Oblig 4 IN3030/IN4330, Spring 2023 Finding the Convex Hull of a set of Points in a Plane: A Recursive Geometric Problem

Delivery deadline: May 3rd 2023, 23:59:00 CEST (local Oslo time)

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Given a set of points P in a plane consisting of n (n >2) different points pi (i =0,1,2,…,n-1) with integer coordinates (i.e., no decimals), where we can assume that *not all* points are on the same line, find the Convex Hull of the set of points. The convex is a polygon consisting of a subset of the points in P and with lines between some of these points such that all the other points are on the inside of this polygon and also so that all inner angles of this polygon are ≤ 180. In simpler terms convex means that there is no inwards dent on the set of lines running around the set of points (fig1).

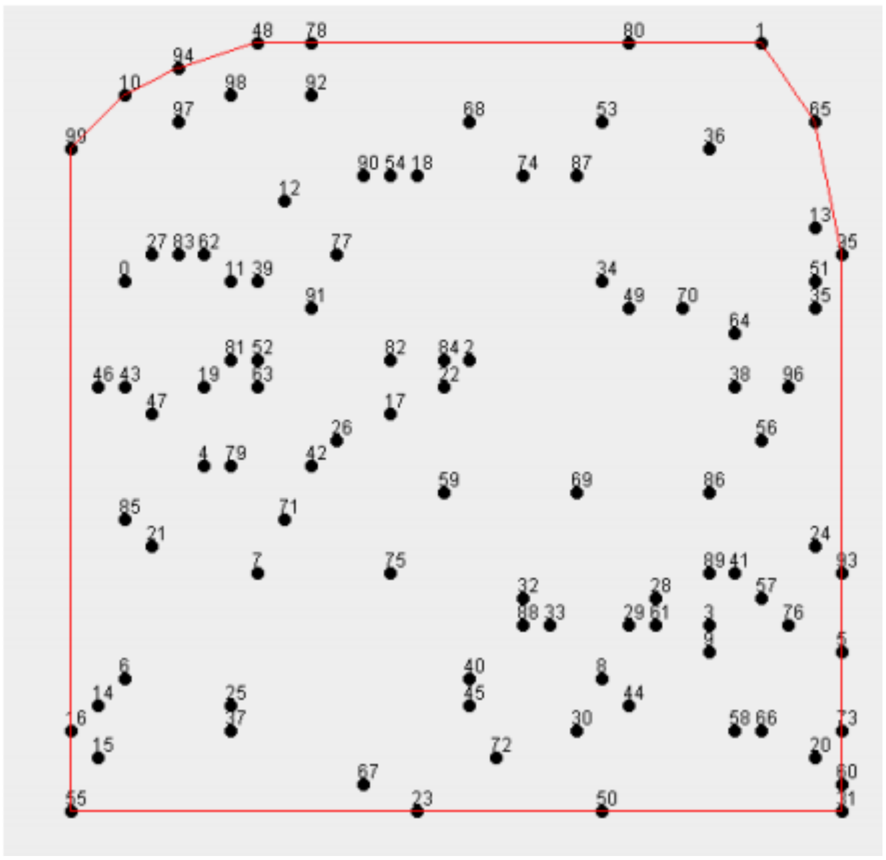


Fig1: The Convex Hull of 100 arbitrary points in a plane (the same as used in the assignment).

When finding such a convex hull, we start with an arbitrary point along the hull and jot down the following points that make up the hull *counter-clockwise*. In fig1, it could for instance be 16, 55, 23, 50, 31, 60, 73, 5, 93, 95, 65, 1, 80, 78, 48, 94, 10, 99. Notice that if there are more points on a line (for instance, 31, 60, 73, 5, 93, and 95), all of those points should be included in the hull. You are in other words not allowed to go straight from point 31 to 95, you have to specify all the lines 31-60, 60-73, 73-5, 5-93 and 93-95. We thus require that all points in P that are on one of the lines of the polygon, are part of the hull. This makes the subset of points included in the convex hull unique.
What point should you start with? Note that any point that has either the highest or lowest x or y value will always be on the hull, so any such point can be used as a starting point. For example points 31 or 55 in fig1.

How do we find the Convex Hull of n points?

We can start with two points that we can see are obviously on the convex hull: the point that has the lowest value for x and the point that has the highest value for x. If there are more points with the same values, just arbitrarily choose one of those with the lowest and one of those with the highest values for x (in the example, for example 16 and 5). The rest of the algorithm is based on simple geometry, which is reviewed below.

Equation for a line

Each line from a point p1 (x1, y1) to p2 (x2, y2) can be written as:

ax + **b**y + **c** = 0

Where: **a** = y1 − y2, **b** = x2 − x1 and **c** = y2 ⋆ x1 − y1 ⋆ x2

Notice that this is a **straight line *from* p1 *to* p2.**

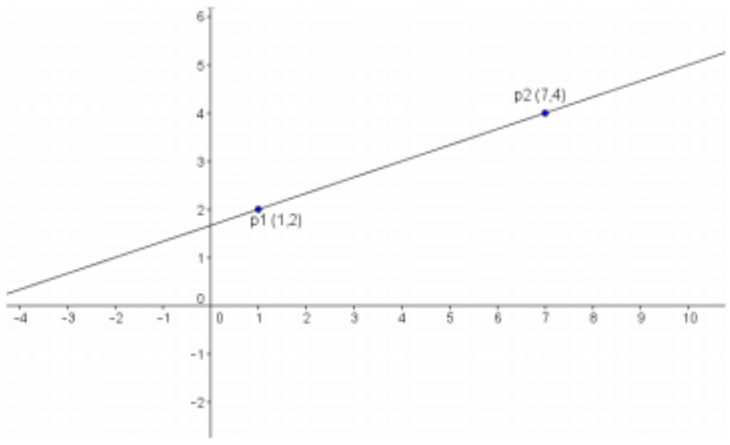


Fig2: A line from p1 (1,2) to p2 (7,4) has the line equation:
(2 − 4)x + (7 − 1)y + (4 ⋆ 1 − 2 ⋆ 7) = 0; i.e. − 2x + 6y − 10 = 0

The line equation means that all points on this line satisfy the equation. We can also say that this line splits the plane in two: the points to the left side seen from the direction of the line - from p1 to p2 - and the points on the right side in the direction from p1 to p2.

The distance from the line to other points

If we consider points that are not on the line p1-p2, we can see that all points, for example p3 (5,1) on the right side of the line gives a number <0 in the line equation ax + by + c, og all points to the left of the line, such as p4 (3,6) gives a number >0.

The further from the line the points are, the larger negatives or positive values they will have. Generally speaking we can say that the distance perpendicular down from a point p (x,y) to a line ax + by + c is:

$$d = \frac{ax + by + c}{\sqrt{a^2 + b^2}}$$

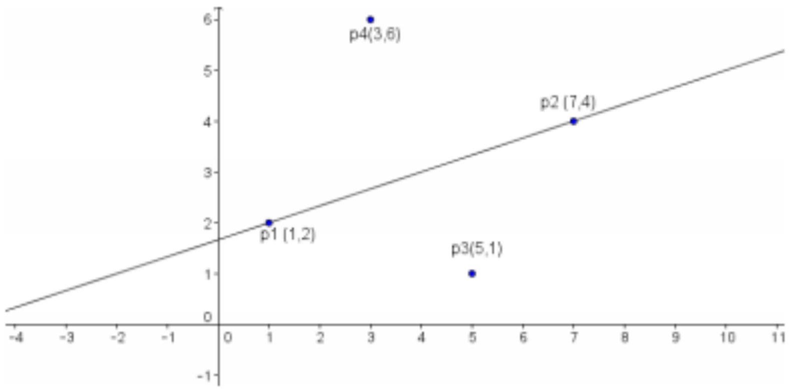


Fig3: The distance from a point to a line. The line divides the plane in two; those points with negative distance (to the right) and those with positive distance (to the left) from the line.

Consider two points in fig3: p3 (5,1) and p4 (3,6), we can calculate the distance from p4 to the line p1-p2 as:

$$\frac{-2*3+6*6-10}{\sqrt{(-2)^2+6^2}} = \frac{20}{\sqrt{40}} = 3{,}16..,$$

While the distance from p3 (5,1) to p1-p2 is:

$$\frac{-2*5+6*1-10}{\sqrt{40}} = \frac{-14}{6{,}32} = -\,2{,}21..$$

The Algorithm for the Convex Hull

After making another observation, we can formulate the algorithm for the convex hull. Observation: The point that has the longest (largest) negative distance from a line pi-pj is itself a point on the convex hull (see fig4, point P and line I - A).

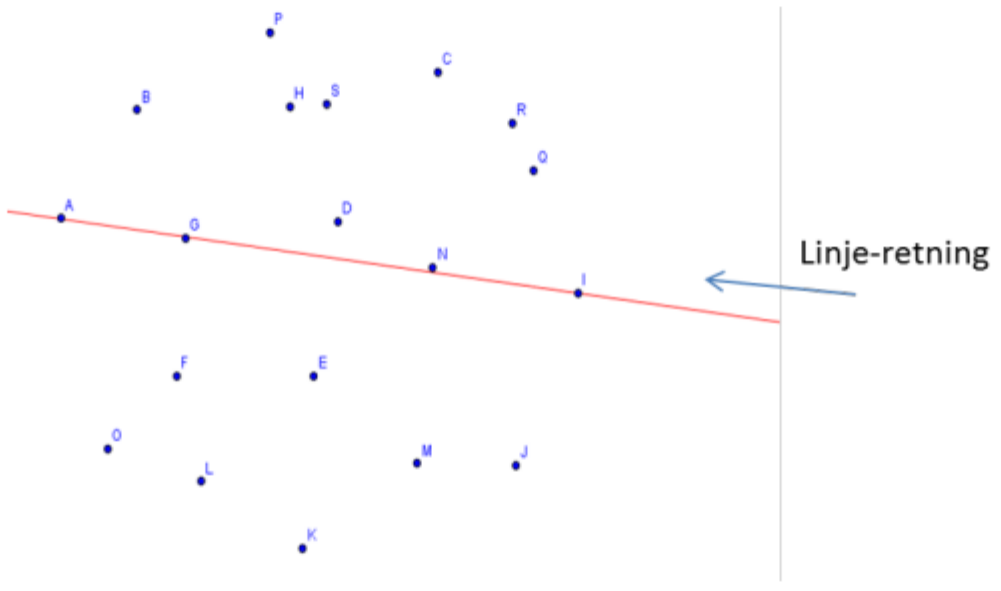


Fig.4. Starten på å finne innhyllinga fra maxx (I) til minx (A).

Fig4: The start to finding the convex hull from max_x (I) to min_x (A). **("line-retning" translates to "direction of line"**

Convex Hull Algorithm:

1. Draw a line between the two points we know are on the hull from max_x to min_x (I to A in fig4)
2. Find the point with the largest negative distance (or 0) from the line (in fig4 it's P). If multiple points have the same distance, just choose one arbitrarily.
3. Draw a line from the two points on the line to this new point on the hull (in fig5: I-P and P-A)
4. Continue recursively from the two new lines and for each of them find a new point on the hull with the largest non-positive distance (less than or equal to 0)
5. Repeat step 3 and 4 until there are no more points on the outside of the lines
6. Repeat steps 2-4 for the line min_x-max_x, and find all points on the hull under it.

This process is illustrated in fig5. You can see the process of finding the points on the hull above the line I-A. Expect to find about 1.4 * sqrt(n) elements in the convex hull from n randomly chosen points on the plane, although all values from 3 to n are possible (we get n, if all points are on a circle or a triangle or a rectangle or any convex polygon).

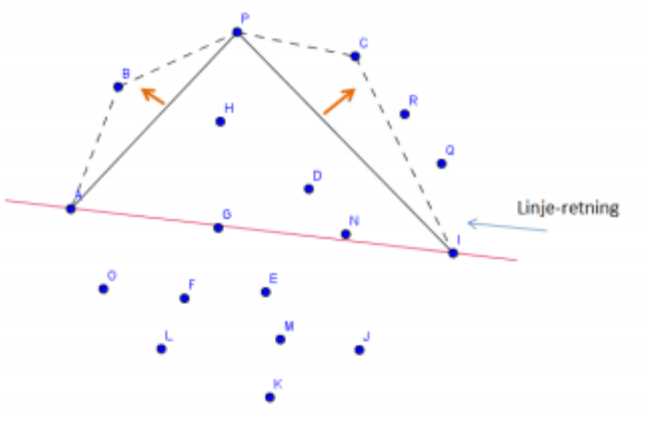


Fig5: We recursively find the point P on the hull from the line I-A (that with the largest negative distance from max_x-min_x), then C from P-I, R from I-C and lastly Q from I-R.

First, you are supposed to program this recursive algorithm sequentially, and then make a parallel version and measure speedups for n=100, 1000, 10 000, …, 10 million.

On generating n different points

a) The data structure for your n points should be two int arrays x and y, each of them with length = n.
```
int []x = new int[n], y = new int[n];
```
In order to fill these arrays with x and y values that does not create equal points, you will use the pre-written class NPunkter which you will find in the Oblig5 file.

b) For each value of n in your program, you should first create an object of the class NPunkter **(NPunkter translates to NPoints)**:
```
NPunkter p = new NPunkter(n);
```
and then fill x[] and y[] with a call to the method in the object p:
```
p.fyllArrayer(int [] x, int [] y);
```
("fyllArrayer" translates to "fillArrays)

The reason why you must use this class, which you must not alter, is that all the deliveries for a given value of n should have the same points. Then it will be much easier for the correctors to correct the obligs (additionally, it is not easy to relatively quickly generate up to 10 million random points that you are certain are not equal, but that task is not a part of Oblig5).

c) The process of generating n points should be excluded from the reported execution times of the algorithm.

Efficiency

a) We are not really interested in the actual distance from a point to a line, it is just used as a relative measure to see which point "wins", i.e., is furthest away from the line. The formula for the distance d can as such be simplified by avoiding the division.
b) It will be time-consuming if you have to run through the entire set of points P all the time to find the few points that are on the outside of a line. That is why you should let your program test on smaller and smaller sets of points
c) Remember to be critical regarding using threads instead of recursion (think "layers", like quicksort). Threads should only be used at the top of the recursive tree.
d) Is `ArrayList<Integer>` fast enough to hold large sets of points, or should you use the class IntList with the "same" methods you need, and where the integers you are supposed to store resides in a simple integer-array?

The Assignment

In Oblig 5 you must:

- Program a sequential version of the algorithm described above.
- Parallelize the sequential implementation preferably using one of the two methods that will be described in the course lectures: either method 1 or 2.
- Use the precode *NPunkter17.java* and *Oblig5Precode.java* published separately. You must use the `drawGraph()` to draw the graph.
- It must be possible to specify the seed for the precode on the command line when starting the program.
- The program should be able to print the resulting convex hull to a file using the `writeHullPoints()` in the precode for n < 10000.
- Write a report on your findings. Report recommendations are on the course web.
- IN4330 students must in addition: run the Java Benchmarking Harness multiple times, and comment on what improvements you make to your code, and what the harness shows as empiric evidence of improvement.

Requirements

Oblig 5 Devilry deadline: May 11th 2023 23:59:00. The delivery should consist of:

a) The code for both the sequential and parallel solution, wrapped in a .zip file.
b) An output of the convex hull for n = 1000.
c) A report with both a table and a curve showing speedup as a function of n (= 100, 1000, … 10 million) and your assessment as to why your runtimes are as low as they are, and why you get the speedups you get. NOTE: runtime for n = 100 milion should be less than 14s and n = 10 million should be less than 2s. Include specifics of the computer you ran your tests on (modell, clock frequency, number of cores) and the size of the main memory, and if possible the size of the caches and delay (run the program latency.exe if you are on a Windows computer) and gladly a picture created by TegnUt when n=100.

Oblig 5 report recommendations have been published on the course page.

Tips

The following points are not requirements but describe areas where you might run into some trouble.

1) **The Sequential solution**
    a) **How to represent a point**
    Use the index in x[] and y[] - the contents of those will not change during runtime; they are only read.
    b) **Debugging**
    Very few, if any, manage to get their code right on the first run. We need to debug the code and that might be difficult on a graphical problem like this one. You can for instance use TegnUt to draw your lines and use a small number for n while debugging (n < 200). The call to TegnUt is made as such:

    ```
    TegnUt tu = new TegnUt (this, koHyll);
    ```

    The first line is to draw the points and the convex hull that is expected to be added in to an `IntList coHull`.

    c) **Find points on the convex hull in the correct order**
    Tip: you are using two methods for the sequential solution (assuming here that you are using ArrayList, but this can trivially be changed by swapping it with a faster IntList):

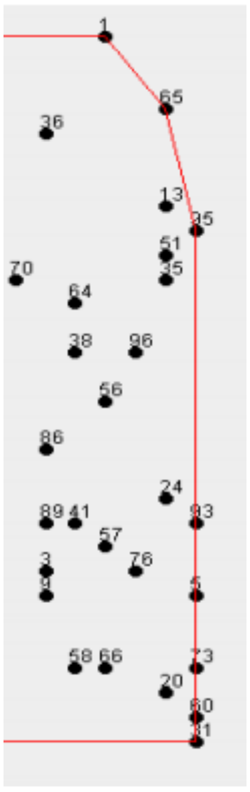
    seqMethod() which finds min_x, max_x, and starts the recursion. If you start on max_x-min_x, add max_x to the list of hull points (but not min_x yet), and start your recursion with two calls to the recursive method:

    seqRec(int p2, int p2, int p3, ArrayList m, ArrayList koHyll) which receives a set of points m (containing all points above or below the line p1-p2) and p3 that has already been found as the point with the largest negative or positive distance from p1-p2. koHyll is the set to which you should add the points from the convex hull in the correct order.

    Notice that you first “do a recursion” on the right side (line I-P in fig5) and then on the left side (line P-A) because we want the points to be added in a counter-clockwise order.

    You can let each call to seqRec add one point: p3 to the list of points on the hull, but where in your code should you do so? When do you add min_x to the list?

d) **Include all points on the hull in cases where multiple points are on the same line (distance = 0), such as on the right side in fig6, and in correct order.**
Remember that if you find that the largest negative distance == 0, there is no need to include p1 or p2 as possible new points (they have already been found). Say that you have found p1=35 and p2=5 and you want to find all the points on the line between p1 and p2 (60 and 73) and you then have to test if a new point p3 has both x and y coordinates corresponding to those of p1 and p2. Then you can find one of the points with a call to seqRec above the line p1-p2 (31-5). Repeat recursively until there no longer are any points between the new p1 and p2.
The other points on the line (for instance, 5-95) will be found recursively similarly to 60 and 73.



2) The parallel solution

a) You could consider dividing the parallel case in two methods. The first parallel call from the main-thread (between the two lines where you time the parallel implementation). It will then find out how far down the call tree we should use threads. If we have k cores on our computer, it might be reasonable to move down to a layer in the call tree where there are approximately k nodes horizontally in the tree. Example: if we have 8 cores, a call the top of the tree level 1, then level 4 will have 8 nodes. Up until this level new threads will replace recursive calls, and from that level regular recursive calls can be used from each of the threads we started.
b) The first parallel method called from and being run from the main-method creates two threads. The following threads are born from these two threads and their offspring-threads. These are started in the parallel method, which is called from the run()-methods.
c) The other parallel method has the same parameters as the sequential one: p1, p2, p3, and the set of points to look within in addition to two more: the level of the tree and the set (IntList) of points on the hull that the thread has found.
d) The other parallel, recursive method decides whether to keep on creating threads or if we are going to start using recursive calls. Prior to that, we have found the point that has the largest negative distance and the set of points for the next call and we have also declared and generated to sets; one for each of those to recursions that are supposed to hold the convex hull that particular call finds.
e) This method strongly resembles the sequential method, and after we have reached the level where we seize to make new threads, we can actually just use the sequential recursive method.

f) Remember that when we start threads we have to first create both of them, and then wait (for instance by using join()) for them. If we do not, we do not get paralyzation.

Finding the points of the convex hull in the correct order (parallel solution)

Another problem you will face is adding points in the correct order. While it is pretty clear when to add points to the convex hull in the sequential solution, it is not as predictable in the parallel solution. A way to solve this is to use:

1. Threads started at the top of the recursion tree only traverse down to a certain level for all nodes of the tree (the level depends on the number of cores at our disposal)..
2. After we can no longer start new threads, each thread moves on to the sequential solution (on its side of data)
3. From each of these sequential executions we will get the points in the correct order (counter-clockwise)
4. The problem then is to join these sets of points that are individually in the correct order. Do this: add first the points the right-hand thread has produced, then the point found by the node itself, and then the points from the left-hand thread.
5. Point 4 is then executed (upon return after finding its points) upwards until we reach the top and is done.