

Privacy

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Introduction

Database access models

Privacy in databases

k-anonymity

Differential privacy



Just because they're the problem,
doesn't mean we aren't.

Privacy in statistical disclosure.

- ▶ Public analysis of sensitive data.
- ▶ Publication of “anonymised” data.

Not about cryptography

- ▶ Secure communication and computation.
- ▶ Authentication and verification.

An issue of trust

- ▶ Who to trust and how much.
- ▶ With what data to trust them.
- ▶ What you want out of the service.

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Databases

Example 1 (Typical relational database in a tax office)

ID	Name	Salary	Deposits	Age	Postcode	Prof
1959060783	Li Pu	150,000	1e6	60	1001	Polit
1946061408	Sara Lee	300,000	-1e9	72	1001	Rent
2100010101	A. B. Student	10,000	100,000	40	1001	Time

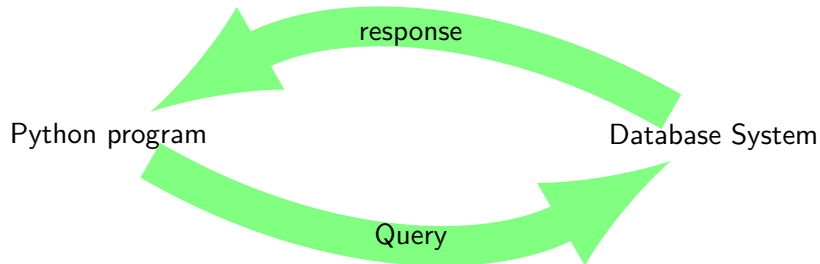
Database access

- ▶ When owning the database: Direct look-up.
- ▶ When accessing a server etc: Query model.

Databases

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Queries in SQL

The SELECT statement

- ▶ `SELECT column1, column2 FROM table;`
- ▶ `SELECT * FROM table;`

Selecting rows

```
SELECT * FROM table WHERE column = value;
```

Arithmetic queries

- ▶ `SELECT COUNT(column) FROM table WHERE condition;`
- ▶ `SELECT AVG(column) FROM table WHERE condition;`
- ▶ `SELECT SUM(column) FROM table WHERE condition;`

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Anonymisation

Example 2 (Typical relational database in Tinder)

Birthday	Name	Height	Weight	Age	Postcode	Profession
06/07	Li Pu	190	80	60-70	1001	Politician
06/14	Sara Lee	185	110	70+	1001	Rentier
01/01	A. B. Student	170	70	40-60	6732	Time Tra

Anonymisation

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Birthday	Name	Height	Weight	Age	Postcode	Profession
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The simple act of hiding or using random identifiers is called anonymisation.

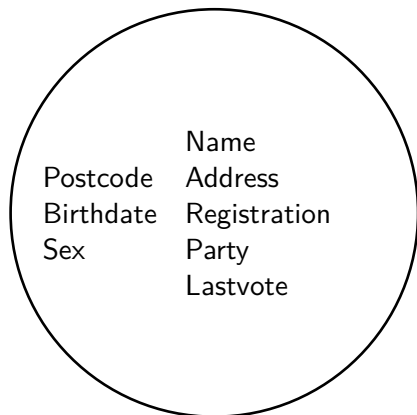
Record linkage

Ethnicity
Date
Diagnosis
Procedure
Medication
Charge
Postcode
Birthdate
Sex



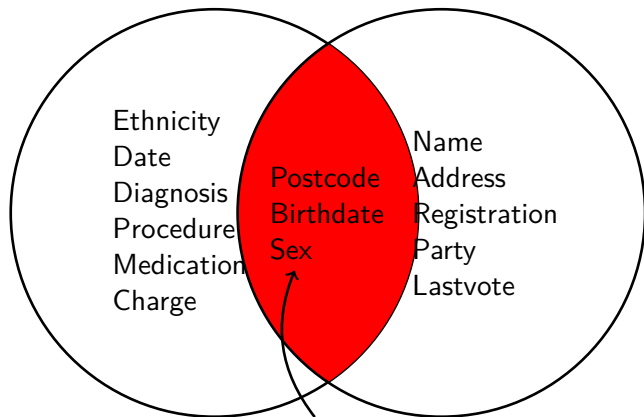
Bill Weld, R-MA

Record linkage



Bill Weld, R-MA

Record linkage



Bill Weld, R-MA

87% of Americans identifiable

Example 3 (Typical relational database in a tax office)

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Example 4 (Typical relational database in a tax office)

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k-anonymity



(a) Samarati



(b) Sweeney

Definition 5 (*k*-anonymity)

A database provides *k*-anonymity if for every person in the database is indistinguishable from $k - 1$ persons with respect to *quasi-identifiers*.

It's the analyst's job to define quasi-identifiers

Birthday	Name	Height	Weight	Age	Postcode	Pr
06/07	Li Pu	190	80	60+	1001	Po
06/14	Sara Lee	185	110	60+	1001	Re
06/12	Nikos Papadopoulos	170	82	60+	1243	Po
01/01	A. B. Student	170	70	40-60	6732	Ti
05/08	Li Yang	175	72	30-40	6910	Ti

Table: 1-anonymity.

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05/08		175	72	30-40	6910	Policeman

1-anonymity

Birthday	Name	Height	Weight	Age	Postcode	Profession
06/07		180-190	80+	60+	1*	
06/14		180-190	80+	60+	1*	
06/12		170-180	60+	60+	1*	
01/01		170-180	60-80	20-60	6*	
05/08		170-180	60-80	20-60	6*	

1-anonymity

Birthday	Name	Height	Weight	Age	Postcode	Profession
		180-190	80+	60+	1*	
		180-190	80+	60+	1*	
		170-180	60-80	60+	1*	
		170-180	60-80	20-60	6*	
		170-180	60-80	20-60	6*	

Table: 2-anonymity: the database can be partitioned in sets of at least 2 records

x_1  x 

Figure: If two people contribute their data $x = (x_1, x_2)$ to a medical database, and an algorithm π computes some public output a from x , then it should be hard to infer anything about the data from the public output.

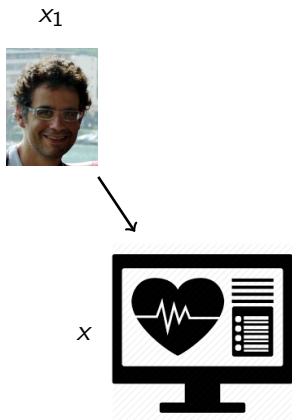


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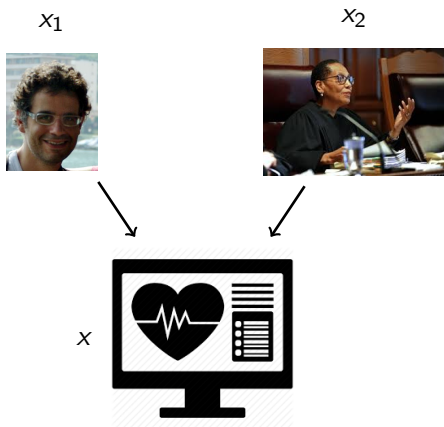


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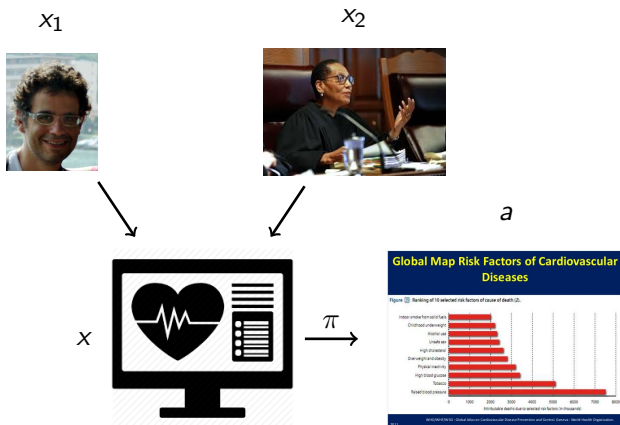


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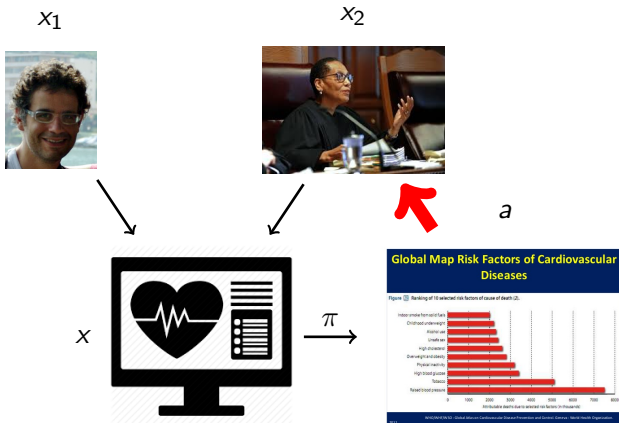


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Privacy desiderata

We wish to calculate something on some private data and publish a **privacy-preserving**, but **useful**, version of the result.

- ▶ Anonymity: Individual participation remains hidden.
- ▶ Secrecy: Individual data x_i is not revealed.
- ▶ Side-information: Linkage attacks are not possible.
- ▶ Utility: The calculation remains useful.

Example: The prevalence of drug use in sport

- ▶ n athletes
- ▶ Ask whether they have doped in the past year.
- ▶ Aim: calculate % of doping.
- ▶ How can we get truthful / accurate results?

Write responses in class: Age, Gender, Tobacco use

Example: The prevalence of drug use in sport

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Algorithm for randomising responses about drug use

1. Flip a coin.
2. If it comes heads, respond truthfully.
3. Otherwise, flip another coin and respond yes if it comes heads and no otherwise.

Exercise 1

Assume that the observed rate of positive responses in a sample is p , that everybody follows the protocol, and the coin is fair. Then, what is the true rate q of drug use in the population?

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Solution.

Since the responses are random, we will deal with expectations first

$$\mathbb{E} p = \frac{1}{2} \times \frac{1}{2} + q \times \frac{1}{2}$$



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$$\mathbb{E} p = \frac{1}{2} \times \frac{1}{2} + q \times \frac{1}{2} = \frac{1}{4} + \frac{q}{2}$$
$$q = 2 \mathbb{E} p - \frac{1}{2}.$$



The randomised response mechanism

Definition 6 (Randomised response)

The i -th user, whose data is $x_i \in \{0, 1\}$, responds with $a_i \in \{0, 1\}$ with probability

$$\pi(a_i = j \mid x_i = k) = p, \quad \pi(a_i = k \mid x_i = k) = 1 - p,$$

where $j \neq k$.

The randomised response mechanism

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Given the complete data x , the mechanism's output is $a = (a_1, \dots, a_n)$. Since the algorithm independently calculates a new value for each data entry, the output is

$$\pi(a \mid x) = \prod_i \pi(a_i \mid x_i)$$

Exercise 1

Let the adversary have a prior $\xi(x = 0) = 1 - \xi(x = 1)$ over the values of the true response of an individual. we use the randomised response mechanism with p and the adversary observes the randomised data $a = 1$ for that individual, then what is $\xi(x = 1 \mid a = 1)$?

The local privacy model

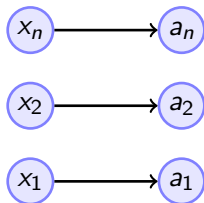
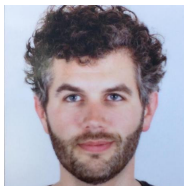


Figure: The local privacy model

Differential privacy.



Definition 7 (ϵ -Differential Privacy)

A stochastic algorithm $\pi : \mathcal{X} \rightarrow \mathcal{A}$, where \mathcal{X} is endowed with a neighbourhood relation N , is said to be ϵ -differentially private if

$$\left| \ln \frac{\pi(a | x)}{\pi(a | x')} \right| \leq \epsilon, \quad \forall x N x'. \quad (5.1)$$

Defining neighbourhoods

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Table: Data x

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Table: 1-Neighbour x'

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Table: 2-Neighbour x'

The definition of differential privacy

- ▶ First rigorous mathematical definition of privacy.
- ▶ Relaxations and generalisations possible.
- ▶ Connection to learning theory and reproducibility.

Current uses

- ▶ Apple.
- ▶ Google.
- ▶ Uber.
- ▶ US 2020 Census.

Open problems

- ▶ Complexity of differential privacy.
- ▶ Verification of implementations and queries.

Remark 1

The randomised response mechanism with $p \leq 1/2$ is $(\ln \frac{1-p}{p})$ -DP.

Proof.

Consider $x = (x_1, \dots, x_j, \dots, x_n)$, $x' = (x_1, \dots, x'_j, \dots, x_n)$. Then

$$\pi(a \mid x)$$



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$$\begin{aligned} \pi(a \mid x) &= \prod_i \pi(a_i \mid x_i) \\ &= \pi(a_j \mid x_j) \prod_{i \neq j} \pi(a_i \mid x_i) \end{aligned}$$



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$\pi(a_j = k \mid x_j = k) = 1 - p$ so the ratio is
 $\max\{(1-p)/p, p/(1-p)\} \leq (1-p)/p$ for $p \leq 1/2$.

Remark 1

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Consider $x = (x_1, \dots, x_j, \dots, x_n)$, $x' = (x_1, \dots, x'_j, \dots, x_n)$. Then

$$\begin{aligned} \pi(a | x) &= \prod_i \pi(a_i | x_i) \\ &= \pi(a_j | x_j) \prod_{i \neq j} \pi(a_i | x_i) \\ &\leq \frac{p}{1-p} \pi(a_j | x'_j) \prod_{i \neq j} \pi(a_i | x_i) \\ &= \frac{1-p}{p} \pi(a | x') \end{aligned}$$



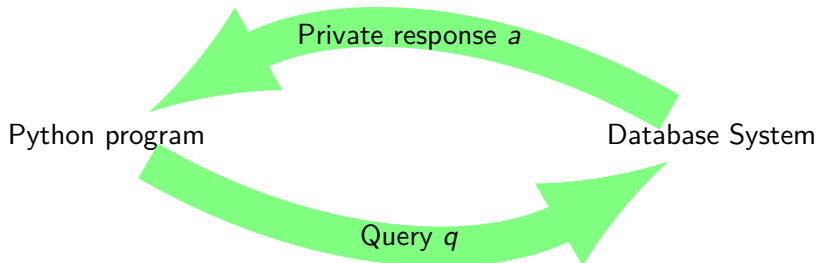


Figure: Private database access model

Response policy

The policy defines a distribution over responses a given the data x and the query q .

$$\pi(a \mid x, q)$$

Differentially private queries

The DP-SELECT statement

- ▶ DP-SELECT ϵ column1, column2 FROM table;
- ▶ DP-SELECT ϵ * FROM table;

Selecting rows

DP-SELECT ϵ * FROM table WHERE column = value;

Arithmetic queries

- ▶ DP-SELECT ϵ COUNT(column) FROM table WHERE condition;
- ▶ DP-SELECT ϵ AVG(column) FROM table WHERE condition;
- ▶ DP-SELECT ϵ SUM(column) FROM table WHERE condition;

Composition

If we answer T queries with an ϵ -DP mechanism, then our cumulative privacy loss is ϵT .

Exercise 2

Adversary knowledge

$$\mathbf{x} = (x_1, \dots, x_j = 0, \dots, x_n)$$

$$\mathbf{x}' = (x_1, \dots, x_j = 1, \dots, x_n).$$

$$\xi(\mathbf{x}) = 1 - \xi(\mathbf{x}')$$

What can we say about the posterior distribution of the adversary $\xi(\mathbf{x} \mid a, \pi)$ after having seen the output, if π is ϵ -DP?

Exercise 2

Adversary knowledge

$$\mathbf{x} = (x_1, \dots, x_j = 0, \dots, x_n)$$

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$$\xi(\mathbf{x}) = 1 - \xi(\mathbf{x}')$$

$$a_t, \quad \pi(a_t | \mathbf{x}_t) \Rightarrow \begin{cases} \pi(a_t | \mathbf{x}_t = \mathbf{x}) \\ \pi(a_t | \mathbf{x}_t = \mathbf{x}') \end{cases}$$

What can we say about the posterior distribution of the adversary $\xi(\mathbf{x} | a, \pi)$ after having seen the output, if π is ϵ -DP?

Dealing with multiple attributes.

Independent release of multiple attributes.

For n users and k attributes, if the release of each attribute i is ϵ -DP then the data release is $k\epsilon$ -DP. Thus to get ϵ -DP overall, we need ϵ/k -DP per attribute.

The Laplace mechanism.

Definition 8 (The Laplace mechanism)

For any function $f: \mathcal{X} \rightarrow \mathbb{R}$,

$$\pi(a \mid x) = \mathcal{L}aplace(f(x), \lambda), \quad (5.2)$$

where the Laplace density is defined as

$$p(\omega \mid \mu, \lambda) = \frac{1}{2\lambda} \exp\left(-\frac{|\omega - \mu|}{\lambda}\right).$$

and has mean μ and variance $2\lambda^2$.

.

Example 9 (Calculating the average salary)

- ▶ The i -th person receives salary x_i
- ▶ We wish to calculate the average salary in a private manner.

Local privacy model

- ▶ Obtain $y_i = x_i + \omega$, where $\omega \sim \mathcal{Laplace}(\lambda)$.
- ▶ Return $a = n^{-1} \sum_{i=1}^n y_i$.

Centralised privacy model

Return $a = n^{-1} \sum_{i=1}^n x_i + \omega$, where $\omega \sim \mathcal{Laplace}(\lambda')$.

How should we add noise in order to guarantee privacy?

The centralised privacy model

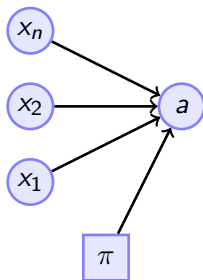


Figure: The centralised privacy model

Assumption 1

*The data x is collected and the result a is published by a **trusted curator***

DP properties of the Laplace mechanism

Definition 10 (Sensitivity)

The sensitivity of a function f is

$$\mathbb{L}(f) \triangleq \sup_{x, x'} |f(x) - f(x')|$$

Example 11

If $f: \mathcal{X} \rightarrow [0, B]$, e.g. $\mathcal{X} = \mathbb{R}$ and $f(x) = \min\{B, \max\{0, x\}\}$, then

DP properties of the Laplace mechanism

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Example 12

If $f: [0, B]^n \rightarrow [0, B]$ is $f = \frac{1}{n} \sum_{t=1}^n x_t$, then

DP properties of the Laplace mechanism

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Example 12

If $f: [0, B]^n \rightarrow [0, B]$ is $f = \frac{1}{n} \sum_{t=1}^n x_t$, then $\mathbb{L}(f) = B/n$.

Theorem 13

The Laplace mechanism on a function f with sensitivity $\mathbb{L}(f)$, ran with $\mathcal{Laplace}(\lambda)$ is $\mathbb{L}(f)/\lambda$ -DP.

Proof.

$$\frac{\pi(a | x)}{\pi(a | x')} = \frac{e^{|a-f(x')|/\lambda}}{e^{|a-f(x)|/\lambda}} \leq \frac{e^{|a-f(x)|/\lambda + \mathbb{L}(f)/\lambda}}{e^{|a-f(x)|/\lambda}} = e^{\mathbb{L}(f)/\lambda}$$

□

So we need to use $\lambda = \mathbb{L}(f)/\epsilon$ for ϵ -DP. What is the effect of applying the Laplace mechanism in the local versus centralised model?

Interactive queries

- ▶ System has data x .
- ▶ User asks query q .
- ▶ System responds with a .
- ▶ There is a common utility function $U: \mathcal{X}, \mathcal{A}, \mathcal{Q} \rightarrow \mathbb{R}$.

We wish to maximise U with our answers, but are constrained by the fact that we also want to preserve privacy.

The Exponential Mechanism.

Definition 14 (The Exponential mechanism)

For any utility function $U : \mathcal{Q} \times \mathcal{A} \times \mathcal{X} \rightarrow \mathbb{R}$, define the policy

$$\pi(a \mid x) \triangleq \frac{e^{\epsilon U(q,a,x)/\mathbb{L}(U(q))}}{\sum_{a'} e^{\epsilon U(q,a',x)/\mathbb{L}(U(q))}} \quad (5.3)$$

What happens when $\epsilon \rightarrow \infty$? What about when $\epsilon \rightarrow 0$?

The unfortunate practice of adaptive analysis

Prior

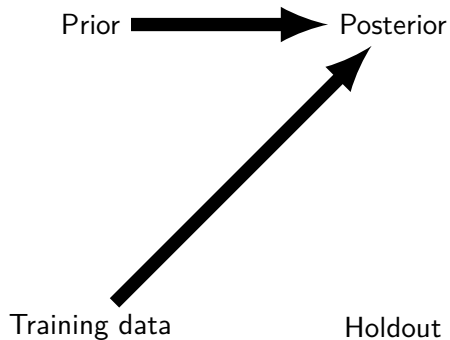
The unfortunate practice of adaptive analysis

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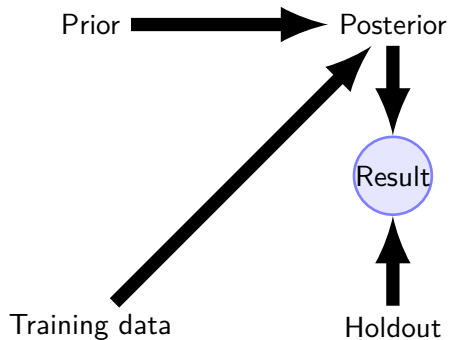
Training data

Holdout

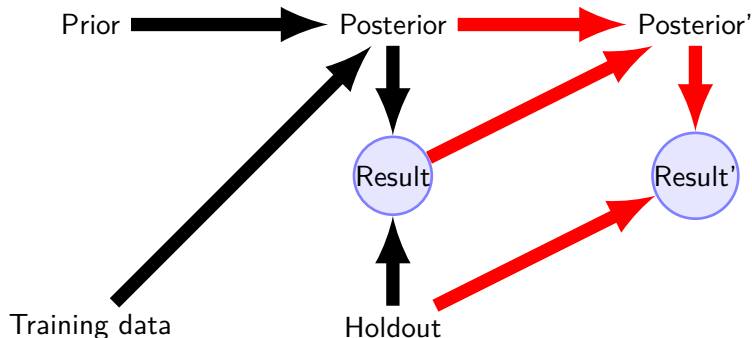
The unfortunate practice of adaptive analysis



The unfortunate practice of adaptive analysis



The unfortunate practice of adaptive analysis



The reusable holdout?¹

Algorithm parameters

- ▶ Performance measure f .
- ▶ Threshold τ .
- ▶ Noise σ .
- ▶ Budget B .

Algorithm idea

Run algorithm λ on data D_T and get e.g. classifier parameters θ .

Run a DP version of the function $f(\theta, D_H) = \mathbb{I}\{U(\theta, D_T) \geq \tau U(\theta, D_H)\}$.

¹Also see

Available privacy toolboxes

k-anonymity

- ▶ <https://github.com/qiyuangong/Mondrian> Mondrian *k*-anonymity

Differential privacy

- ▶ <https://github.com/bmcmenamein/thresholdOut-explorations> Threshold out
- ▶ <https://github.com/steven7woo/Accuracy-First-Differential-Privacy> Accuracy-constrained DP
- ▶ <https://github.com/menisadi/pydp> Various DP algorithms
- ▶ <https://github.com/haiphanNJIT/PrivateDeepLearning> Deep learning and DP

Learning outcomes

Understanding

- ▶ Linkage attacks and k -anonymity.
- ▶ Inferring data from summary statistics.
- ▶ The local versus global differential privacy model.
- ▶ False discovery rates.

Skills

- ▶ Make a dataset satisfy k -anonymity with respect to identifying attributes.
- ▶ Apply the randomised response and Laplace mechanism to data.
- ▶ Apply the exponential mechanism to simple decision problems.
- ▶ Use differential privacy to improve reproducibility.

Reflection

- ▶ How can potentially identifying attributes be chosen to achieve