**GEF2500 - OBLIGATORY EXERCISE 4, SPRING 2014**
**All symbols are defined in the GEF2500 Lecture Notes**

**Problem 1**

a) What balance of forces do we have in the Ekman layer?

b) What do we mean by a no-slip condition for viscous flow at a rigid boundary?

c) An "infinitely" large horizontal ice sheet is drifting with constant velocity $v_0$ along the $y$-axis; see the figure below.

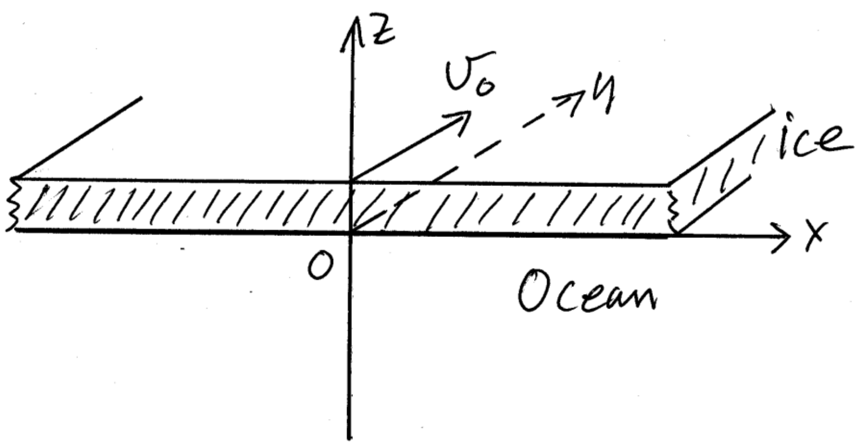


*Figure sketch*

The ocean has constant density, and there are no horizontal pressure gradients in the problem. Assume that $u = u(z), v = v(z), w = 0$, and show that the equations for the motion become

$$\frac{d^2u}{dz^2} + \frac{fv}{A^{(z)}} = 0, \qquad (1.1)$$
$$\frac{d^2v}{dz^2} - \frac{fu}{A^{(z)}} = 0.$$

d) Introduce the complex velocity $W = u + iv$, and show that

$$\frac{d^2W}{dz^2} - a^2W = 0\,, \qquad (1.2)$$

where $a^2 = \dfrac{if}{A^{(z)}}$.

e) Show that the solution to this equation can be written

$$W = C_1 \exp\left(\left(1+i\right)\frac{\pi z}{D_E}\right) + C_2 \exp\left(-(1+i)\frac{\pi z}{D_E}\right), \tag{1.3}$$

where we have defined

$$D_E = \pi\left(\frac{2A^{(z)}}{f}\right)^{1/2}$$

f) What does $D_E$ signify?

g) Use the no-slip condition at $z = 0$, and the fact that the velocity must vanish for large negative values of $z$ to determine the complex coefficients $C_1$ and $C_2$.

h) Show that the velocity components in this problem can be written

$$\begin{aligned} u &= -v_0 \exp\left(\frac{\pi z}{D_E}\right)\sin\left(\frac{\pi z}{D_E}\right), \\ v &= v_0 \exp\left(\frac{\pi z}{D_E}\right)\cos\left(\frac{\pi z}{D_E}\right). \end{aligned} \tag{1.4}$$

i) Discuss the variation with depth of the current (1.4).

j) Find the Ekman transport components $(q^{(x)}, q^{(y)})$ under the ice in this problem by integrating (1.4) from $z = -\infty$ to $z = 0$.

k) Compute the tangential stresses $\tau_0^{(x)} = \rho_r A^{(z)} du/dz$, and $\tau_0^{(y)} = \rho_r A^{(z)} dv/dz$ in the water just below the ice (at $z = 0$) from (1.4). Could you use this result as an alternative to (j) in obtaining the Ekman transport in the water under the ice? (explain).

l) The ice sheet (thickness $h$, density $\rho_i$) is driven by a wind stress $\vec{\tau}_w$. The frictional stress from the water *on* the ice is $(-\tau_0^{(x)}, -\tau_0^{(y)})$ in (k). Sketch the balance of forces per unit area on the ice sheet (remember Nansen).