Taylor series for cos x

The Taylor appoximation to cos x can be written as

$$\cos x \approx S(x;n) = \sum_{j=0}^{n} (-1)^j \frac{x^{2j}}{(2n)!}$$

To formulate this sum as a difference equation, we write the sum as $S(x;n) = \sum_{j=0}^{n} a_j$, and find a relation between two consecutive terms. We have

$$a_j = (-1)^j \frac{x^{2j}}{(2j)!},$$

and by a few manipulations we get

$$\begin{aligned} a_j &= (-1)^j \frac{x^{2j}}{(2j)!} = (-1)^j \frac{x^{2(j-1)+2}}{(2j)(2j-1)(2j-2)\ldots} \\ &= (-1)(-1)^{j-1} \frac{x^2 x^{2(j-1)}}{(2j)(2j-1)((2(j-1))!)} = (-1) \frac{x^2}{2j(2j-1)} a_{j-1}. \end{aligned}$$

This relation is on the same form as equation A.61 from the book, and we can apply the steps from exercise A.14 directly.