Oblig 3

IN2080

April 22, 2021

Hand-in and deadline

Hand in a single PDF file in Devilry. The deadline is **May 11, at 23:59**.

We recommend LaTeX, but all major text editors allows exporting to PDF. You can get help with LaTeX at the group sessions. You can also download the LaTeX source (`.tex`) for this assignment at the assignments page.

Definitions

A *literal* is a formula on the form x or $\overline{x}$, where x is a variable. A formula φ is on *Conjunctive Normal Form* (CNF) if

$$\varphi = \left(l_1^1 \vee \cdots \vee l_{k_1}^1\right) \wedge \cdots \wedge \left(l_1^n \vee \cdots \vee l_{k_n}^n\right),$$

where l_i^j is the i-th literal of the j-th clause, and k_m is the number of literals in the m-th clause. A formula φ is on *Disjunctive Normal Form* (DNF) if

$$\varphi = \left(l_1^1 \wedge \cdots \wedge l_{k_1}^1\right) \vee \cdots \vee \left(l_1^n \wedge \cdots \wedge l_{k_n}^n\right).$$

We define the following languages:

$$\begin{aligned}
\mathsf{CNFSAT} &= \{\varphi \mid \varphi \text{ is on CNF, and } \varphi \text{ is satisfiable}\}\\
\mathsf{DNFSAT} &= \{\varphi \mid \varphi \text{ is on DNF, and } \varphi \text{ is satisfiable}\}\\
\mathsf{CNFUNSAT} &= \{\varphi \mid \varphi \text{ is on CNF, and } \varphi \text{ is unsatisfiable}\}\\
\mathsf{DNFUNSAT} &= \{\varphi \mid \varphi \text{ is on DNF, and } \varphi \text{ is unsatisfiable}\}\\
\mathsf{CNFTAUT} &= \{\varphi \mid \varphi \text{ is on CNF, and } \varphi \text{ is a tautology}\}\\
\mathsf{DNFTAUT} &= \{\varphi \mid \varphi \text{ is on DNF, and } \varphi \text{ is a tautology}\}
\end{aligned}$$

The complexity class coNP consists of complements of languages in NP. More formally, $\mathsf{coNP} = \{\overline{A} \mid A \in \mathsf{NP}\}$.

A language A is coNP-complete if

i) $A \in \mathsf{coNP}$

ii) For every language $B \in \mathsf{coNP}$, $B \leq_p A$.

You may use, without proof, that SAT and $\mathsf{3SAT}$ are NP-complete.

Problem 1

At least one of the above languages is in P. Identify them, and prove that they are in P.

Problem 2

At least one of the above languages is NP-complete. Identify them, and prove that they are NP-complete.

Problem 3

a) Show that for all languages A and B, $A \leq_p B$ implies $\overline{A} \leq_p \overline{B}$.

b) Show that a language A is NP-complete if and only if $\overline{A}$ is coNP-complete.

c) At least one of the above languages is coNP-complete. Identify them, and, using what you have shown in 3a) and 3b), prove that they are coNP-complete.