GEF2610 - Problem set 1
Due October 3, 2018

Problem 1

1. Describe four major sources or sinks of freshwater in the surface oceans. Explain how such freshwater fluxes change the salinity of the upper ocean?

2. Explain briefly what parameters the density of seawater depend on? In which direction do each of these properties impact density (explain why it makes sense)?

3. What is potential temperature? Is potential temperature higher or lower compared to in situ temperature in the ocean, and what is causing this difference?

4. What is potential density? What is the difference between σ_t and σ_θ?

5. Explain how the buoyancy frequency is a measure of the stability in a water column. What is a fluid's response to an unstable density stratification? How do you explain this in terms of what happens to the buoyancy frequency?

6. Write down the mass conservation equation. What do we assume when we apply the Boussinesq approximation? Show how applying Boussinesq turns the equation for conservation of mass into an equation for conservation of volume (the continuity equation).

Problem 2

For the three vertical density profiles in Figure 1, show (calculate) that the potential energies (PE) take on the values shown in the right in the figure.

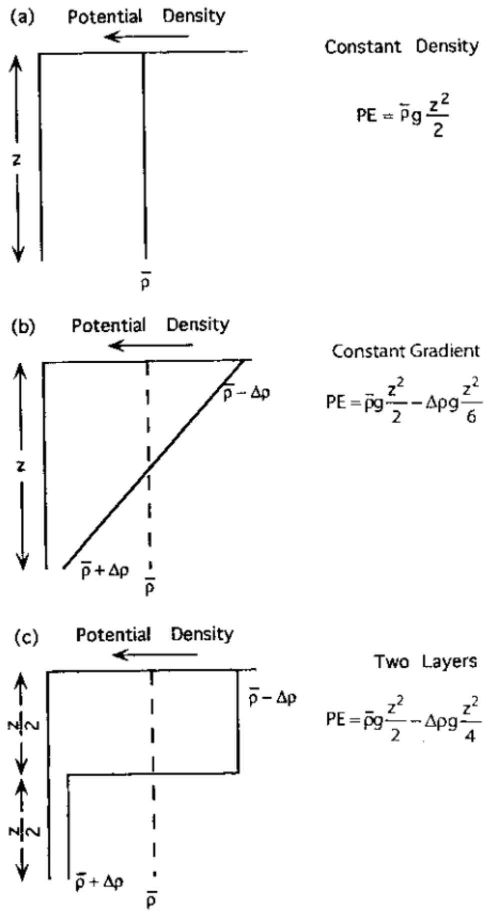

Figure 1: Three different vertical density stratifications (same total mass).

Problem 3

For a slanted density stratification, as shown in Figure 2, the potential energy change ΔPE due to an exchange of particles is

$$\Delta PE = -g\frac{\partial\overline{\rho}}{\partial z}\left(\Delta x\right)^2\left(s_{ex} - s_\rho\right)s_{ex},$$

where g is the gravitational acceleration, $\partial\overline{\rho}/\partial z$ is the background vertical density stratification and s_{ex} and s_ρ are the slopes of the particle exchange and of isopycnals (lines of constant density), respectively. We assume a stable vertical stratification, i.e. $\partial\overline{\rho}/\partial z < 0$.

1. Define and discuss the three regimes: $\Delta PE < 0$, $\Delta PE = 0$ and $\Delta PE > 0$.

2. Show that the maximum release of PE is achieved when $s_{ex} = s_\rho/2$. (Hint: Here you need to differentiate to find the maximum of the expression).

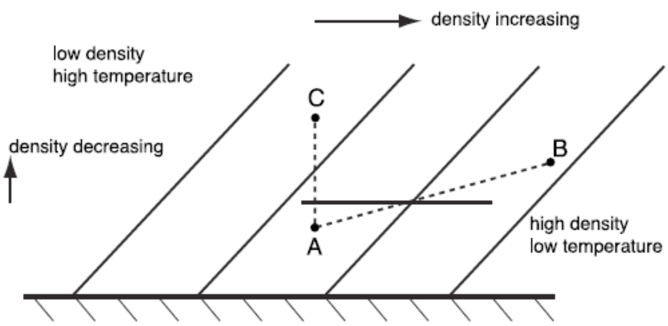

Figure 2: The exchange of fluid parcels in a slanted density stratification.

Problem 4

Download temperature and salinity data from a hydrographic station in the Greenland Sea (73.03°N, 2.83°W). The text file 'wod_003308505O.dat' can be downloaded from the course web pages. It contains three columns of data: z (depth, negative since it is below the sea surface), S (salinity on the 'practical salinity scale') and T (in situ temperature). Plot the hydrographic data as a function of depth and also in a T-S diagram. Do you find any evidence of warming at great depths due to pressure?

Problem 5

The current equation of state for seawater, which describes water density as a function of temperature, salinity and pressure, is called TEOS-10. This new description replaces an earlier one, called EOS-80.

1. Use the information given at www.teos-10.org (and, if you like, other sources) to discuss some differences between TEOS-10 and EOS-80. Comment, for example, on the differences between the new 'absolute salinity' S_A and the old 'practical salinity' S_P and on the difference between 'potential temperature' θ and 'conservative temperature' Θ.

2. Use software found on www.teos-10.org or, if you don't know any of the programing languages Matlab, Fortran or C, use the web interface at https://monrecifamoi.saulme.fr/salinite/sea_water_calculator_teos10.php to calculate and plot the density for the T-S data of problem 4. (If you use the web interface, you'll have to enter value by value, and then it is sufficient to do the calculations for a subset of the pressure values.)

Problem 6

We have mentioned in class how a freshwater flux through the sea surface can change the upper-ocean salinity, but have not gone into details. To arrive at a direct relationship we imagine an ocean surface layer having salinity S, density ρ and thickness H. It receives fresh water (rain) FW per unit time per unit horizontal area (having units $\mathrm{kg\,s^{-1}m^{-2}}$) as shown in Figure ?. Assume that salinity is well-mixed vertically all the time so that there is no vertical salinity or density gradient. Show that the salinity of the layer changes as

$$\frac{\partial S}{\partial t} = -S\frac{FW}{\rho H}.$$

Hint: The salinity of the layer is $S = m_s/(M + m_s)$, where m_s is the mass of the salt in the water and M is the mass of freshwater so that $(M + m_s) = \rho H$ is the total mass of the water (per unit horizontal area). Now take the time derivative of S.

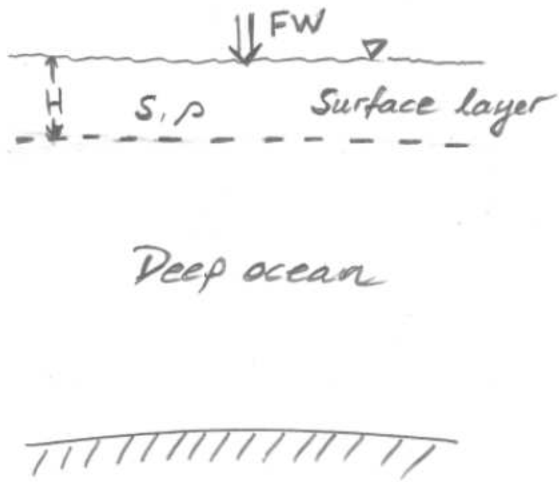

Figure 3: The flux of freswater FW (from rain) into an ocean surface layer of thickness H and having salinity S and density ρ.

Problem 7

A column of water, 100 m thick, has upward-directed vertical velocities of 2 mm/s and 0.5 mm/s at the top and the bottom, respecively. What is the convergence of the horizontal flow needed to sustain this 'stretching' of the water column?

Problem 8

Molecular diffusion of heat or salt in the ocean is a tremendously slow process, and it can usually be negleced. But turbulence increases the effective diffusivity by orders of magnigude and is therefore an important process for maintaining the observed density stratification. When studying the effect of turbulence we can replace the molecular diffusivity in the conservation equations with a (much larger) turbulent diffusivity value.

Imagine an ocean region in which the potential temperature is horizontally homogeneous but decays exponentially as

$$\theta(z) = \theta(0\,\mathrm{m}) \exp\left(z/z_d\right),$$

where the surface temperature is $\theta(0\,\mathrm{m}) = 20°\mathrm{C}$ and the vertical decay scale is $z_d = 200\,\mathrm{m}$. Assume that turbulence creates a constant effective diffusivity of $\kappa_T \sim 10^{-4}\mathrm{m}^2\mathrm{s}^{-1}$.

1. What is the temperature at 500 m depth ($z = -500\,\mathrm{m}$)? And what is the diffusive temperature flux? In what direction is it directed (upward or downward)?

2. If turbulent diffusion acts alone, what will be the time rate of change of temperature at 500 m depth? How long would it take to take to change the temperature there by 1°C?

3. If the vertical diffusion of potential temperature is instead balanced by vertical advection of temperature, so that $\partial\theta/\partial t = 0$, what vertical velocity is found at 500 m depth? Is the vertical velocity upwards or downwards? Is it large or small compared to what you would expect?

Problem 9

A relationship between depth and pressure in the ocean is found by use of the so-called *hydrostatic balance*, an assumption that for the vertical momentum equation there is an approximate balance between the vertical pressure gradient and the gravitational force,

$$\frac{dp}{dz} = -\rho g.$$

So vertical acceleration Dw/Dt as well as frictional stresses are assumed to be small (even tiny) in comparison. Assuming density is approximately constant, $\rho = 1027\,\mathrm{kg\,m}^{-3}$, integrate this equation to find the pressure in dbar (1 dbar = 0.1 bar = 10^4 Pa) at 1000 m depth. You can assume that the pressure at the ocean surface is 0 dbar.

Problem 10

The last problem is about wind-driven currents in a non-rotating reference frame. In the ocean this problem is more complex, since rotational effects kick in and limit the depth to which the wind have a direct influence, but the analytical treatment of the wind stress is the same.

We have a fluid of constant density ρ_0, dynamic viscosity μ, and let the surface be given by $z = 0$ and with a flat bottom at $z = -H$. The momentum equation for the fluid is then

$$\rho_0 \frac{D\boldsymbol{v}}{dt} = -\nabla p - \rho_0 g \boldsymbol{k} + \mu \nabla^2 \boldsymbol{v},$$

where $\boldsymbol{k}$ is the unit vector in the z-direction. Furthermore, we assume that the velocity is given

$$\boldsymbol{v} = (u(z), 0, 0)\,,$$

that is, the only flow is in the x-direction and it only varies with depth.

1. Show that the z-component of the momentum equation becomes

$$\frac{\partial p}{\partial z} = -\rho_0 g.$$

2. The pressure at the surface is taken to be constant, i.e. $p(z = 0) = p_0$. Show that the x-component of the momentum equation becomes

$$\mu \frac{\partial^2 u}{\partial z^2} = 0.$$

3. There is a wind stress τ_{wind} at the surface, which means that the upper boundary condition is

$$\mu \frac{\partial u}{\partial z}(z = 0) = \tau_{\text{wind}}.$$

   At the bottom we have a no-slip condition, i.e. $u(z = -H) = 0$. Find the solution for $u(z)$.

4. What is the stress acting on the bottom due to the fluid flow?