1.18 a) $1(0 \cup 1)^* 0$

b) $(0 \cup 1)^* 1 (0 \cup 1)^* 1 (0 \cup 1)^* 1 (0 \cup 1)^*$

c) $(0 \cup 1)^* 0101 (0 \cup 1)^*$

d) $(0 \cup 1)(0 \cup 1) 0 (0 \cup 1)^*$

e) $0((0 \cup 1)(0 \cup 1))^* \cup 1(0 \cup 1)((0 \cup 1)(0 \cup 1))^*$

f) $(0 \cup 10)^*(\varepsilon \cup 111^*)$

g) $(\varepsilon \cup 0 \cup 1)(\varepsilon \cup 0 \cup 1)(\varepsilon \cup 0 \cup 1)(\varepsilon \cup 0 \cup 1)(\varepsilon \cup 0 \cup 1)$

h) $\varepsilon \cup 1 \cup 0(0 \cup 1)^* \cup 10(0 \cup 1)^* \cup 110(0 \cup 1)^* \cup 111(0 \cup 1)(0 \cup 1)^*$

i) $1((0 \cup 1)1)^*$

j) $000^* \cup 1000^* \cup 000^*1 \cup 00^*100^*$

k) $\varepsilon \cup 0$

l) $(1^*01^*01^*)^* \cup 0^*10^*10^*$

m) $\emptyset$

n) $(0 \cup 1)(0 \cup 1)^*$

1.49

a) Note first that $F \cap ab^*c^* = \{ab^nc^n \mid n \geq 0\}$.

We first show that $E = \{ab^nc^n \mid n \geq 0\}$ is not regular.
Suppose the language is regular, and let p be the pumping length given by the pumping lemma.
Consider the string $s = ab^pc^p$. Consider a decomposition $s = xyz$. By condition 3 of the pumping lemma, $|xy| \leq p$, so we have one of two choices for y:

1) $y = ab^n$ w/ $0 \leq n < p$
2) $y = b^n$ w/ $0 < n < p$

In case 1), xy^2z consists of 2 a's, and so $xy^2z \notin E$.
In case 2), $xy^2z = ab^{p+n}c^p$. Since $n > 0$, $p+n \neq p$, and so $xy^2z \notin E$.
So for all choices of y, $xy^2z \notin E$. This contradicts the pumping lemma, and thus E is not regular.
Since ab^*c^* is regular, if F is regular, then $E = F \cap ab^*c^*$ is regular. Since this is a contradiction F is not regular.

b) We let the pumping length $p = 2$.
Let $s = a^ib^jc^k \in F$ of length ≥ 2. Then at least one of $i > 0$, $j > 0$, $k > 0$. We consider the cases:

1) $s = a^ib^jc^k$, $i > 0$. If $i = 2$, $x = \varepsilon$, $y = a^2$, $z = b^jc^k$.
Then $xy^rz = a^{2r}b^jc^k \in F$ as $2r \neq 1$ for all r.
If $i \neq 1$, $x = \varepsilon$, $y = a$, $z = a^{i-1}b^kc^k$. Then $xy^rz = a^{r+i-1}b^kc^k \in F$.
2) $s = b^jc^k$, $j > 0$. Set $x = \varepsilon$, $y = b$, $z = b^{j-1}c^k$.
Then $xy^rz = b^{r+j-1}c^k \in F$.
2) $s = c^k$, $k > 0$. Set $x = \varepsilon$, $y = c$, $z = c^{k-1}$.
Then $xy^rz = c^{r+k-1} \in F$.

So, for any string $s \in F$ of length ≥ 2, there exists a division of s into three pieces, $s = xyz$, satisfying the conditions of the pumping lemma.

c) The pumping lemma states that every regular language behaves in a certain way, but says nothing about nonregular languages. So if a language satisfies the conditions of the pumping lemma, we can not conclude that the language is regular, and therefore there is no contradiction.

1.51

a) We wish to show that the language $A = \{0^n 1^m 0^n \mid n, m \geq 0\}$ is not regular. Suppose it is regular. By the pumping lemma, there is a pumping length p. Let $s = 0^p 1 0^p \in A$. Suppose $s = xyz$ according to the pumping lemma. Since $|xy| \leq p$, we must have $x = 0^k$, $y = 0^n$, $z = 0^{p-(k+n)} 1 0^p$, $k \geq 0$, $n > 0$. Then $xy^2z = 0^k 0^{2n} 0^{p-(k+n)} 1 0^p = 0^{p+n} 1 0^p$. Since $n > 0$, $xy^2z \notin A$, which contradicts the pumping lemma.

b) See the textbook.

c) We show that the language $C = \{w \mid w \in \{0,1\}^* \text{ is not a palindrome}\}$ is not regular by showing that $\overline{C} = \{w \mid w \in \{0,1\}^* \text{ is a palindrome}\}$ is not regular. Assume that $\overline{C}$ is regular, and get a pumping length p from the pumping lemma. Let $s = 0^p 11 0^p \in \overline{C}$. By the same argument as in a), $xy^2z = 0^{p+n} 11 0^p$ for $n > 0$, and $0^{p+n} 11 0^p \neq 0^p 11 0^{p+n}$, so $xy^2z \notin \overline{C}$, which is a contradiction. Therefore $\overline{C}$ is not regular, and neither is C.

d) To show that $D = \{wtw \mid w, t \in \{0,1\}^+\}$ is not regular, assume it is and let p be the pumping length given by the pumping lemma.
Consider the string $s = \underbrace{0^p 1^p}_{w} \underbrace{01}_{t} \underbrace{0^p 1^p}_{w} \in D$.

By the pumping lemma, in a similar way to a) and c) $s' = 0^{p+n} 1^p 01 0^p 1^p \in D$ for some $n > 0$.
Then w starts with a 0 and ends with a 1, since s' starts with w, $w = 0^{p+n} v 1$, $v \in \{0,1\}^*$.
Since 0^{p+n} is not a substring of the rest of s', neither can w, and thus $s' \neq wtw$ which shows that $s' \notin D$. This is a contradiction, and thus D is not regular.