Fairness

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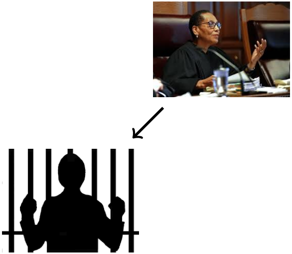
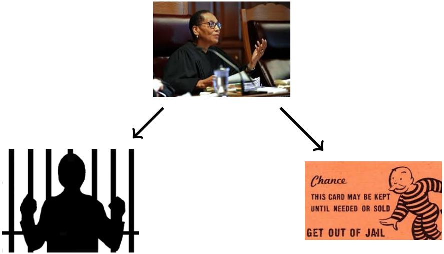
September 27, 2018

Bail decisions

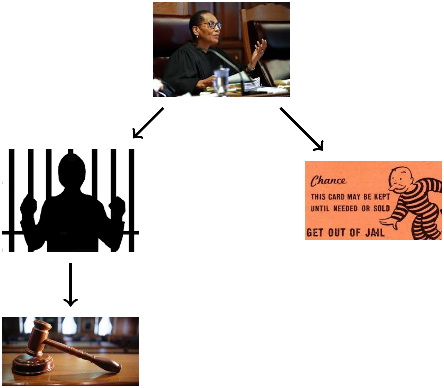
Bail decisions



Bail decisions



Bail decisions



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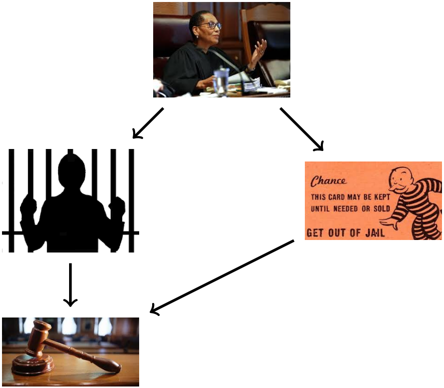

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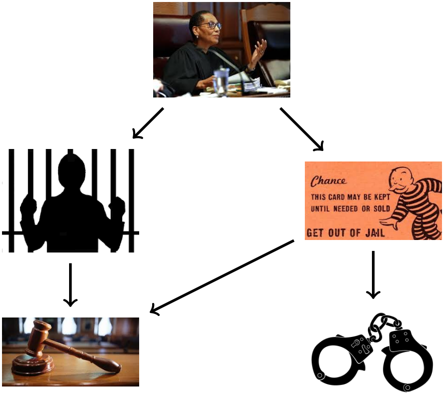

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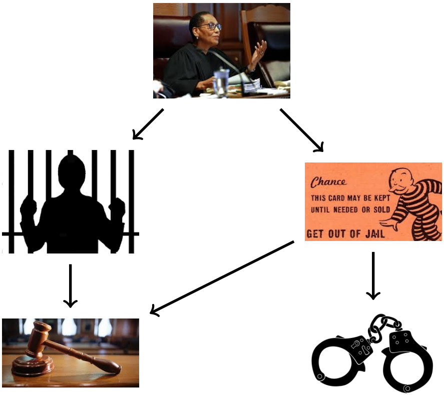
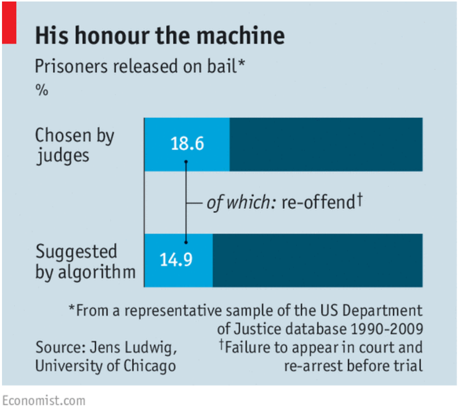
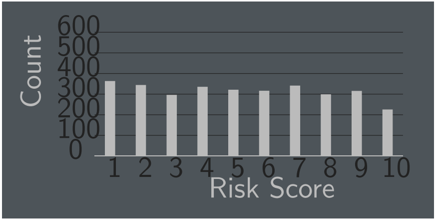
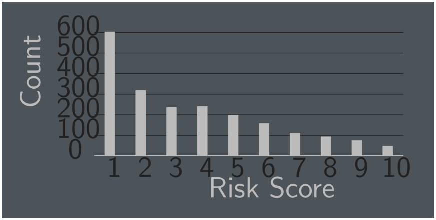




Whites get lower scores than blacks[1]



Black



White

Figure: Apparent bias in risk scores towards black versus white defendants.

[1] Pro-publica, 2016

But scores equally accurately predict recidivsm[2]

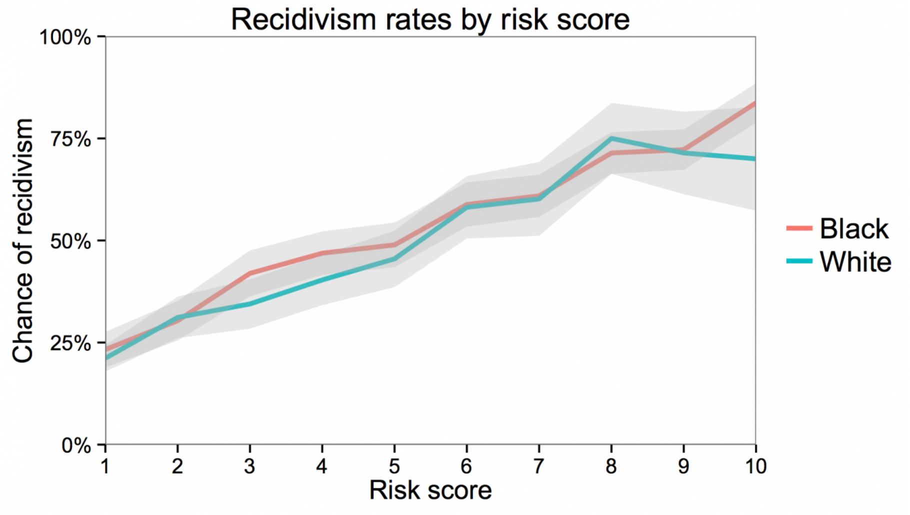


Figure: Recidivism rates by risk score.

[2]Washington Post, 2016

But non-offending blacks get higher scores

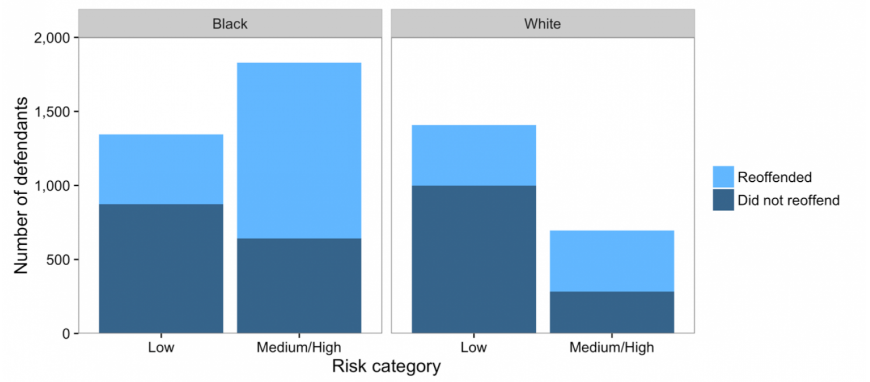


Figure: Score breakdown based on recidivism rates.

Graphical models and independence

- Why is it not possible to be fair in all respects?
- Different notions of conditional independence.
- Can only be satisfied rarely simultaneously.

Graphical models

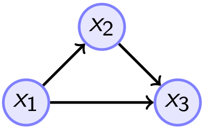


Figure: Graphical model (directed acyclic graph) for three variables.

Joint probability

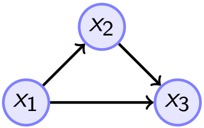
Let $\mathbf{x} = (x_1, \ldots, x_n)$. Then $\mathbf{x} : \Omega \to X$, $X = \prod_i X_i$ and:

$$\mathbb{P}(\mathbf{x} \in A) = P(\{\omega \in \Omega \mid \mathbf{x}(\omega) \in A\}).$$

Factorisation

$$\mathbb{P}(\mathbf{x}) = \mathbb{P}(\mathbf{x}_B \mid \mathbf{x}_C)\, \mathbb{P}(\mathbf{x}_C), \qquad B, C \subset [n]$$

Graphical models



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$$\mathbb{P}(\mathbf{x} \in A) = P(\{\omega \in \Omega \mid \mathbf{x}(\omega) \in A\}).$$

Factorisation

So we can write any joint distribution as

$$\mathbb{P}(x_1)\,\mathbb{P}(x_2 \mid x_1)\,\mathbb{P}(x_3 \mid x_1, x_2) \cdots \mathbb{P}(x_n \mid x_1, \ldots, x_{n-1}).$$

Directed graphical models

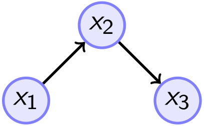


Figure: Graphical model for the factorisation $\mathbb{P}(x_3 \mid x_2) \, \mathbb{P}(x_2 \mid x_1) \, \mathbb{P}(x_1)$.

Conditional independence

We say x_i is conditionally independent of $\mathbf{x}_B$ given $\mathbf{x}_D$ and write $x_i \mid \mathbf{x}_D \perp\!\!\!\perp \mathbf{x}_B$ iff

$$\mathbb{P}(x_i, \mathbf{x}_B \mid \mathbf{x}_D) = \mathbb{P}(x_i \mid \mathbf{x}_D) \, \mathbb{P}(\mathbf{x}_D \mid \mathbf{x}_B).$$

Example 1 (Smoking and lung cancer)

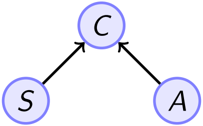


Figure: Smoking and lung cancer graphical model, where S: Smoking, C: cancer, A: asbestos exposure.

Explaining away

Even though S, A are independent, they become dependent once you know C.

Example 2 (Time of arrival at work)

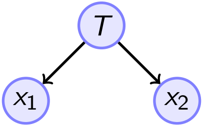


Figure: Time of arrival at work graphical model where T is a traffic jam and x_1 is the time John arrives at the office and x_2 is the time Jane arrives at the office.

Conditional independence

Even though x_1, x_2 are correlated, they become independent once you know T.

Example 3 (Treatment effects)

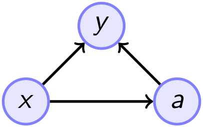


Figure: Kidney treatment model, where x: severity, y: result, a: treatment applied

	Treatment A	Treatment B
Small stones	87	270
Large stones	263	80

Severity	Treatment A	Treatment B
Small stones)	93%	87%
Large stones	73%	69%
Average	78%	83%

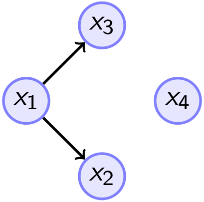
Example 4 (School admission)

Figure: School admission graphical model, where z: gender, s: school applied to, a: whether you were admitted.

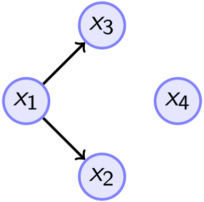
School	Male	Female
A	62%	82%
B	63%	68%
C	37%	34%
D	33%	35%
E	28%	24%
F	6%	7%
Average	*45%*	*38%*

Exercise 1

Factorise the following graphical model.



Exercise 1

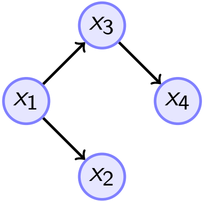


Factorise the following graphical model.

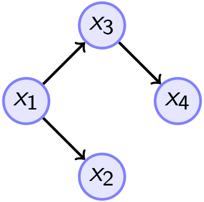
$$\mathbb{P}(\mathbf{x}) = \mathbb{P}(x_1)\,\mathbb{P}(x_2 \mid x_1)\,\mathbb{P}(x_3 \mid x_1)\,\mathbb{P}(x_4)$$

Exercise 2

Factorise the following graphical model.



Exercise 2



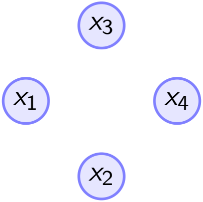
Factorise the following graphical model.

$$Xb\,\mathbb{P}(\mathbf{x}) = \mathbb{P}(x_1)\,\mathbb{P}(x_2 \mid x_1)\,\mathbb{P}(x_3 \mid x_1)\,\mathbb{P}(x_4 \mid x_3)$$

Exercise 3

What dependencies does the following factorisation imply?

$$\mathbb{P}(\mathbf{x}) = \mathbb{P}(x_1)\,\mathbb{P}(x_2 \mid x_1)\,\mathbb{P}(x_3 \mid x_1)\,\mathbb{P}(x_4 \mid x_2, x_3)$$



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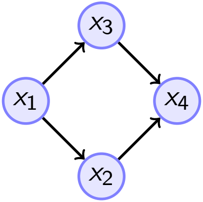


Deciding conditional independence

There is an algorithm for deciding conditional independence of any two variables in a graphical model.

Measuring independence

Theorem 5

If $x_i \mid \mathbf{x}_D \perp\!\!\!\perp \mathbf{x}_B$ *then*

$$\mathbb{P}(x_i \mid \mathbf{x}_B, \mathbf{x}_D) = \mathbb{P}(x_i \mid \mathbf{x}_D)$$

Example 6

$$\| \mathbb{P}(a \mid y, z) - \mathbb{P}(a \mid y) \|_1$$

which for discrete a, y, z is:

$$\max_{i,j} \| \mathbb{P}(a \mid y = i, z = j) - \mathbb{P}(a \mid y = i) \|_1 = \max_{i,j} \| \sum_k \mathbb{P}(a = k \mid y = i, z = j) -$$

Measuring independence

Theorem 5

If $x_i \mid \mathbf{x}_D \perp\!\!\!\perp \mathbf{x}_B$ *then*

$$\mathbb{P}(x_i \mid \mathbf{x}_B, \mathbf{x}_D) = \mathbb{P}(x_i \mid \mathbf{x}_D)$$

This implies

$$\mathbb{P}(x_i \mid \mathbf{x}_B = b, \mathbf{x}_D) = \mathbb{P}(x_i \mid \mathbf{x}_B = b', \mathbf{x}_D)$$

so we can measure independence by seeing how the distribution of x_i changes when we vary $\mathbf{x}_B$, keeping $\mathbf{x}_D$ fixed.

Example 6

$$\| \mathbb{P}(a \mid y, z) - \mathbb{P}(a \mid y) \|_1$$

which for discrete a, y, z is:

$$\max_{i,j} \| \mathbb{P}(a \mid y = i, z = j) - \mathbb{P}(a \mid y = i) \|_1 = \max_{i,j} \| \sum_k \mathbb{P}(a = k \mid y = i, z = j) -$$

Coin tossing, revisited

Example 7

The Beta-Bernoulli prior

Figure: Graphical model for a Beta-Bernoulli prior

$$\theta \sim \mathcal{B}eta(\xi_1, \xi_2), \quad \text{i.e. } \xi \text{ are Beta distribution parameters} \tag{2.1}$$

$$x \mid \theta \sim \mathcal{B}ernoulli(\theta), \quad \text{i.e. } P_\theta(x) \text{ is a Bernoulli} \tag{2.2}$$

Example 8

An alternative model for coin-tossing This is an elaboration of Example **??** for hypothesis testing.

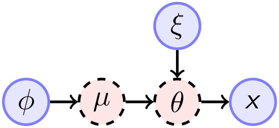


Figure: Graphical model for a hierarchical prior

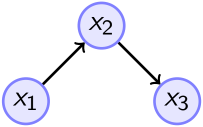
- μ_1: A Beta-Bernoulli model with $\mathcal{B}eta(\xi_1, \xi_2)$
- μ_0: The coin is fair.

$$\theta \mid \mu = \mu_0 \sim \mathcal{D}(0.5), \qquad \text{i.e. } \theta \text{ is always } 0.5 \tag{2.3}$$

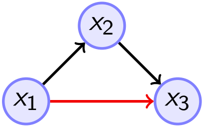
$$\theta \mid \mu = \mu_1 \sim \mathcal{B}eta(\xi_1, \xi_2), \qquad \text{i.e. } \theta \text{ has a Beta distribution} \tag{2.4}$$

$$x \mid \theta \sim \mathcal{B}ernoulli(\theta), \qquad \text{i.e. } P_\theta(x) \text{ is Bernoulli} \tag{2.5}$$

Bayesian testing of independence



(a) Θ_0 assumes independence



(b) Θ_1 does not assume independence

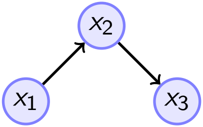
Example 9

Assume data $D = \{x_1^t, x_2^t, x_3^t \mid t = 1, \ldots, T\}$ with $x_i^t \in \{0, 1\}$.

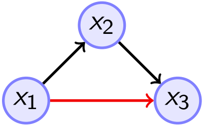
$$P_\theta(D) = \prod_t P_\theta(x_3^t \mid x_2^t) P_\theta(x_2^t \mid x_1^t) P_\theta(x_1^t), \qquad \theta \in \Theta_0 \tag{2.6}$$

$$P_\theta(D) = \prod_t P_\theta(x_3^t \mid x_2^t, x_1^t) P_\theta(x_2^t \mid x_1^t) P_\theta(x_1^t), \qquad \theta \in \Theta_1 \tag{2.7}$$

Bayesian testing of independence



(a) $\mathit{\Theta}_0$ assumes independence



(b) $\mathit{\Theta}_1$ does not assume independence

Example 9

$$\theta_1 \triangleq P_\theta(x_1^t = 1) \qquad (\mu_0, \mu_1)$$

$$\theta_{2|1}^i \triangleq P_\theta(x_2^t = 1 \mid x_1^t = i) \qquad (\mu_0, \mu_1)$$

$$\theta_{3|2}^j \triangleq P_\theta(x_3^t = 1 \mid x_2^t = j) \qquad (\mu_0)$$

$$\theta_{3|2,1}^{i,j} \triangleq P_\theta(x_3^t = 1 \mid x_2^t = j, x_1^t = i) \qquad (\mu_1)$$

Figure: Hierarchical model.

$$\mu_i \sim \phi \tag{2.6}$$

$$\theta \mid \mu = \mu_i \sim \xi_i \tag{2.7}$$

Marginal likelihood

$$\mathbb{P}_\phi(D) = \phi(\mu_0)\,\mathbb{P}_{\mu_0}(D) + \phi(\mu_1)\,\mathbb{P}_{\mu_1}(D) \tag{2.8}$$

$$\mathbb{P}_{\mu_i}(D) = \int_{\Theta_i} P_\theta(D)\,\mathrm{d}\xi_i(\theta). \tag{2.9}$$

Figure: Hierarchical model.

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$$\mathbb{P}_{\mu_i}(D) = \int_{\Theta_i} P_{\theta}(D)\, \mathrm{d}\xi_i(\theta). \tag{2.7}$$

Model posterior

$$\phi(\mu \mid D) = \frac{\mathbb{P}_{\mu}(D)\phi(\mu)}{\sum_i \mathbb{P}_{\mu_i}(D)\phi(\mu_i)} \tag{2.8}$$

Calculating the marginal likelihood

Monte-Carlo approximation

$$\int_{\Theta} P_\theta(D) \, \mathrm{d}\xi(\theta) \approx \sum_{n=1}^{N} P_{\theta_n}(D) + O(1/\sqrt{N}), \qquad \theta_n \sim \xi \tag{2.9}$$

Importance sampling

$$\int_{\Theta} P_\theta(D) \, \mathrm{d}\xi(\theta) \tag{2.10}$$

Calculating the marginal likelihood

Monte-Carlo approximation

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Importance sampling

$$\int_{\Theta} P_{\theta}(D) \, \mathrm{d}\xi(\theta) = \int_{\Theta} P_{\theta}(D) \frac{\mathrm{d}\psi(\theta)}{\mathrm{d}\psi(\theta)} \, \mathrm{d}\xi(\theta) \tag{2.10}$$

Calculating the marginal likelihood

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Sequential updating of the marginal likelihood

$$\mathbb{P}_\xi(D) \tag{2.14}$$

Example 10 (Beta-Bernoulli)

$$\mathbb{P}_\xi(x_t = 1 \mid x_1, \ldots, x_{t-1}) = \frac{\alpha_t}{\alpha_t + \beta_t},$$

with $\alpha_t = \alpha_0 + \sum_{n=1}^{t-1} x_n, \quad \beta_t = \beta_0 + \sum_{n=1}^{t-1}(1 - x_n)$

Sequential updating of the marginal likelihood

$$\mathbb{P}_\xi(D) = \mathbb{P}_\xi(x_1, \ldots, x_T) \tag{2.14}$$

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Sequential updating of the marginal likelihood

$$\mathbb{P}_\xi(D) = \mathbb{P}_\xi(x_1, \ldots, x_T) \tag{2.11}$$
$$= \mathbb{P}_\xi(x_2, \ldots, x_T \mid x_1)\, \mathbb{P}_\xi(x_1)$$

(2.14)

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Sequential updating of the marginal likelihood

$$\mathbb{P}_\xi(D) = \mathbb{P}_\xi(x_1, \ldots, x_T) \tag{2.11}$$

$$= \mathbb{P}_\xi(x_2, \ldots, x_T \mid x_1)\, \mathbb{P}_\xi(x_1) \tag{2.12}$$

$$= \prod_{t=1}^{T} \mathbb{P}_\xi(x_t \mid x_1, \ldots, x_{t-1})$$

$$\tag{2.14}$$

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$$= \prod_{t=1}^{T} \mathbb{P}_\xi(x_t \mid x_1, \ldots, x_{t-1}) \tag{2.13}$$
$$= \prod_{t=1}^{T} \int_\Theta P_{\theta_n}(x_t)\, \mathrm{d}\underbrace{\xi(\theta \mid x_1, \ldots, x_{t-1})}_{\text{posterior at time } t} \tag{2.14}$$

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with $\alpha_t = \alpha_0 + \sum_{n=1}^{t-1} x_n, \quad \beta_t = \beta_0 + \sum_{n=1}^{t-1} (1 - x_n)$

Further reading

Python sources

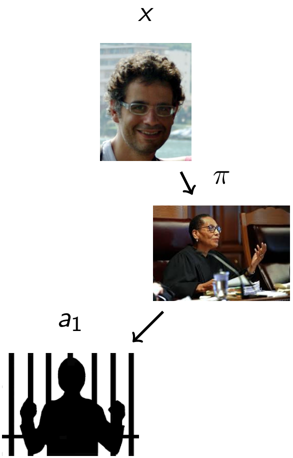
- A simple python measure of conditional independence
  `src/fairness/ci_test.py`
- A simple test for discrete Bayesian network
  `src/fairness/DirichletTest.py`
- Using the PyMC package
  `https://docs.pymc.io/notebooks/Bayes_factor.html`

Bail decisions, revisited

x

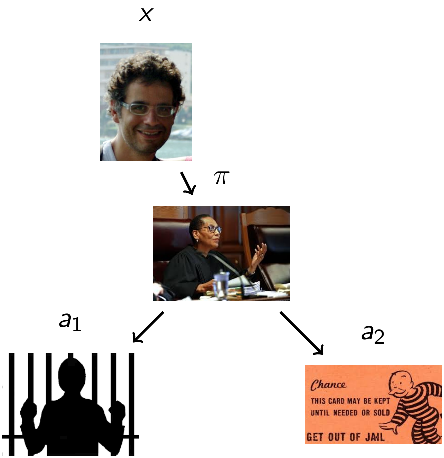
$\searrow$ π

Bail decisions, revisited



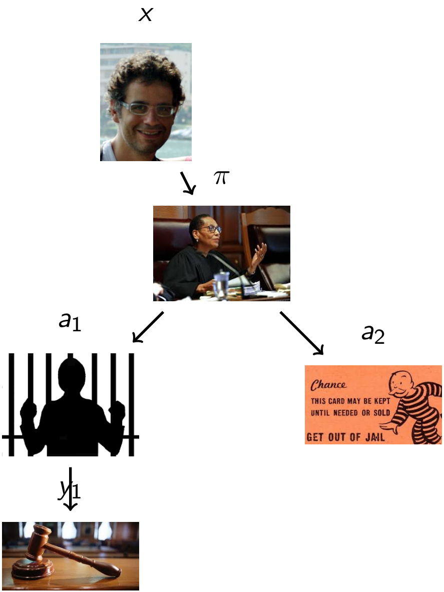
$\pi(a \mid x)$ (policy)

Bail decisions, revisited



$$\pi(a \mid x) \qquad \text{(policy)}$$

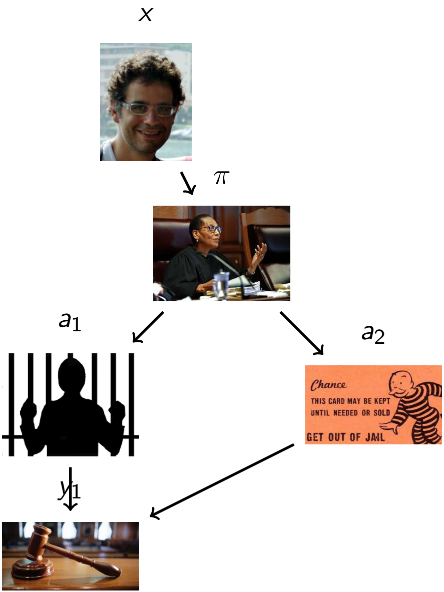
Bail decisions, revisited



$$\pi(a \mid x) \qquad \text{(policy)}$$

$$\mathbb{P}(y \mid a, x) \qquad \text{(outcome)}$$

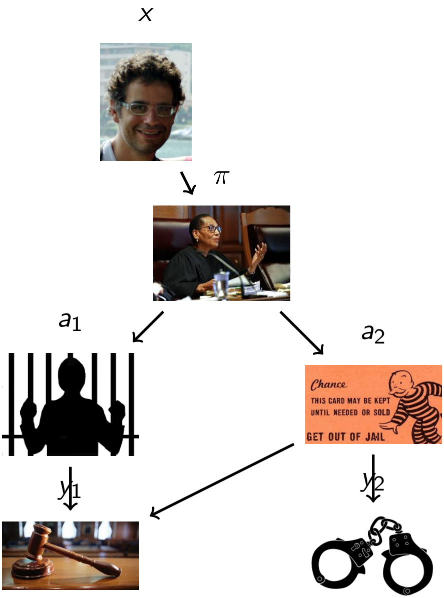
Bail decisions, revisited



$$\pi(a \mid x) \qquad \text{(policy)}$$

$$\mathbb{P}(y \mid a, x) \qquad \text{(outcome)}$$

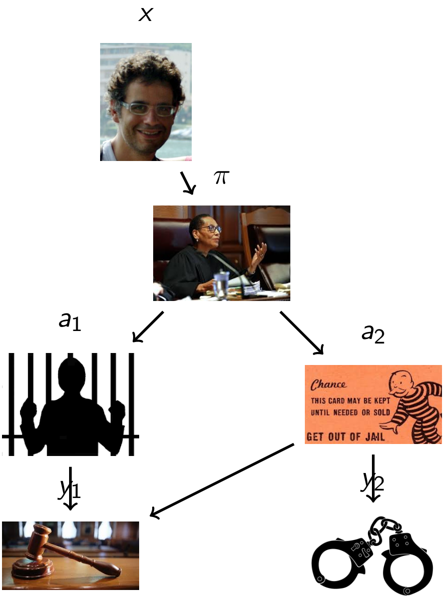
Bail decisions, revisited



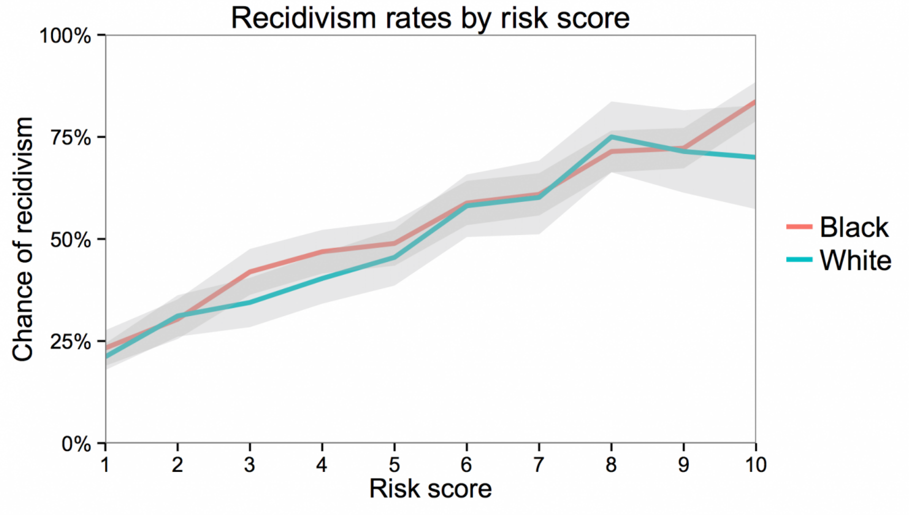
$$\pi(a \mid x) \qquad \text{(policy)}$$

$$\mathbb{P}(y \mid a, x) \qquad \text{(outcome)}$$

Bail decisions, revisited



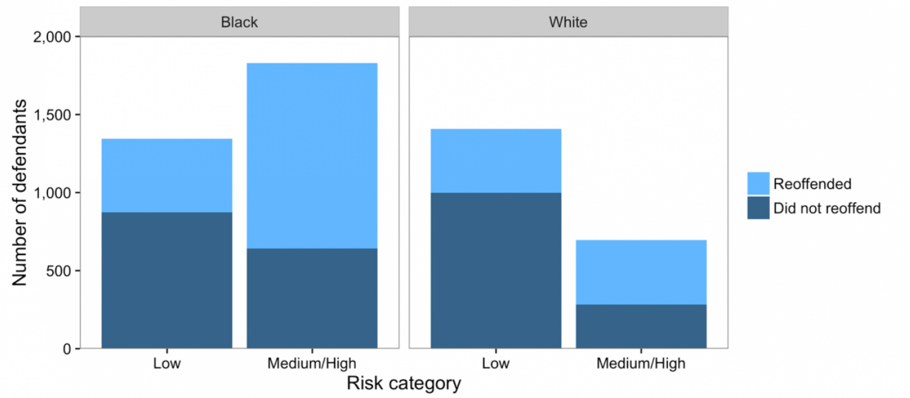
$\pi(a \mid x)$	(policy)
$\mathbb{P}(y \mid a, x)$	(outcome)
$U(a, y)$	(utility)



- y Result.
- a Assigned score.
- z Race.

$$\mathbb{P}^\pi(y \mid a, z) = \mathbb{P}^\pi(y \mid a) \qquad \text{(calibration)}$$

$$\mathbb{P}^\pi(a \mid y, z) = \mathbb{P}^\pi(a \mid y) \qquad \text{(balance)}$$



y Result.

a Assigned score.

z Race.

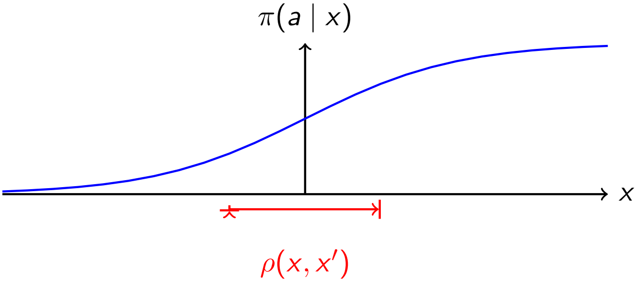
$$\mathbb{P}^\pi(y \mid a, z) = \mathbb{P}^\pi(y \mid a) \qquad \text{(calibration)}$$

$$\mathbb{P}^\pi(a \mid y, z) = \mathbb{P}^\pi(a \mid y) \qquad \text{(balance)}$$

Meritocratic decision

$$a_t(\theta, x_t) \in \arg\max_a \mathbb{E}_\theta(U \mid a, x_t) = \int_{\mathcal{Y}} U(a_t, y) \, \mathbb{E}_\theta(U \mid a_t, x_t) \tag{3.1}$$

$$D[\pi(a \mid x), \pi(a \mid x')] \leq \rho(x, x'). \tag{3.2}$$



The Bayesian approach to fairness

The value of a policy

Let λ represent the trade-off between utility and fairness.

$$V(\lambda, \theta, \pi) = \lambda \overbrace{U(\theta, \pi)}^{\text{utility}} - \underbrace{(1 - \lambda) F(\theta, \pi)}_{\text{fairness violation}} \tag{3.3}$$

$$V(\lambda, \xi, \pi) = \int_{\Theta} V(\lambda, \theta, \pi) \, \mathrm{d}\xi(\theta). \tag{3.4}$$

Online resources

- COMPAS analysis by propublica
  `https://github.com/propublica/compas-analysis`
- Open policing database `https://openpolicing.stanford.edu/`