Final Exam: GEF4500

Permitted aids (hjelpemidler):

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Calculator

Problem 1: Perturbation method

Consider the following equation, with $\epsilon \ll 1$:

$$\frac{\partial \phi}{\partial t} + \epsilon \frac{\partial}{\partial x}(\phi^2) - a \frac{\partial^3 \phi}{\partial x^3} = 0$$

a) Expand ϕ in a perturbation series in ϵ and find an equation for ϕ_0

b) Use a (real) sinusoidal solution in the first order equation to find the phase speed. Are the waves dispersive? In which direction do they propagate?

c) Write the equation for ϕ_1. Notice this can be expressed as a forced wave equation. What is the frequency of the forcing?

Problem 2: The PV equation

The PV equation in log-pressure coordinates is given by:

$$(\frac{\partial}{\partial t} + \vec{u}_g \cdot \nabla)\,[\nabla^2 \psi + \frac{1}{\rho_0} \frac{\partial}{\partial z^*}(\frac{\rho_0 f_0^2}{N^2} \frac{\partial \psi}{\partial z^*}) + \beta y] = 0$$

a) Write the expression for ρ_0.

b) Write the equation in the case that the scale height is much greater than the depth of the fluid, D, and the Brunt-Vaisala frequency is constant.

c) Simplify the equation for a barotropic fluid.

d) Compare this equation with the barotropic PV equation that we discussed in class. What is missing? Where do these terms come from?

Problem 3: Geostrophic contours

Consider a portion of a closed basin in the ocean, indicated by the box in Fig. (1).

a) Assume the ocean is barotropic and that $\beta = 0$. Write the potential vorticity equation (including forcing terms).

b) The geostrophic contours, q_s, are the stationary (time-independent) part of the potential vorticity. What is q_s here?

c) Assume there is no wind forcing or Ekman drag. Is a time mean flow possible in the box? If so, in what direction?

d) Now imagine that the box boundaries are solid walls. Is a mean flow possible without wind forcing?

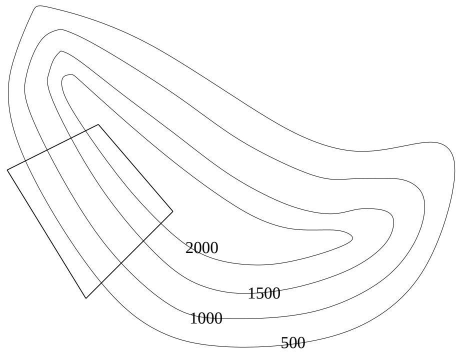

Figure 1: An ocean basin. The contours are lines of constant depth, and the region considered in the problem is indicated by the box.

e) Now imagine that there is a negative wind stress curl over the box (again with solid walls). Which way does the interior flow move relative to the depth contours?

f) Where would you expect to find a frictional boundary current in the box?

Problem 4: Instability

a) Consider a barotropic flow without forcing or topography and a mean flow:

$$U = const.$$

Linearize the potential vorticity equation, assuming $\beta \neq 0$. Is instability possible?

b) Consider a barotropic flow without forcing or topography and a mean flow:

$$U = Ay$$

Linearize the potential vorticity equation, assuming $\beta \neq 0$. Is instability possible?

c) Consider a barotropic flow without forcing or topography and a mean flow:

$$U = Ay^2$$

Linearize the potential vorticity equation, assuming $\beta \neq 0$. Is instability possible?

d) Consider a *baroclinic* flow without forcing or topography, with $\beta \neq 0$. and a mean flow:

$$U = Ay + Bz$$

Assume that the scale height is much greater than the depth of the fluid, D, and the Brunt-Vaisala frequency is constant. Linearize the potential vorticity equation and the temperature equation on the boundaries, assuming two rigid plates ($w = 0$) at $z = 0$ and $z = D$. Is instability possible?