HYD2010/2020

Formelhefte

Statistiske formler

Gjennomsnitt for observasjoner:

$$\hat{\mu} = m_X = \frac{1}{N}\sum_{i=1}^{N} x_i \qquad \text{(CB2-1)}$$

Varians for observasjoner:

$$\hat{\sigma}^2 = S_X{}^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - m_X\right)^2 \qquad \text{(CB2-5)}$$

Variasjonskoeffisient for observasjoner:

$$C\hat{V}_X = \frac{S_X}{m_X} \qquad \text{(CB2-6)}$$

Korrelasjon mellom to observerte variabler:

$$\hat{\rho}_{X,Y} = r_{X,Y} = \frac{\sum_{i=1}^{N}\left(x_i - m_X\right)\cdot\left(y_i - m_Y\right)}{\left(N-1\right)\cdot S_X \cdot S_Y} \qquad \text{(CB5-1)}$$

Kumulativ sannsynlighetsfordeling:

$$P_X\left(x_i\right) \equiv \Pr\left\{X \le x_i\right\} = \sum_{alle\ X \le x_i} p_x(x_i) \qquad \text{(C-6)}$$

Empirisk estimat av kumulativ sannsynlighetsfordeling:

$$q = \frac{i}{N+1} \qquad \text{(CB1-3)}$$

Viss hendelsene A, B , C , ... ikke overlapper så er:

$$\Pr\left\{A \quad eller \quad B \quad eller \quad C \quad eller \quad ...\right\} = \Pr\left\{A\right\} + \Pr\left\{B\right\} + \Pr\left\{C\right\}... \qquad \text{(C-2)}$$

Viss hendelsene A, B , C , ... er uavhengige så er:

$$\Pr\left\{A \quad og \quad B \quad og \quad C \quad og \quad ...\right\} = \Pr\left\{A\right\}\Pr\left\{B\right\}\Pr\left\{C\right\}... \qquad \text{(C-3)}$$

Vannets egenskaper

Vekttetthet:

$$\gamma \equiv \frac{mg}{V} = \rho g \qquad \text{(B-2)}$$

Kapillær stigning:

$$h_{cr} = \frac{2\sigma \cos\theta_C}{\gamma_w r} \qquad \text{(B-7)}$$

Dynamisk viskositet:

$$\mu \equiv \frac{\tau(y)}{du(y)/dy} \qquad \text{(B-10)}$$

Varmekapasitet:

$$c_p \equiv \frac{\Delta H}{M \cdot \Delta T} \qquad \text{(B-19)}$$

Verdier ved 0 °C (her brukes subskript w for vann og subskript i for is)

Tetthet,	$\rho_w = 1000$ kg m^{-3}
Vekttetthet,	$\gamma_w = 9799$ N m^{-3}
Dynamisk viskositet,	$\mu_w = 1.787 \cdot 10^{-3}$ Pa·s (Nsm^{-2})
Overflatespenning,	$\sigma_w = 7.56 \cdot 10^{-2}$ N m^{-1}
Varmekapasitet for vann,	$c_w = 4216$ J kg^{-1} K^{-1}
Varmekapasitet for is,	$c_i = 2102$ J kg^{-1} K^{-1}
Latent varme, smeltevarme	$\lambda_f = 3.34 \cdot 10^5$ J kg^{-1}
fordampningsvarme	$\lambda_v = 2.495 \cdot 10^6$ J kg^{-1}
Varmeledningsevne,	$k_\theta = 3.37 \cdot 10^{-3}$ J s^{-1}m^{-1}K^{-1}

Meteorologiske elementer

Mettet vanndampdtrykk, e_{sat}:

$$e_s^* = e_{sat}(T) = 0.611 \cdot \exp\left(\frac{17.3 \cdot T_s}{T_s + 237.3}\right) \qquad \text{(7-4)}$$

der vanndamptrykket er i kPa og temperaturen i °C

Psykrometerkonstanten:

$$\gamma = \frac{c_a P}{0.622 \lambda_v} \qquad \text{(7-13)}$$

Helning av metningsdamptrykket:

$$\Delta = \frac{e_s^* - e_a^*}{T_s - T_a} \qquad (7B1-3)$$

Penmans formel:

$$E = \frac{\Delta * (K + L) + \gamma * K_E * \rho_w * \lambda_v * v_a * e_a^* * (1 - W_a)}{\rho_w * \lambda_v * (\Delta + \gamma)} \qquad (7-33)$$

Vindhastighet i høyden z over bakken:

$$v_a = \frac{1}{k} \cdot u_* \cdot \ln\left(\frac{z - z_d}{z_0}\right), z > z_d + z_0 \qquad (D-22)$$

Energibalanse

Følbar varme, H:

$$H = c_a \rho_a \frac{k^2}{\left\{\ln\left(\frac{z_m - z_d}{z_0}\right)\right\}^2} \cdot v_a \cdot (T_s - T_a) \qquad (7-9/7-10)$$

$der\ k = 0.4\ (von\ Karmans\ konstant)$

$$H = K_H\ v_a\ (T_s - T_a) \qquad (7-9)$$

Latent varme, LE:

$$LE = \lambda_V \frac{0.622 \rho_a}{P} \frac{k^2}{\left\{\ln\left(\frac{z_m - z_d}{z_0}\right)\right\}^2} \cdot v_a \cdot (e_s - e_a) \qquad (7-2/7-7)$$

$$LE = \rho_w \cdot \lambda_v \cdot E \qquad (7-7)$$

$$E = K_E \cdot v_a \cdot (e_s - e_a) \qquad (7-1)$$

Bowen forholdet:

$$B = \frac{H}{LE} = \frac{c_a \cdot P \cdot (T_s - T_a)}{0.622 \cdot \lambda_v \cdot (e_s - e_a)} = \gamma \cdot \frac{(T_s - T_a)}{(e_s - e_a)} \qquad \text{(7-11/12/13)}$$

$$B = \frac{H}{LE} = \frac{c_a \cdot P \cdot (T_1 - T_2)}{0.622 \cdot \lambda_v \cdot (e_1 - e_2)} = \gamma \frac{(T_1 - T_2)}{(e_1 - e_2)} \qquad \text{(7-78)}$$

Innkommende langbølga stråling:

$$L_{in} = \varepsilon_{at} \cdot \sigma \cdot (T_{at} + 273.2)^4$$
$$der\, \sigma\, er\ Stefan - Boltzman\, kons\tan t = 4.90 \cdot 10^{-9}\, MJm^{-2} day^{-1} K^{-4} \qquad \text{(5.35)}$$

Utgående langbølga stråling:

$$L_{out} = \varepsilon_s \cdot \sigma \cdot (T_s + 273.2)^4 + (1 - \varepsilon_s) \cdot L_{in}$$
$$der\ _s\ står\ for\ ved\ overflaten\ generelt\ og \qquad \text{(5.36)}$$
$$\sigma\ er\ Stefan - Boltzman\, kons\tan t = 4.90 \cdot 10^{-9}\, MJm^{-2} day^{-1} K^{-4}$$

Lagret termal energi i en innsjø:

$$\Delta Q = \frac{c_w \rho_w}{A_L}(V_2 T_{L2} - V_1 T_{L1}) \qquad \text{(7-32)}$$

Fordampning

$$LE\left[ET^{-1}\right] = \lambda_v \left[EM^{-1}\right] \cdot E\left[MT^{-1}\right] \qquad \text{(D-13a)}$$
$$LE\left[EL^{-2}T^{-1}\right] = \lambda_v \left[EM^{-1}\right] \rho_w \left[ML^{-3}\right] \cdot E\left[LT^{-1}\right] \qquad \text{(D-14a)}$$

$[EL^{-2}T^{-1}]$ gis i SI enheter som $Jm^{-2}s^{-1} = Wm^{-2}$

Penmans ligning:

$$E = \frac{\Delta \cdot (K + L) + \rho_a \cdot c_a \cdot C_{at} \cdot e_a^* \cdot [1 - W_a]}{\rho_W \lambda_V \{\Delta + \gamma\}}$$
$$der \qquad \text{(7-33/7-55)}$$
$$\rho_a \cdot c_a \cdot C_{at} = \gamma \cdot K_E \cdot \rho_w \cdot \lambda_v \cdot v_a$$

Atmosfærisk ledningsevne:

$$C_{at} \equiv \frac{v_a}{6.25\left\{\ln\left[(z_m - z_d) \big/ z_0\right]\right\}^2} \tag{7-49}$$

Penman-Monteiths ligning:

$$E = \frac{\Delta \cdot (K + L) + \rho_a \cdot c_a \cdot C_{at} \cdot e_a^* \cdot [1 - W_a]}{\rho_W \cdot \lambda_V \cdot \left\{\Delta + \gamma\left[1 + C_{at} \big/ C_{can}\right]\right\}} \tag{7.56}$$

Snø

Kuldeinhold:

$$Q_{cc} = -c_i \cdot \rho_w \cdot h_m \cdot (T_s - T_m) \tag{5.13}$$

Flytende vanninnhold i snø:

$$h_{wret} = \theta_{ret} \cdot h_s \tag{5.14}$$

Smelteenergi for snø:

$$\Delta Q = \rho_w \cdot \lambda_f \cdot \Delta h_w \tag{5.21}$$

Markvann

Volumvekt:

$$\rho_b \equiv \frac{M_m}{V_s} \tag{6-2}$$

Darcys ligning:

$$q_x = -K_h \cdot \frac{d\left(z + p \big/ \gamma_w\right)}{dx} \tag{6-8a}$$

der

$\gamma_w = \rho_w g$

Grunnvann

Darcys ligning:

$$V_x \equiv \frac{Q}{A_x} = -K_{hx}\frac{d\left[z + \frac{p}{\gamma_w}\right]}{dx} \qquad (8\text{-}1/2)$$

Spesifikt magasin

$$S_s = \frac{S}{H} \qquad (8\text{-}7)$$

der S er magasinkoeffesienten

Boussinesqs ligning:

$$\frac{\partial}{\partial x}\left(K_{hx}\cdot\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_{hy}\cdot\frac{\partial h}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_{hz}\cdot\frac{\partial h}{\partial z}\right)=S_s\cdot\frac{\partial h}{\partial t} \qquad (8\text{-}14)$$

Fersk-saltvann grense

$$z_s(x) = \frac{\gamma_f}{\gamma_s - \gamma_f}h_f(x) \qquad (8B2\text{-}3/8B2\text{-}4)$$

To dimensjonal form av (8-14) for radiell strømning i en homogen og isotropisk aquifer (polare koordinater):

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial h}{\partial r} = \frac{S_y}{K_h h_0}\cdot\frac{\partial h}{\partial t} \qquad (8\text{-}59)$$

Theis løsning til ligning (8-59):

$$h_0 - h(r,t) = \frac{Q_w}{4\cdot\pi\cdot K_h\cdot h_0}\cdot W[u(r,t)] \qquad (8\text{-}60)$$

der $W[u(r,t)]$ *er brønnfunksjonen, og*

$$u(r,t) \equiv \frac{S_y \cdot r^2}{4\cdot K_h\cdot h_0\cdot t} \qquad (8\text{-}62)$$

Frost i jord

Teledyp:

$$z_t = \sqrt{\frac{2k_\theta F(t)}{\rho_w \theta \lambda_f}}$$ (Fra frostkompendium)

Vannføring

Vertikalt hastighetsprofil:

$$u(y_i) = 2.5 \cdot u_{*i} \cdot \ln\left(\frac{y_i}{y_{0i}}\right) \quad \text{(F-5)}$$

Friksjonshastighet:

$$u_{*i} = \left(g \cdot Y_i \cdot S_c\right)^{1/2} \quad \text{(F-6)}$$

Vannføring for et v-profil:

$$Q = C_W \cdot g^{1/2} \cdot \tan\left(\frac{\theta_v}{2}\right) \cdot \left(Z_W - Z_V\right)^{5/2} \quad \text{(F-17)}$$

Flom og ekstremverdianalyse

Gjentaksintervall:

$$TR_X(x) = \frac{1}{1 - F_X(x)}$$ (C-32)

Gumbelfordeling:

$$\Pr\{X \le x\} = F_X(x) = \exp\left[-\exp\left(-\left(\frac{x-\xi}{\alpha}\right)\right)\right]$$ Tabell C-1

$$\mu_x = \xi + 0.5772\alpha$$
$$\sigma_x^2 = 1.645\alpha^2$$

Bruk av frekvensfaktor, K, for bestemmelse av verdier med gjentaksintervall TR:

$$x(TR) = E(X) + K(TR)S(X)$$ (Side 9 i flomkompendiet)

for Gumbelfordeling gjelder:

$$K(10) = 1.30$$
$$K(100) = 3.14$$
$$K(1000) = 5.00$$