GEF4610 – OBLIGATORY EXERCISE 1
All symbols are defined in the GEF4610 Lecture Notes

Problem 1

The equations governing the Ekman currents are:

$$0 = fv + A^{(z)} \frac{d^2 u}{dz^2}, \qquad (1.1)$$
$$0 = -fu + A^{(z)} \frac{d^2 v}{dz^2}.$$

a) State the assumptions behind this set of equations.

b) We apply a constant wind stress τ along the y-axis at the surface $z = 0$, and a no-slip bottom $(u = v = 0)$ at $z = -H$. Solve (1.1) and show that the complex solution $W = u + iv$ may be written

$$W(z) = \frac{i\tau}{\rho_r a A^{(z)}} \frac{\sinh((z+H)a)}{\cosh(Ha)}, \qquad (1.2)$$

where $a^2 = if / A^{(z)}$.

c) We now consider the angle α between the wind direction and the surface velocity. Express (1.2) in terms of the Ekman depth $D_E = \pi(2A^{(z)}/f)^{1/2}$, and apply a numerical software, e.g. matlab, to plot α for various values of the ratio D_E / H. Discuss your findings.

Problem 2

a) State the assumptions leading to the equations

$$u_t + uu_x + vu_y - fv = -g\eta_x - P_{Sx}/\rho,$$
$$v_t + uv_x + vv_y + fu = -g\eta_y - P_{Sy}/\rho, \qquad (2.1)$$
$$\eta_t + \left((H+\eta)u\right)_x + \left((H+\eta)v\right)_y = 0,$$

and derive the conservation equation for the potential vorticity:

$$\frac{DQ}{dt} \equiv \frac{D}{dt} \frac{f+\zeta}{H+\eta} = 0. \qquad (2.2)$$

This means that Q is constant for a given fluid column. We will use these equations on an f-plane, and consider steady flow in this problem.

b) We have constant geostrophic flow U_0 in the x-direction in a layer of constant depth H_0. What are the surface pressure gradients in this case?

c) We introduce a small sub-sea ridge $q(x)$ of width $2L$ which is infinitely long in the y-direction. The fluid depth then becomes $H = H_0 - q$; see Fig. 1.

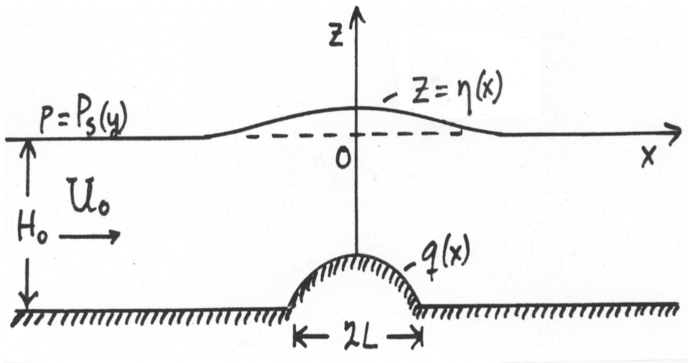

Figure 1. Figure sketch.

The flow will now be modified. The velocities in the x- and y-directions can be written $U_0 + u(x)$ and $v(x)$, respectively, where u and v are small compared to U_0. Assume that the velocity far upstream of the ridge is U_0, the depth is H_0, and the relative vorticity is zero. Show from (2.2) that

$$v_x = \frac{f}{H_0}(\eta - q). \tag{2.3}$$

d) Show that the linearized momentum equations in (2.1) becomes

$$\begin{aligned} U_0 u_x - fv &= -g\eta_x, \\ U_0 v_x + fu &= 0. \end{aligned} \tag{2.4}$$

e) Combine (2.3) and (2.4), and show that

$$(1 - F_0^2)\eta_{xx} - \frac{1}{a^2}\eta = -F_0^2 q_{xx} - \frac{1}{a^2} q, \tag{2.5}$$

where

$$F_0 = \frac{U_0}{(gH_0)^{1/2}}, \quad a = \frac{(gH_0)^{1/2}}{f}. \tag{2.6}$$

F_0 and a are important fluid parameters. What is their significance?

f) Assume subcritical flow, and take that $\eta \to 0,\ x \to \pm\infty$. Use that d/dx^2 scales as $1/L^2$ in order to simplify (2.5). Discuss the surface elevation over the ridge in the extreme cases $L^2 << a^2$, and $L^2 >> a^2$.

g) Sometimes the rigid lid approximation is used for flow over a ridge (the surface is horizontal everywhere). Now (2.3) reduces to

$$v_x = -\frac{f}{H_0} q, \tag{2.7}$$

i.e. anti-cyclonic relative vorticity is generated over the entire ridge. Show from (2.7) that v downstream of the ridge is constant, and dependent on the cross-sectional area A of the ridge, given by

$$A = \int_{-L}^{L} q(x)dx. \tag{2.8}$$

h) Use the continuity equation in (2.1), and find u in this case.

i) Show that the angle between the current upstream of the ridge and the current downstream of the ridge in the case of a rigid lid is given by

$$\theta = \arctan\left(\frac{fA}{U_0 H_0}\right). \tag{2.9}$$