GEF2610: final exam 2018

December 2018

1 The equation of state for sea water

1. Explanation of pressure effect on temperature and definition of potential temperature (in text or as expression), which leads to one profile appearing to be unstable. Point out that comparison of potential density using common reference level must be done.

2. Temperature is destabilizing, salinity is stabilizing. Should note that salinity dominates since density profile is stable. Coastal water is cold and fresh, and in the top layer.

2 Observations and modeling

1. Thermal wind (shear) equations. Equations are derived from the assumption of hydrostatic pressure distribution and geostrophic balance. Explicit derivation gives full points.

2. Density is determined from temperature, salinity (obtained using conductivity) and pressure. These measurements are obtained using a CTD.

3. It should be noted that if the density is known everywhere, then the velocity *shear* is known everywhere according to these equations. Hence if we obtain a measurement of the velocity at some depth, we can integrate the thermal wind equations from this depth to obtain the velocities everywhere in the water column. Full points for showing the integration, e.g.
$$v_g(z) = v_g(z_r) - \frac{g}{f\rho_0}\int_{z_r}^{z} \frac{\partial \rho}{\partial x} dz',$$
where z_r is the reference depth and $v_g(z_r)$ is the reference velocity we measure.

4. There are several ways to measure or estimate the currents at some level, e.g. (i) assuming zero velocities at some depth (so-called "level of no motion", not often used nowadays), (ii) using drifters or an ADCP for direct measurements, (iii) use satellite observations of sea surface height and calculate geostrophically balanced surface currents.

3 Large scale wind-driven flows

1. Need to start from continuity equation, which is integrated in the vertical. Need to point out that vertical velocity at surface is zero, which leaves the vertical velocity $w(z_0)$ at the base of the Ekman layer. Substitute with expressions for Ekman transport.

2. Note that friction is assumed zero, i.e. $R = 0$, which leaves yields the Sverdrup balance as the only terms remaining in the equation.

3. The total horizontal transport is assumed divergence free, i.e.

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0,$$

and that V is known. We then have a first order differential equation for U with only one possibility to set the proper boundary condition $U = 0$ at either the east or the west coast. Second, note that the Stommel equation requires that

$$\frac{\partial v}{\partial x} < 0$$

in order to balance a northward transport V, which means that the boundary current is along the western coast.

4 Waves

1. Since the wave number k is inversely proportional to the wave length, we may use kH as a parameter to distinguish between waves in deep or shallow water. Use $\tanh(kH) \approx 1$ for $kH \gg 1$ (deep water waves) and $\tanh(kH) \approx kH$ for $kH \ll 1$ (shallow water waves). Explicitly show the dispersion relation for these approximations.

2. Show the definitions of phase speed $c = \omega/k$ and group speed $c_g = \partial\omega/\partial k$. Note that the phase speed is the speed of a wave crest and that the energy travels with the group speed. Full points requires calculating the derivatives in the expressions for the group speeds, and also noting that DWW are dispersive while SWW are non-dispersive.

3. Wind waves are typically short and steep and are generated by the local wind. Swell waves are typically long and gentle, and have propagated away from the area where they were generated (and hence can have phase velocities that are larger than the local wind). Need to note the transition from DWW to SWW as the depth decreases, the shortening and steepening of the SWW as the waves propagate closer to the shoreline, and that they eventually break when they become steep enough. Cf. Fig. 88 in compendium.

4. The Kelvin waves decay exponentially away from the coast on a horizontal scale given by the Rossby radius of deformation $L_d = c/f$, where $c = \sqrt{gH}$. The Rossby radius decreases towards the poles and with decreasing depth.

5. To the right.

5 Tides

1. Need to point out that the earth-moon system rotates about its common center of gravity. The two maxima on either side of the earth (disregarding the continents and friction) are the results of the local difference between the centrifugal force and the gravitational pull. The diurnal tides are the results of the earth's rotation axis not being perpendicular to the earth-moon axis (obliquity). A sketch is recommended.

2. Need to define spring = maximum tidal amplitudes and neap = minimum tidal amplitudes, and to point out the connection between the positions of the sun and the moon relative to earth. A sketch like Fig. 95 in the compendium should ideally include *all* relevant positions of the moon, i.e. not only between the earth and sun for spring tide, but also on the far side of the earth.

3. The figure shows a wave propagating with the coast to the right, indicative of a Kelvin wave. It also shows that the phase velocity is slower in the shallowest parts of the basin.

4. Tides can be both Poincaré and Kelvin waves. For Poincaré waves we have $\omega > |f|$, hence for a given tidal period (e.g. semi-diurnal) there exists a critical latitude north (or south) of which this relation does not hold. At higher latitudes the tides can therefore only propagate as Kelvin waves.