H}\right)\\ &= 10\text{km}\cdot\exp\left(\frac{-z}{10}\right)\end{aligned} \tag{51}$$

and inserting for $z = 40$, 30, 20 and 10km, we get optical thicknesses

z [km]	σ_λ
40	0.18
30	0.50
20	1.35
10	3.68

d. The transmissivity τ from the top of atmosphere down to the heights specified are given by

$$\tau = \frac{E_\lambda}{E_{\lambda,\infty}} = \exp(-\sigma) \tag{52}$$

so we have

z [km]	σ_λ	τ
40	0.18	0.835
30	0.5	0.607
20	1.35	0.259
10	3.68	0.025

e. For an atmosphere with no scattering, the absorbtivity is related only to transmissivity: $a = 1 - \tau$, giving

z [km]	σ_λ	τ	a
40	0.18	0.835	0.165
30	0.5	0.607	0.393
20	1.35	0.259	0.741
10	3.68	0.025	0.975

f. Looking at where the change in absorbtivity is largest, we see that this is between 20 and 30km.

z [km]	σ_λ	τ	a	Δa
40	0.18	0.835	0.165	
				0.228
30	0.5	0.607	0.393	
				0.348
20	1.35	0.259	0.741	
				0.243
10	3.68	0.025	0.975	

The maximum change in absorbtivity is located where the optical thickness close to unity. In fact, it can be shown that the largest change in absorbtivity is at an optical depth of unity.