OBLIG3-fasit

Computation of p_1:

$$\frac{\partial p_1}{\partial z} = -g\rho_1$$
$$\Rightarrow \quad p_1 = -g\int \rho_1 dz + C_1 = -g\rho_1 z + C_1$$

Boundary condition:

$$p_1 = P_0, \quad z = 0 \quad \Rightarrow \quad C_1 = P_0$$
$$\Rightarrow \quad p_1 = -g\rho_1 z + P_0$$

Computation of p_2:

Use the hydrostatic approximation in the wedge. Apply the dynamic boundary condition at the material interface $z = -Hx/L$, and determine the pressure p_2 in the coastal water.

$$\frac{\partial p_2}{\partial z} = -g\rho_2 = -g\rho_1\left(1 - a\left(z + \frac{H}{L}x\right)\right)$$
$$\Rightarrow \quad p_2 = -g\int \rho_2 dz + C_2 = -g\rho_1 z + g\rho_1 a\left(\frac{1}{2}z^2 + \frac{H}{L}xz\right) + C_2$$

Boundary condition:

$$p_1 = p_2, \quad z = -\frac{H}{L}x \quad \Rightarrow \quad C_2 = P_0 + \frac{g\rho_1 a H^2}{2L^2}x^2$$
$$\Rightarrow \quad p_2 = -g\rho_1 z + \frac{g\rho_1 a}{2}\left(z + \frac{H}{L}x\right)^2 + P_0$$