Problem set 2: GEF4500: Due: 7 Oct., 2010

Problem 1: QG expansion for the ocean.

Derive the oceanic QG vorticity equation (94), using the Boussinesq and β-plane approximations. Then use this with the QG density equation to derive the QGPV equation in terms of the streamfunction, $\psi = p/(\rho_c f_0)$.

Problem 2: A larger domain

What if the range of latitude is so large so that $\beta L \approx f_0$? What can you infer about the size of the vertical velocity? What does that imply about the Lagrangian derivative? You can use the Boussinesq approximation to keep things simple.

Integrate the expression that you now have for $\frac{\partial}{\partial z} w$, assuming the fluid is barotropic. Take the bottom to be flat, and ignore the bottom Ekman layer. Say there is a surface Ekman layer. What does the equation tell you if there is a negative wind stress curl?

Problem 3: Free surface

Assume the layer is between z_0 and z_1. Assume the bottom is flat and there are no Ekman layers. Now let the upper surface *move*. Use the condition on w on the upper surface, then rewrite the vorticity equation. Assume the *undisturbed* surface is at $z = 0$ and the perturbed surface is at $z = \eta$. What assumption do you need to make about η to be consistent with quasi-geostrophy?

Make that assumption, and determine what the new conserved quantity is following the fluid motion. What happens when the surface is depressed (pushed down) if there is no vorticity initially?