Fairness

Christos Dimitrakakis

October 3, 2019

Fairness

What is it?

Fairness

What is it?

- Meritocracy.

Fairness

What is it?

- Meritocracy.
- Proportionality and representation.

Fairness

What is it?

- Meritocracy.
- Proportionality and representation.
- Equal treatment.

Fairness

What is it?

- Meritocracy.
- Proportionality and representation.
- Equal treatment.
- Non-discrimination.

Meritocracy

Meritocracy

Example 1 (College admissions)

- Student A has a grade 4/5 from Gota Highschool.
- Student B has a grade 5/5 from Vasa Highschool.

Meritocracy

Example 1 (College admissions)

- Student A has a grade 4/5 from Gota Highschool.
- Student B has a grade 5/5 from Vasa Highschool.

Example 2 (Additional information)

- 70% of admitted Gota graduates with 4+ get their degree.
- 50% of admitted Vasa graduates with 5 get their degree.

Meritocracy

Example 1 (College admissions)

- Student A has a grade $4/5$ from Gota Highschool.
- Student B has a grade $5/5$ from Vasa Highschool.

Example 2 (Additional information)

- 70% of admitted Gota graduates with $4+$ get their degree.
- 50% of admitted Vasa graduates with 5 get their degree.

We still don't know how a specific student will do!

Solutions

Meritocracy

Example 1 (College admissions)

- Student A has a grade $4/5$ from Gota Highschool.
- Student B has a grade $5/5$ from Vasa Highschool.

Example 2 (Additional information)

- 70% of admitted Gota graduates with $4+$ get their degree.
- 50% of admitted Vasa graduates with 5 get their degree.

We still don't know how a specific student will do!

Solutions

- Admit everybody?

Meritocracy

Example 1 (College admissions)

- Student A has a grade $4/5$ from Gota Highschool.
- Student B has a grade $5/5$ from Vasa Highschool.

Example 2 (Additional information)

- 70% of admitted Gota graduates with $4+$ get their degree.
- 50% of admitted Vasa graduates with 5 get their degree.

We still don't know how a specific student will do!

Solutions

- Admit everybody?
- Admit randomly?

Meritocracy

Example 1 (College admissions)

- Student A has a grade $4/5$ from Gota Highschool.
- Student B has a grade $5/5$ from Vasa Highschool.

Example 2 (Additional information)

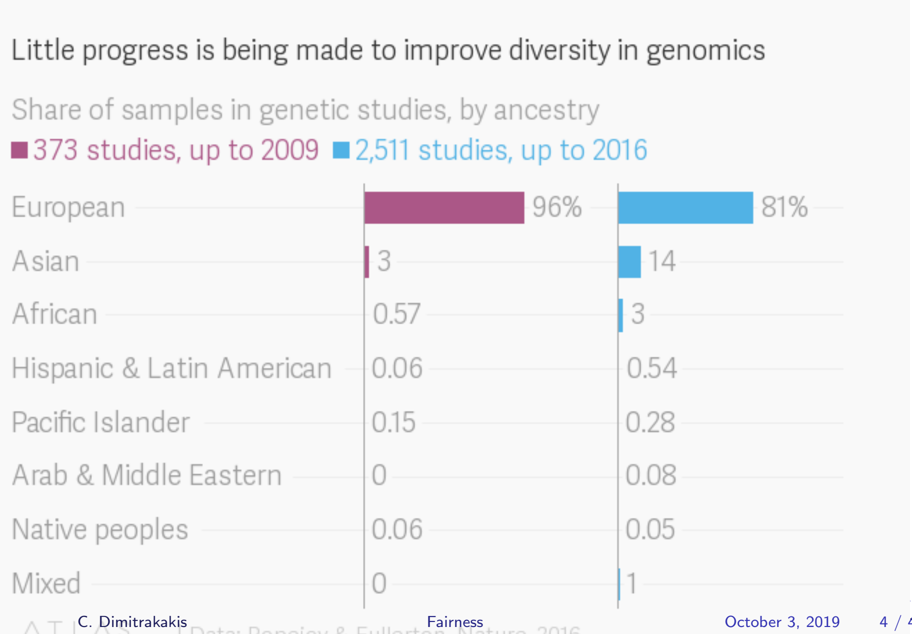
- 70% of admitted Gota graduates with $4+$ get their degree.
- 50% of admitted Vasa graduates with 5 get their degree.

We still don't know how a specific student will do!

Solutions

- Admit everybody?
- Admit randomly?
- Use prediction of individual academic performance?

Proportional representation

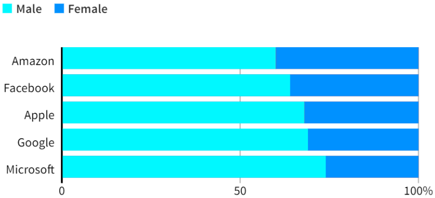


Hiring decisions

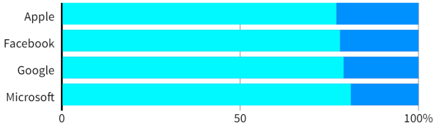
Dominated by men

Top U.S. tech companies have yet to close the gender gap in hiring, a disparity most pronounced among technical staff such as software developers where men far outnumber women. Amazon's experimental recruiting engine followed the same pattern, learning to penalize resumes including the word "women's" until the company discovered the problem.

GLOBAL HEADCOUNT



EMPLOYEES IN TECHNICAL ROLES



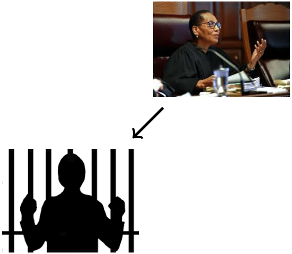
Fairness and information

Example 3 (College admissions data)

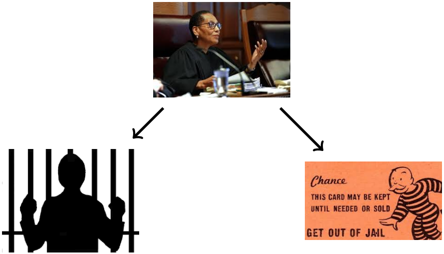
School	Male	Female
A	62%	82%
B	63%	68%
C	37%	34%
D	33%	35%
E	28%	24%
F	6%	7%
Average	*45%*	*38%*

Bail decisions

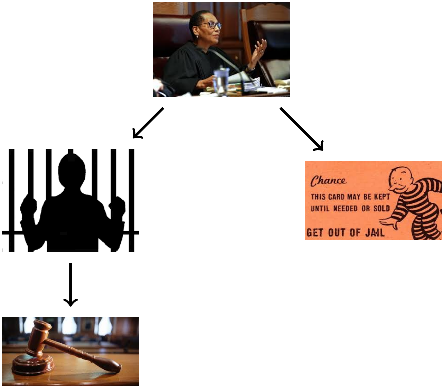
Bail decisions



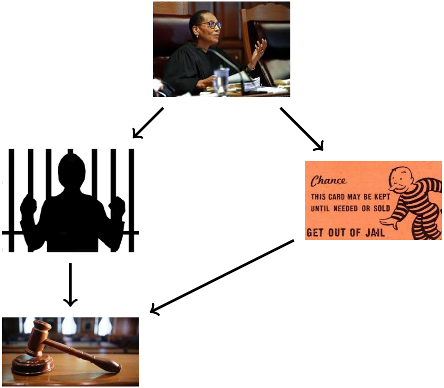
Bail decisions



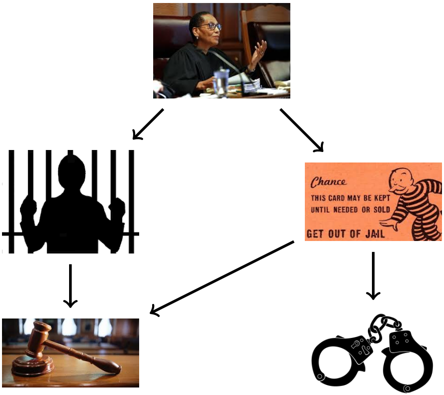
Bail decisions



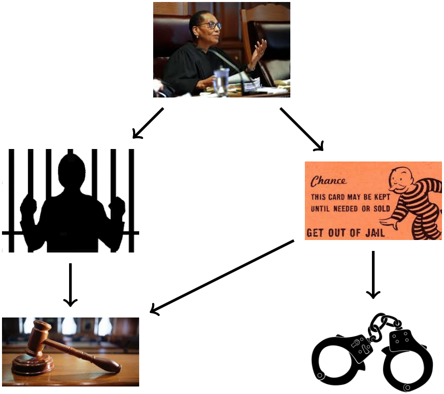
Bail decisions

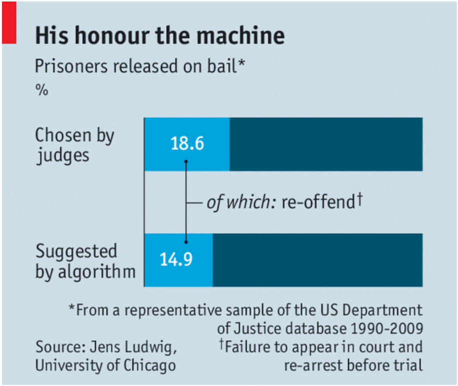


Bail decisions



Bail decisions





Economist.com

Whites get lower scores than blacks[1]

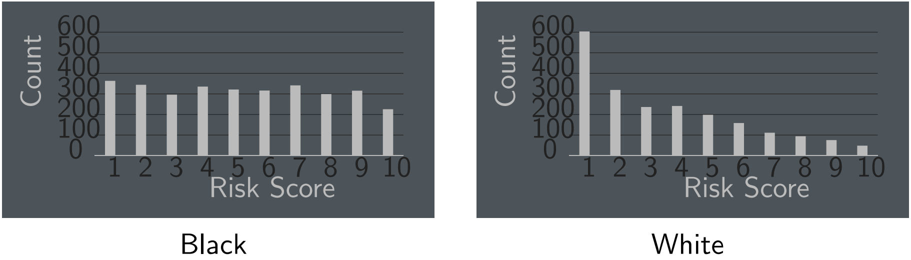


Figure: Apparent bias in risk scores towards black versus white defendants.

[1] Pro-publica, 2016

But scores equally accurately predict recidivsm[2]

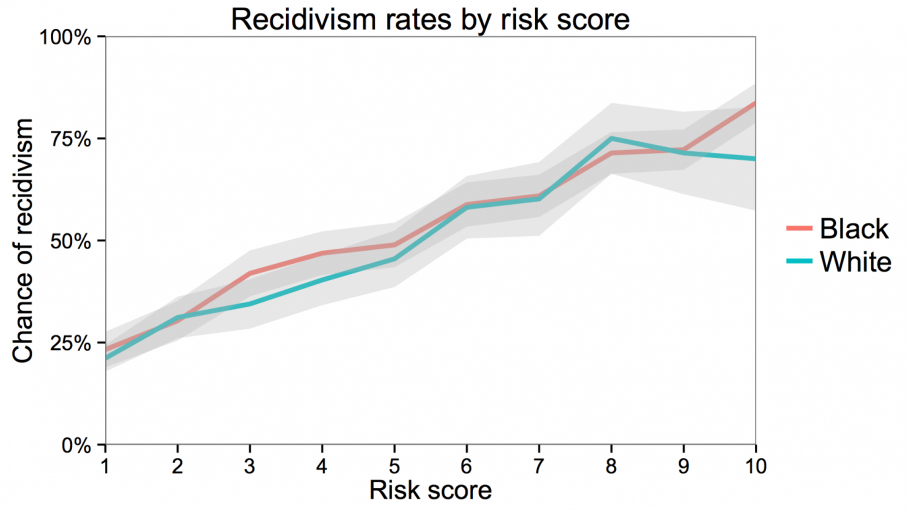


Figure: Recidivism rates by risk score.

[2]Washington Post, 2016

But non-offending blacks get higher scores

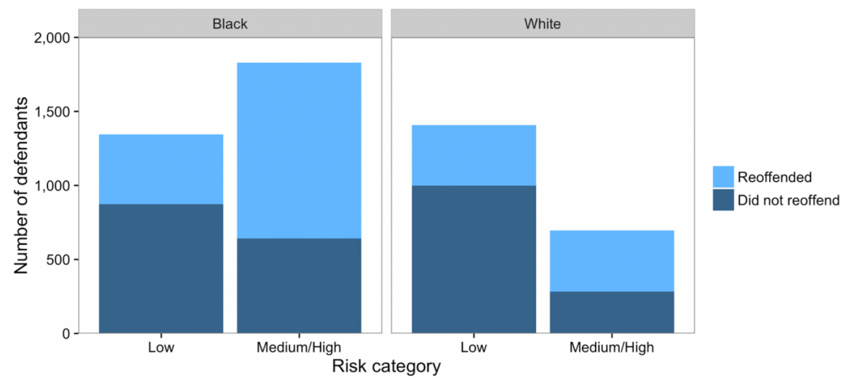


Figure: Score breakdown based on recidivism rates.

Graphical models and independence

- Why is it not possible to be fair in all respects?
- Different notions of conditional independence.
- Can only be satisfied rarely simultaneously.

Graphical models

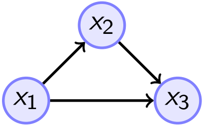


Figure: Graphical model (directed acyclic graph) for three variables.

Joint probability

Let $\boldsymbol{x} = (x_1, \ldots, x_n)$. Then $\boldsymbol{x} : \Omega \to X$, $X = \prod_i X_i$ and:

$$\mathbb{P}(\boldsymbol{x} \in A) = P(\{\omega \in \Omega \mid \boldsymbol{x}(\omega) \in A\}).$$

Factorisation

$$\mathbb{P}(\boldsymbol{x}) = \mathbb{P}(\boldsymbol{x}_B \mid \boldsymbol{x}_C)\,\mathbb{P}(\boldsymbol{x}_C), \qquad B, C \subset [n]$$

Graphical models

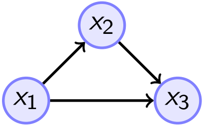


Figure: Graphical model (directed acyclic graph) for three variables.

Joint probability

Let $\boldsymbol{x} = (x_1, \ldots, x_n)$. Then $\boldsymbol{x} : \Omega \to X$, $X = \prod_i X_i$ and:

$$\mathbb{P}(\boldsymbol{x} \in A) = P(\{\omega \in \Omega \mid \boldsymbol{x}(\omega) \in A\}).$$

Factorisation

So we can write any joint distribution as

$$\mathbb{P}(x_1)\,\mathbb{P}(x_2 \mid x_1)\,\mathbb{P}(x_3 \mid x_1, x_2)\cdots\mathbb{P}(x_n \mid x_1, \ldots, x_{n-1}).$$

Directed graphical models

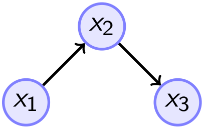


Figure: Graphical model for the factorisation $\mathbb{P}(x_3 \mid x_2)\,\mathbb{P}(x_2 \mid x_1)\,\mathbb{P}(x_1)$.

Conditional independence

We say x_i is conditionally independent of $\boldsymbol{x}_B$ given $\boldsymbol{x}_D$ and write $x_i \mid \boldsymbol{x}_D \perp\!\!\!\perp \boldsymbol{x}_B$ iff

$$\mathbb{P}(x_i, \boldsymbol{x}_B \mid \boldsymbol{x}_D) = \mathbb{P}(x_i \mid \boldsymbol{x}_D)\,\mathbb{P}(\boldsymbol{x}_B \mid \boldsymbol{x}_D).$$

Example 4 (Smoking and lung cancer)

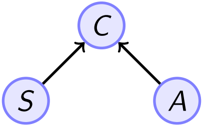


Figure: Smoking and lung cancer graphical model, where S: Smoking, C: cancer, A: asbestos exposure.

Explaining away

Even though S, A are independent, they become dependent once you know C.

Example 5 (Time of arrival at work)

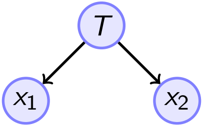


Figure: Time of arrival at work graphical model where T is a traffic jam and x_1 is the time John arrives at the office and x_2 is the time Jane arrives at the office.

Conditional independence

Even though x_1, x_2 are correlated, they become independent once you know T.

Example 6 (Treatment effects)

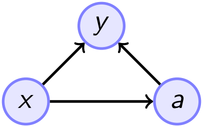
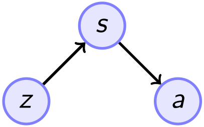


Figure: Kidney treatment model, where x: severity, y: result, a: treatment applied

	Treatment A	Treatment B
Small stones	87	270
Large stones	263	80

Severity	Treatment A	Treatment B
Small stones)	93%	87%
Large stones	73%	69%
Average	78%	83%

Example 7 (School admission)



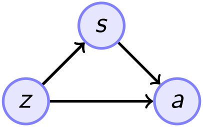
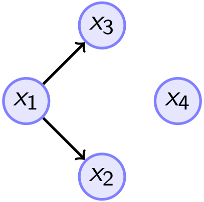


Figure: School admission graphical model, where z: gender, s: school applied to, a: whether you were admitted.

School	Male	Female
A	62%	82%
B	63%	68%
C	37%	34%
D	33%	35%
E	28%	24%
F	6%	7%
Average	*45%*	*38%*

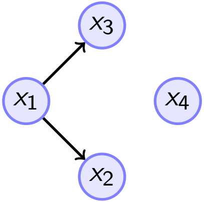
Exercise 1

Factorise the following graphical model.



Exercise 1

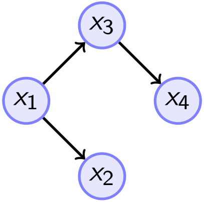
Factorise the following graphical model.



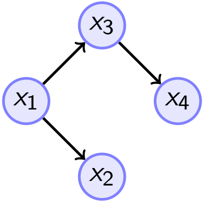
$$\mathbb{P}(\boldsymbol{x}) = \mathbb{P}(x_1)\,\mathbb{P}(x_2 \mid x_1)\,\mathbb{P}(x_3 \mid x_1)\,\mathbb{P}(x_4)$$

Exercise 2

Factorise the following graphical model.



Exercise 2



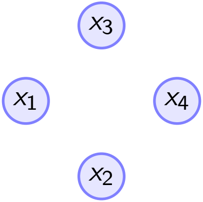
Factorise the following graphical model.

$$\mathbb{P}(\boldsymbol{x}) = \mathbb{P}(x_1)\,\mathbb{P}(x_2 \mid x_1)\,\mathbb{P}(x_3 \mid x_1)\,\mathbb{P}(x_4 \mid x_3)$$

Exercise 3

What dependencies does the following factorisation imply?

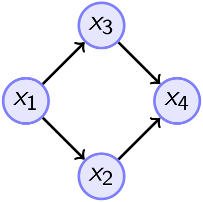
$$\mathbb{P}(\boldsymbol{x}) = \mathbb{P}(x_1)\,\mathbb{P}(x_2 \mid x_1)\,\mathbb{P}(x_3 \mid x_1)\,\mathbb{P}(x_4 \mid x_2, x_3)$$



Exercise 3

What dependencies does the following factorisation imply?

$$\mathbb{P}(\boldsymbol{x}) = \mathbb{P}(x_1)\,\mathbb{P}(x_2 \mid x_1)\,\mathbb{P}(x_3 \mid x_1)\,\mathbb{P}(x_4 \mid x_2, x_3)$$



Deciding conditional independence

There is an algorithm for deciding conditional independence of any two variables in a graphical model.

Inference and prediction in graphical models

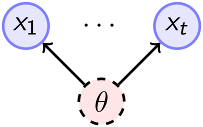


Figure: Inference and prediction in a graphical model.

Inference of latent variables

$$\mathbb{P}(\theta \mid x_1, \ldots, x_t)$$

- Model parameters.
- System states.

Inference and prediction in graphical models

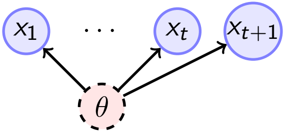


Figure: Inference and prediction in a graphical model.

Prediction

$$\mathbb{P}(x_{t+1} \mid x_1, \ldots, x_t) = \int_{\Theta} \mathbb{P}(x_{t+1} \mid \theta) \, \mathrm{d} \, \mathbb{P}(\theta \mid x_1, \ldots, x_t)$$

Predictions are testable.

Coin tossing, revisited

Example 8

The Beta-Bernoulli prior

Figure: Graphical model for a Beta-Bernoulli prior

$$\theta \sim \mathcal{Beta}(\xi_1, \xi_2), \quad \text{i.e. } \xi \text{ are Beta distribution parameters} \tag{3.1}$$

$$x \mid \theta \sim \mathcal{Bernoulli}(\theta), \quad \text{i.e. } P_\theta(x) \text{ is a Bernoulli} \tag{3.2}$$

Example 9

The *n*-meteorologists problem (continuation of Exercise **??**)

- Meteorological models $\mathcal{M} = \{\mu_1, \ldots, \mu_n\}$
- Rain predictions at time t: $p_{t,\mu} \triangleq P_\mu(x_t = \mathrm{rain})$.
- Prior probability $\xi(\mu) = 1/n$ for each model.
- Decision a, resulting in utility $U(a, x_{t+1})$

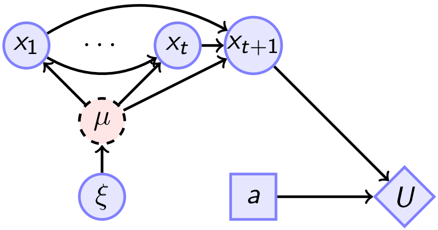


Figure: Inference, prediction and decisions in a graphical model.

Measuring independence

Theorem 10

If $x_i \mid \boldsymbol{x}_D \perp\!\!\!\perp \boldsymbol{x}_B$ *then*

$$\mathbb{P}(x_i \mid \boldsymbol{x}_B, \boldsymbol{x}_D) = \mathbb{P}(x_i \mid \boldsymbol{x}_D)$$

Example 11

$$\| \mathbb{P}(a \mid y, z) - \mathbb{P}(a \mid y) \|_1$$

which for discrete a, y, z is:

$$\max_{i,j} \| \mathbb{P}(a \mid y = i, z = j) - \mathbb{P}(a \mid y = i) \|_1 = \max_{i,j} \| \sum_k \mathbb{P}(a = k \mid y = i, z = j) -$$

Measuring independence

Theorem 10

If $x_i \mid \boldsymbol{x}_D \perp\!\!\!\perp \boldsymbol{x}_B$ *then*

$$\mathbb{P}(x_i \mid \boldsymbol{x}_B, \boldsymbol{x}_D) = \mathbb{P}(x_i \mid \boldsymbol{x}_D)$$

This implies

$$\mathbb{P}(x_i \mid \boldsymbol{x}_B = b, \boldsymbol{x}_D) = \mathbb{P}(x_i \mid \boldsymbol{x}_B = b', \boldsymbol{x}_D)$$

so we can measure independence by seeing how the distribution of x_i changes when we vary $\boldsymbol{x}_B$, keeping $\boldsymbol{x}_D$ fixed.

Example 11

$$\| \mathbb{P}(a \mid y, z) - \mathbb{P}(a \mid y) \|_1$$

which for discrete a, y, z is:

$$\max_{i,j} \| \mathbb{P}(a \mid y = i, z = j) - \mathbb{P}(a \mid y = i) \|_1 = \max_{i,j} \| \sum_k \mathbb{P}(a = k \mid y = i, z = j) -$$

Example 12

An alternative model for coin-tossing This is an elaboration of Example **??** for hypothesis testing.

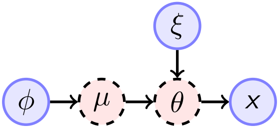
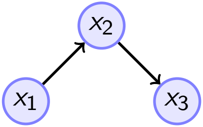


Figure: Graphical model for a hierarchical prior

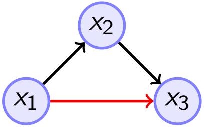
- μ_1: A Beta-Bernoulli model with $\mathcal{B}eta(\xi_1, \xi_2)$
- μ_0: The coin is fair.

$$\theta \mid \mu = \mu_0 \sim \mathcal{D}(0.5), \qquad \text{i.e. } \theta \text{ is always } 0.5 \tag{3.3}$$
$$\theta \mid \mu = \mu_1 \sim \mathcal{B}eta(\xi_1, \xi_2), \qquad \text{i.e. } \theta \text{ has a Beta distribution} \tag{3.4}$$
$$x \mid \theta \sim \mathcal{B}ernoulli(\theta), \qquad \text{i.e. } P_\theta(x) \text{ is Bernoulli} \tag{3.5}$$

Bayesian testing of independence



(a) Θ_0 assumes independence



(b) Θ_1 does not assume independence

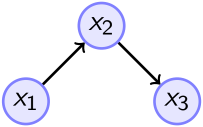
Example 13

Assume data $D = \{x_1^t, x_2^t, x_3^t \mid t = 1, \ldots, T\}$ with $x_i^t \in \{0, 1\}$.

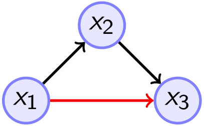
$$P_\theta(D) = \prod_t P_\theta(x_3^t \mid x_2^t) P_\theta(x_2^t \mid x_1^t) P_\theta(x_1^t), \qquad \theta \in \Theta_0 \tag{3.6}$$

$$P_\theta(D) = \prod_t P_\theta(x_3^t \mid x_2^t, x_1^t) P_\theta(x_2^t \mid x_1^t) P_\theta(x_1^t), \qquad \theta \in \Theta_1 \tag{3.7}$$

Bayesian testing of independence



(a) Θ_0 assumes independence



(b) Θ_1 does not assume independence

Example 13

$$\theta_1 \triangleq P_\theta(x_1^t = 1) \qquad (\mu_0, \mu_1)$$

$$\theta_{2|1}^i \triangleq P_\theta(x_2^t = 1 \mid x_1^t = i) \qquad (\mu_0, \mu_1)$$

$$\theta_{3|2}^j \triangleq P_\theta(x_3^t = 1 \mid x_2^t = j) \qquad (\mu_0)$$

$$\theta_{3|2,1}^{i,j} \triangleq P_\theta(x_3^t = 1 \mid x_2^t = j, x_1^t = i) \qquad (\mu_1)$$

Figure: Hierarchical model.

$$\mu_i \sim \phi \tag{3.6}$$

$$\theta \mid \mu = \mu_i \sim \xi_i \tag{3.7}$$

Marginal likelihood

$$\mathbb{P}_{\phi}(D) = \phi(\mu_0)\,\mathbb{P}_{\mu_0}(D) + \phi(\mu_1)\,\mathbb{P}_{\mu_1}(D) \tag{3.8}$$

$$\mathbb{P}_{\mu_i}(D) = \int_{\Theta_i} P_{\theta}(D)\,\mathrm{d}\xi_i(\theta). \tag{3.9}$$

Figure: Hierarchical model.

Marginal likelihood

$$\mathbb{P}_{\phi}(D) = \phi(\mu_0)\,\mathbb{P}_{\mu_0}(D) + \phi(\mu_1)\,\mathbb{P}_{\mu_1}(D) \tag{3.6}$$

$$\mathbb{P}_{\mu_i}(D) = \int_{\Theta_i} P_{\theta}(D)\,\mathrm{d}\xi_i(\theta). \tag{3.7}$$

Model posterior

$$\phi(\mu \mid D) = \frac{\mathbb{P}_{\mu}(D)\phi(\mu)}{\sum_i \mathbb{P}_{\mu_i}(D)\phi(\mu_i)} \tag{3.8}$$

Calculating the marginal likelihood

Monte-Carlo approximation

$$\int_{\Theta} P_\theta(D) \, \mathrm{d}\xi(\theta) \approx \sum_{n=1}^{N} P_{\theta_n}(D) + O(1/\sqrt{N}), \qquad \theta_n \sim \xi \tag{3.9}$$

Importance sampling

$$\int_{\Theta} P_\theta(D) \, \mathrm{d}\xi(\theta) \tag{3.10}$$

Calculating the marginal likelihood

Monte-Carlo approximation

$$\int_{\Theta} P_\theta(D) \, \mathrm{d}\xi(\theta) \approx \sum_{n=1}^{N} P_{\theta_n}(D) + O(1/\sqrt{N}), \qquad \theta_n \sim \xi \tag{3.9}$$

Importance sampling

$$\int_{\Theta} P_\theta(D) \, \mathrm{d}\xi(\theta) = \int_{\Theta} P_\theta(D) \frac{\mathrm{d}\psi(\theta)}{\mathrm{d}\psi(\theta)} \, \mathrm{d}\xi(\theta) \tag{3.10}$$

Calculating the marginal likelihood

Monte-Carlo approximation

$$\int_{\Theta} P_\theta(D)\, \mathrm{d}\xi(\theta) \approx \sum_{n=1}^{N} P_{\theta_n}(D) + O(1/\sqrt{N}), \qquad \theta_n \sim \xi \tag{3.9}$$

Importance sampling

$$\int_{\Theta} P_\theta(D)\, \mathrm{d}\xi(\theta) = \int_{\Theta} P_\theta(D) \frac{\mathrm{d}\xi(\theta)}{\mathrm{d}\psi(\theta)}\, \mathrm{d}\psi(\theta) \tag{3.10}$$

Calculating the marginal likelihood

Monte-Carlo approximation

$$\int_{\Theta} P_{\theta}(D) \, \mathrm{d}\xi(\theta) \approx \sum_{n=1}^{N} P_{\theta_n}(D) + O(1/\sqrt{N}), \qquad \theta_n \sim \xi \tag{3.9}$$

Importance sampling

$$\int_{\Theta} P_{\theta}(D) \, \mathrm{d}\xi(\theta) \approx \sum_{n=1}^{N} P_{\theta}(D) \frac{\mathrm{d}\xi(\theta_n)}{\mathrm{d}\psi(\theta_n)}, \qquad \theta_n \sim \psi \tag{3.10}$$

Sequential updating of the marginal likelihood

$$\mathbb{P}_\xi(D) \tag{3.14}$$

Example 14 (Beta-Bernoulli)

$$\mathbb{P}_\xi(x_t = 1 \mid x_1, \ldots, x_{t-1}) = \frac{\alpha_t}{\alpha_t + \beta_t},$$

with $\alpha_t = \alpha_0 + \sum_{n=1}^{t-1} x_n, \quad \beta_t = \beta_0 + \sum_{n=1}^{t-1} (1 - x_n)$

Sequential updating of the marginal likelihood

$$\mathbb{P}_\xi(D) = \mathbb{P}_\xi(x_1, \ldots, x_T) \tag{3.14}$$

Example 14 (Beta-Bernoulli)

$$\mathbb{P}_\xi(x_t = 1 \mid x_1, \ldots, x_{t-1}) = \frac{\alpha_t}{\alpha_t + \beta_t},$$

with $\alpha_t = \alpha_0 + \sum_{n=1}^{t-1} x_n, \quad \beta_t = \beta_0 + \sum_{n=1}^{t-1} (1 - x_n)$

Sequential updating of the marginal likelihood

$$\mathbb{P}_\xi(D) = \mathbb{P}_\xi(x_1, \ldots, x_T) \tag{3.11}$$
$$= \mathbb{P}_\xi(x_2, \ldots, x_T \mid x_1)\, \mathbb{P}_\xi(x_1)$$

(3.14)

Example 14 (Beta-Bernoulli)

$$\mathbb{P}_\xi(x_t = 1 \mid x_1, \ldots, x_{t-1}) = \frac{\alpha_t}{\alpha_t + \beta_t},$$

with $\alpha_t = \alpha_0 + \sum_{n=1}^{t-1} x_n, \quad \beta_t = \beta_0 + \sum_{n=1}^{t-1} (1 - x_n)$

Sequential updating of the marginal likelihood

$$\mathbb{P}_\xi(D) = \mathbb{P}_\xi(x_1, \ldots, x_T) \tag{3.11}$$
$$= \mathbb{P}_\xi(x_2, \ldots, x_T \mid x_1) \, \mathbb{P}_\xi(x_1) \tag{3.12}$$
$$= \prod_{t=1}^{T} \mathbb{P}_\xi(x_t \mid x_1, \ldots, x_{t-1})$$
$$\tag{3.14}$$

Example 14 (Beta-Bernoulli)

$$\mathbb{P}_\xi(x_t = 1 \mid x_1, \ldots, x_{t-1}) = \frac{\alpha_t}{\alpha_t + \beta_t},$$

with $\alpha_t = \alpha_0 + \sum_{n=1}^{t-1} x_n, \quad \beta_t = \beta_0 + \sum_{n=1}^{t-1} (1 - x_n)$

Sequential updating of the marginal likelihood

$$
\begin{aligned}
\mathbb{P}_\xi(D) &= \mathbb{P}_\xi(x_1, \ldots, x_T) && (3.11)\\
&= \mathbb{P}_\xi(x_2, \ldots, x_T \mid x_1)\, \mathbb{P}_\xi(x_1) && (3.12)\\
&= \prod_{t=1}^{T} \mathbb{P}_\xi(x_t \mid x_1, \ldots, x_{t-1}) && (3.13)\\
&= \prod_{t=1}^{T} \int_\Theta P_{\theta_n}(x_t)\, \mathrm{d}\underbrace{\xi(\theta \mid x_1, \ldots, x_{t-1})}_{\text{posterior at time } t} && (3.14)
\end{aligned}
$$

Example 14 (Beta-Bernoulli)

$$\mathbb{P}_\xi(x_t = 1 \mid x_1, \ldots, x_{t-1}) = \frac{\alpha_t}{\alpha_t + \beta_t},$$

with $\alpha_t = \alpha_0 + \sum_{n=1}^{t-1} x_n, \quad \beta_t = \beta_0 + \sum_{n=1}^{t-1}(1 - x_n)$

Further reading

Python sources

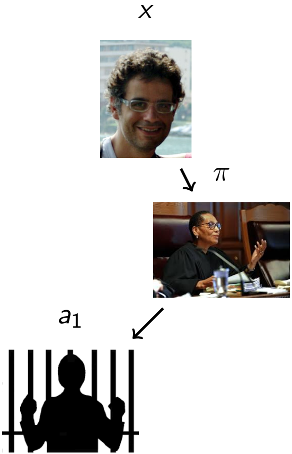
- A simple python measure of conditional independence `src/fairness/ci_test.py`
- A simple test for discrete Bayesian network `src/fairness/DirichletTest.py`
- Using the PyMC package `https://docs.pymc.io/notebooks/Bayes_factor.html`

Bail decisions, revisited

x

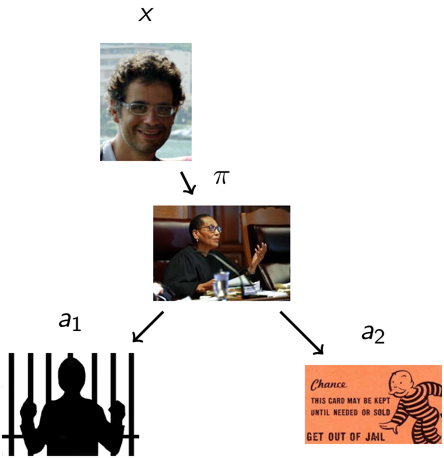
$\searrow$ π

Bail decisions, revisited



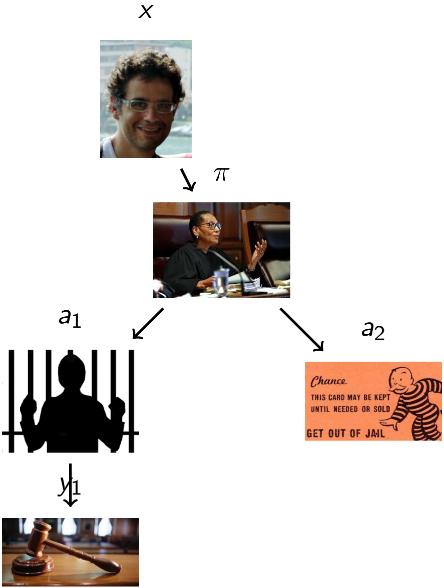
$\pi(a \mid x)$ (policy)

Bail decisions, revisited



$\pi(a \mid x)$ (policy)

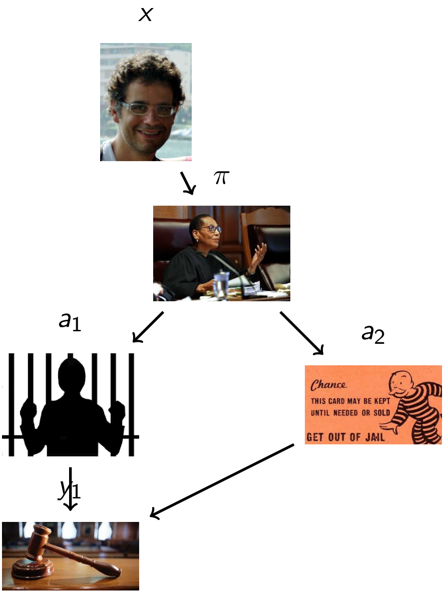
Bail decisions, revisited



$$\pi(a \mid x) \qquad \text{(policy)}$$

$$\mathbb{P}(y \mid a, x) \qquad \text{(outcome)}$$

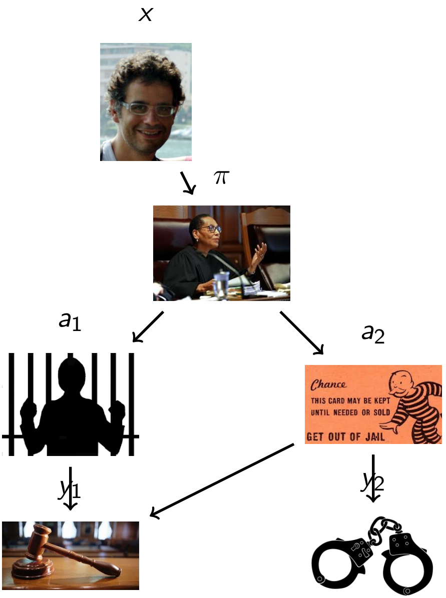
Bail decisions, revisited



$\pi(a \mid x)$ (policy)

$\mathbb{P}(y \mid a, x)$ (outcome)

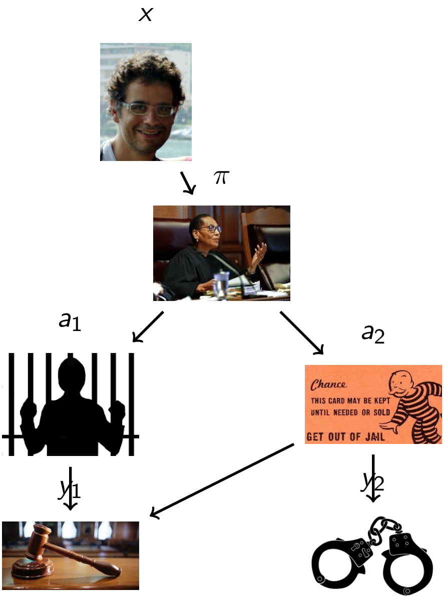
Bail decisions, revisited



$$\pi(a \mid x) \qquad \text{(policy)}$$

$$\mathbb{P}(y \mid a, x) \qquad \text{(outcome)}$$

Bail decisions, revisited

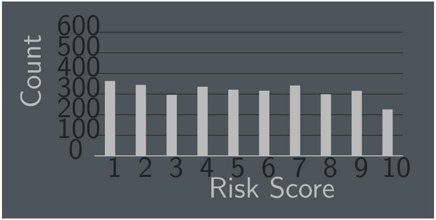


$\pi(a \mid x)$ (policy)

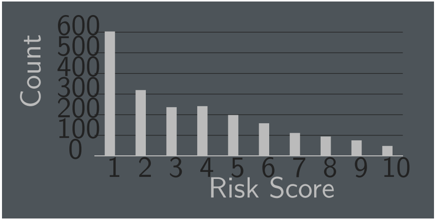
$\mathbb{P}(y \mid a, x)$ (outcome)

$U(a, y)$ (utility)

Independence



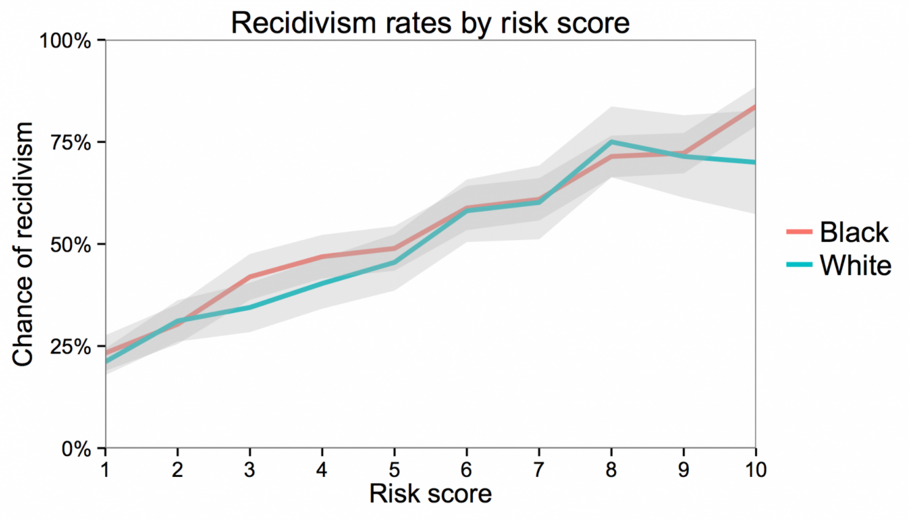
Black



White

Figure: Apparent bias in risk scores towards black versus white defendants.

$$\mathbb{P}^\pi_\theta(a \mid z) = \mathbb{P}^\pi_\theta(a) \qquad \text{(non-discrimination)}$$



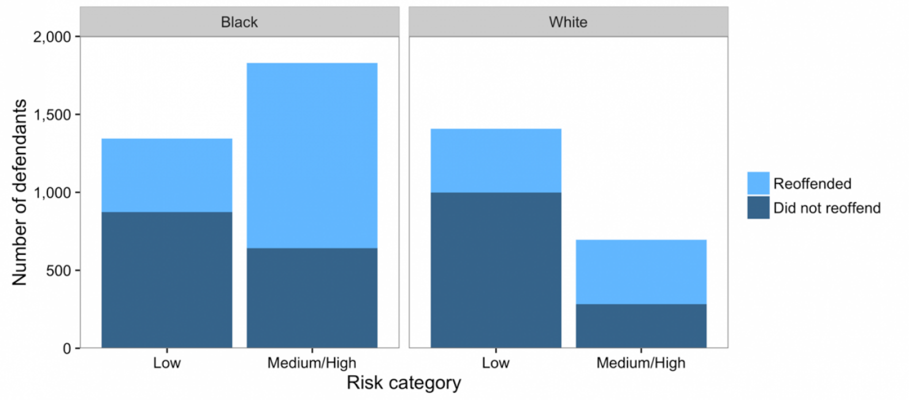
y Result.

a Assigned score.

z Race.

$$\mathbb{P}^\pi(y \mid a, z) = \mathbb{P}^\pi(y \mid a) \qquad \text{(calibration)}$$

$$\mathbb{P}^\pi(a \mid y, z) = \mathbb{P}^\pi(a \mid y) \qquad \text{(balance)}$$



- y Result.
- a Assigned score.
- z Race.

$$\mathbb{P}^\pi(y \mid a, z) = \mathbb{P}^\pi(y \mid a) \qquad \text{(calibration)}$$

$$\mathbb{P}^\pi(a \mid y, z) = \mathbb{P}^\pi(a \mid y) \qquad \text{(balance)}$$

Meritocratic decision

$$a_t(\theta, x_t) \in \arg\max_a \mathbb{E}_\theta(U \mid a, x_t) = \int_{\mathcal{Y}} U(a_t, y) \, \mathbb{E}_\theta(U \mid a_t, x_t) \tag{4.1}$$

Smooth fairness

$$D[\pi(a \mid x), \pi(a \mid x')] \le \rho(x, x'). \tag{4.2}$$

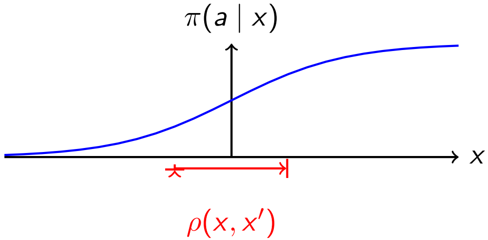


Figure: A Lipschitz function

The constrained maximisation problem

$$\max_\pi \left\{ U(\pi) \mid \rho(x, x') \le \epsilon \right\} \tag{4.3}$$

The value of a policy

Fairness metrics: balance

$$F_{\mathrm{balance}}(\theta, \pi) \triangleq \sum_{y,z,a} |\, \mathbb{P}^{\pi}_{\theta}(a \mid y, z) - \mathbb{P}^{\pi}_{\theta}(a \mid y)|^2 \tag{4.4}$$

The value of a policy

Fairness metrics: balance

$$F_{\text{balance}}(\theta, \pi) \triangleq \sum_{y,z,a} |\mathbb{P}_\theta^\pi(a \mid y, z) - \mathbb{P}_\theta^\pi(a \mid y)|^2 \tag{4.4}$$

Utility: Classification accuracy

$$U(\theta, \pi) = \mathbb{P}_\theta^\pi(y_t = a_t)$$

The value of a policy

Fairness metrics: balance

$$F_{\mathrm{balance}}(\theta, \pi) \triangleq \sum_{y,z,a} |\, \mathbb{P}^\pi_\theta(a \mid y, z) - \mathbb{P}^\pi_\theta(a \mid y)|^2 \tag{4.4}$$

Utility: Classification accuracy

$$U(\theta, \pi) = \mathbb{P}^\pi_\theta(y_t = a_t)$$

Use λ to trade-off utility and fairness

$$V(\lambda, \theta, \pi) = (1 - \lambda) \overbrace{U(\theta, \pi)}^{\text{utility}} - \lambda \underbrace{F(\theta, \pi)}_{\text{unfairness}} \tag{4.5}$$

Model uncertainty

θ is unknown

Theorem 15

A decision rule in the form of a lottery, i.e.

$$\pi(a \mid x) = p_a$$

can be the only way to satisfy balance for all possible θ.

Possible solutions

- Marginalize over θ ("expected" model)
- Use Bayesian reasoning

The value of a policy

Let λ represent the trade-off between utility and fairness.

$$V(\lambda, \theta, \pi) = \lambda \overbrace{U(\theta, \pi)}^{\text{utility}} - \underbrace{(1 - \lambda) F(\theta, \pi)}_{\text{fairness violation}} \tag{4.6}$$

The Bayesian decision problem

The Bayesian value of a policy

$$V(\lambda, \xi, \pi) = \int_{\Theta} V(\lambda, \theta, \pi) \, \mathrm{d}\xi(\theta). \tag{4.7}$$

Online resources

- COMPAS analysis by propublica
  `https://github.com/propublica/compas-analysis`
- Open policing database `https://openpolicing.stanford.edu/`

Learning outcomes

Understanding

- Graphical models and conditional independence.
- Fairness as independence and meritocracy.

Skills

- Specify a graphical model capturing dependencies between variables.
- Testing for conditional independence.
- Verify if a policy satisfies a fairness condition.

Reflection

- Determining is to be fair with respect to sensitive attributes?
- Balancing the needs of individuals, the decision maker and society?
- Does having more data available make it easier to achieve fairness?
- What is the relation to game theory and welfare economics?