GEF2610 - Problem set 2
Due 14 November 2018

October 31, 2018

Problem 1: Thermal wind

Figure 1 shows potential density in the so-called Svinøy hydrographic section off the Norwegian west coast. The hydrography shows the buoyant Norwegian Atlantic Current (NwAC) flowing along the shelf and shelf break and then denser waters offshore.

1. Start with the primitive equations and then present the arguments/assumptions that lead to the a) geostrophic and b) hydrostatic approximations.

2. From the geostrophic x-momentum equation and the hydrostatic approximation, derive the 'thermal wind' relationship for the vertical shear of the northward geostrophic velocity (appropriate along the Norwegian coast):

$$\frac{\partial v_g}{\partial z} = -\frac{g}{\rho_0}\frac{\partial \rho}{\partial x}.$$

3. Using the density field in Figure 1 and the thermal wind relationship above, make a rough estimate of the difference in geostrophic velocity between the surface and a depth of 100 m at station 280 (just over the shelf break).

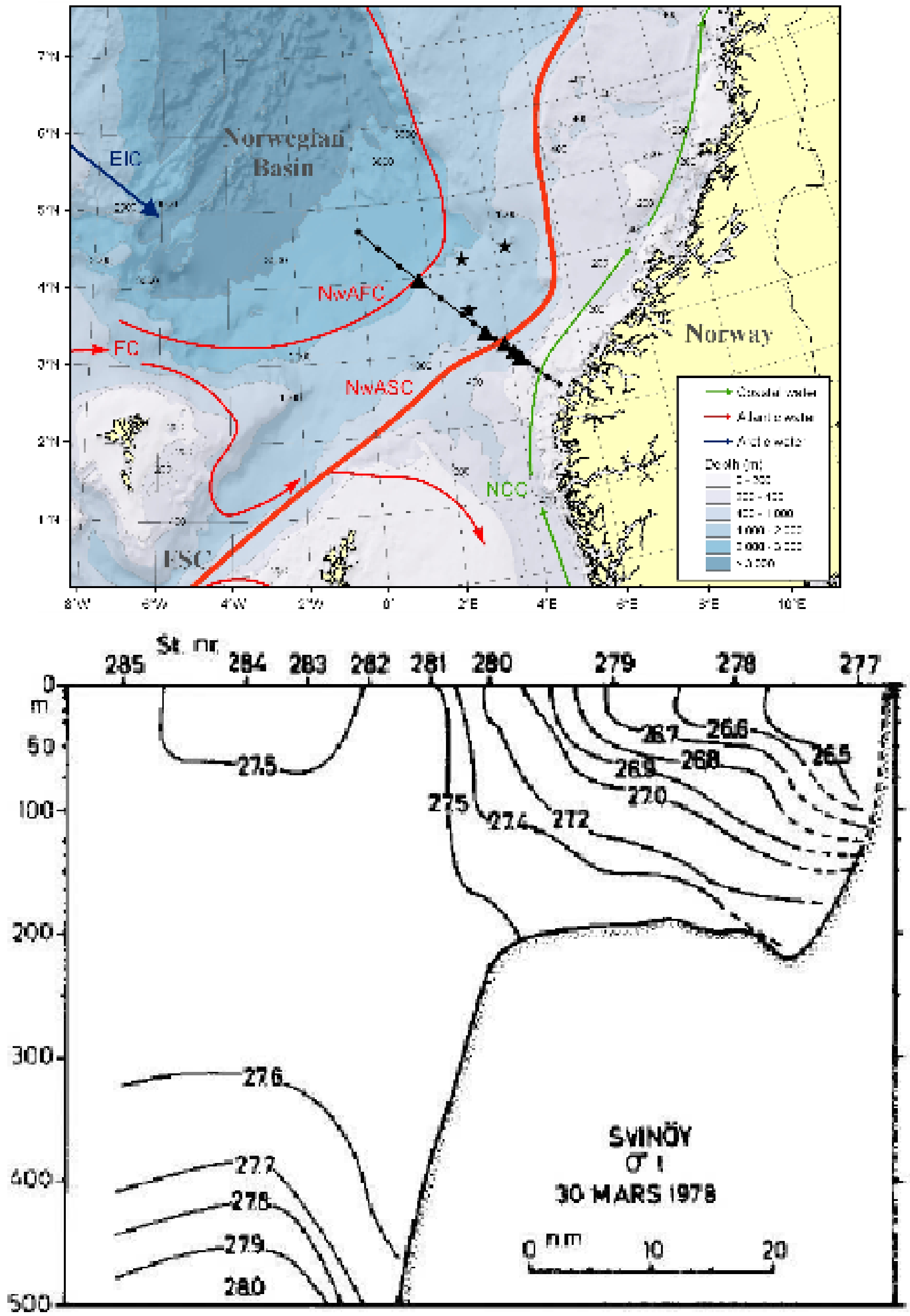

Figure 1: Potential density, σ_0, across the Svinøy section off the west coast of Norway (location shown on top). Density contours are shown at increments of $0.1\,\mathrm{kg\,m^{-3}}$.

Problem 2: Potential vorticity

The concept of potential vorticity is central to understanding large-scale oceanographic flows. From the shallow water equations, we can derive two important conservation equations, both of which depend on the horizontal divergence:

1. Explain the physical significance of the equation

$$\frac{1}{H}\frac{DH}{Dt} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right),$$

   where H is the total water depth.

2. Explain the physical significance of the equation

$$\frac{1}{(f+\zeta)}\frac{D}{Dt}(f+\zeta) = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right),$$

   where $f+\zeta$ is the total vorticity.

3. In Figure 2 we have depicted three water columns. Two of these have non-zero relative vorticity as they have been displaced to either deeper or shallower locations. Explain why their relative vorticity change, and also indicate on which side of the centerline we have deep and shallow water, respectively. Finally, indicate how the two displaced water columns rotate relative to the reference frame.

4. The following quotation is taken from the abstract of a research article (M. Spall and J. Price: Mesoscale Variability in Denmark Strait: The PV Outflow Hypothesis, *Journal of Physical Oceanography*, vol. 28, pp. 1598–1623, 1998) about ocean flows through the Denmark Strait (between Greenland and Iceland):

   > "The [southward] outflow through Denmark Strait shows remarkable mesoscale variability characterized by the continuous formation of intense mesoscale cyclones [positive relative vorticity] just south of the sill. These cyclones have a diameter of about 30 km and have clear signatures at the sea surface and in currents measured near the bottom."

   Find out about the main currents in the Denmark Strait (Google it!) and draw what you think the authors are describing here. Explain, using the concept of concervation of potential vorticity (which you've used above), why they observe all these cyclones in the outflows.

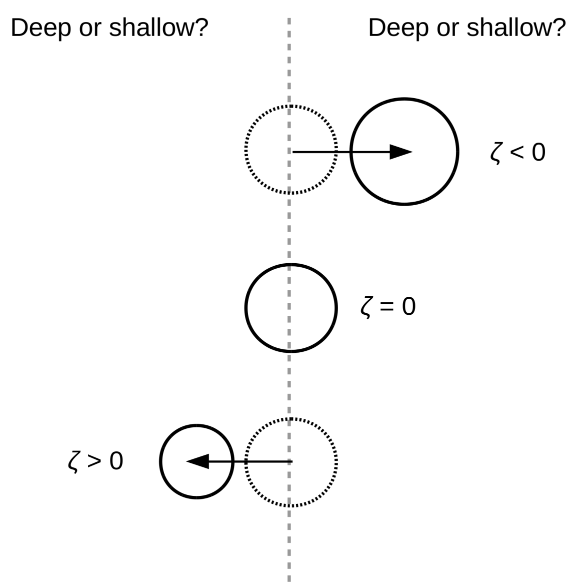

Figure 2: The grey centerline indicate the boundary between deep and shallow waters. Two water columns are displaced away from the centerline, and as a result experience a change in the relative vorticity.

Problem 3: Ekman transport and Ekman pumping

The winds are blowing steadily eastward over the ocean in the northern hemisphere mid-latitudes.

1. The wind stress is $\tau_x = 0.1\,\mathrm{Nm}^{-2}$. Assuming a Coriolis parameter $f = 10^{-4}\,\mathrm{s}^{-1}$ and a water density $\rho_0 = 1000\,\mathrm{kg\,m}^{-3}$, estimate the depth-integrated Ekman transport in the upper ocean. In which direction is it pointing?

2. Assume now that the zonal winds decay linearly in the northward direction and attain zero strength over a distance of 100 km. Explain how this causes a vertical velocity at the bottom of the oceanic Ekman layer. What is the size of this vertical velocity in this particular case? Is it pointing up or down?

3. We have seen how the Ekman pumping out of the surface layer can be balanced by bottom Ekman pumping. Can you think of dynamical processes in the *interior* of the ocean (away from the top or bottom boundaries) that can balance the surface Ekman pumping?

Problem 4: A wind-driven Sverdrup gyre

We want to examine the wind-driven transport in a flat-bottom square basin in the *southern hemisphere*. The basin has dimensions $5000\,\mathrm{km} \times 5000\,\mathrm{km}$ and we assume a so-called 'beta-plane' with $f_0 = -1 \times 10^{-4}\mathrm{s}^{-1}$ and $\beta = 1.5 \times 10^{-11}\mathrm{m}^{-1}\mathrm{s}^{-1}$. We also assume a purely zonal wind stress with the meridional profile

$$\tau_x(y) = \tau_0 \cos\left(\frac{\pi}{L_y} y\right),$$

where y is the distance from the southern boundary, $\tau_0 = 0.1\,\mathrm{Nm}^{-2}$ and $L_y = 5000\,\mathrm{km}$.

1. Draw, qualitatively, the surface Ekman transport and explain in what way this drives flows below the Ekman layer.

2. Write down the Sverdrup balance for the depth-integrated flow in the interior of the basin and describe how the expression has been derived.

3. Is the Sverdrup transport in this gyre, with the given wind stress, northward or southward? Draw, qualitatively, the depth-integrated flow field (with directions), including the position of a Stommel frictional boundary layer.

4. The width of the Stommel boundary layer can be estimated by examining the balance expected to hold in this boundary layer, i.e. a balance between the advection of planetary vorticity and bottom friction:

$$\beta V = R \frac{\partial v}{\partial x}.$$

   Scale this equation and find the representative (zonal) length scale, i.e. the width of the boundary current. Assume that the ocean has depth $H = 3000\,\mathrm{m}$ and that $V = Hv$. Finally, use bottom friction coefficient $R = 1 \times 10^{-3}\mathrm{m\,s}^{-1}$.

5. What is the transport in the boundary current, at $y = 2500\,\mathrm{km}$, i.e. at the center 'latitude' of the domain (magnitude and direction)? (Hint: The meridional transport in the boundary current has to balance the meridional flow in the interior.)