Problem set 4: GEF4500: Due: 22 Nov., 2010

Problem 1: Closed basin

Consider a circular basin, centered at $(x, y) = (0, 0)$. The wind stress is given by $\vec{\tau} = A\, y\, sin(3t)\ \hat{i}$. Find an expression for the mean velocity at a radius R. What is the mean velocity in the limit of weak bottom friction $r \to 0$? Explain the sense of the flow in relation to the surface Ekman transport.

Problem 2: Barotropic instability

We have a region with $0 \leq x < 1$ and $-1 \leq y < 1$. Consider the following velocity profiles:

a) $U = 1 - y^2$
b) $U = exp(-y^2)$
c) $U = sin(\pi y)$
d) $U = \frac{1}{6}y^3 + \frac{5}{6}y$

Which profiles are unstable by the Rayleigh-Kuo criterion if $\beta = 0$? Which profile is stable by the Fjørtoft criterion? How large must β be to stabilize *all* the profiles? Note that the terms here have been non-dimensionalized, so that β can be any number (e.g. an integer).

Problem 3: Mountain waves (Holton, 12.3)

Suppose that a stationary linear Rossby wave is forced by flow over sinusoidal topography with height $h(x) = h_0\, cos(kx)$. Show that the lower boundary condition on the streamfunction can be expressed as:

$$\frac{\partial}{\partial z}\psi = -\frac{hN^2}{f_0} \tag{1}$$

Using this, and an appropriate upper boundary condition, solve for $\psi(x, z)$ in the case $|m| \gg (1/2H)$. How does the position of the trough (low crest) relative to the mountain ridge depend on the sign of m^2 in this limit?