# Model Semantics

Read

- Foundations of Semantic Web Technologies: chapter 3, 2.

## 1 From the lecture

a) What does RDFS add to RDF?

b) What is formal semantics?

c) Why do we need a model semantics for RDF/RDFS?

d) What does a DL-interpretation consist of?

## 2 Definitions

First, some notation and definitions collected from the lecture slides.

### 2.1 Syntax: Triple abbreviations

| Triple pattern | Triple instance | Abbreviation |
|---|---|---|
| `indi prop indi .` | $i_1$ $r$ $i_2$ | $r(i_1, i_2)$ |
| `indi rdf:type class .` | $i_1$ `rdf:type` $C$ | $C(i_1)$ |
| `class rdfs:subClassOf class .` | $C$ `rdfs:subClassOf` $D$ | $C \sqsubseteq D$ |
| `prop rdfs:subPropertyOf prop .` | $r$ `rdfs:subPropertyOf` $s$ | $r \sqsubseteq s$ |
| `prop rdfs:domain class .` | $r$ `rdfs:domain` $C$ | $\mathsf{dom}(r, C)$ |
| `prop rdfs:range class .` | $r$ `rdfs:range` $C$ | $\mathsf{rg}(r, C)$ |

### 2.2 Interpretation

An *interpretation* $\mathcal{I}$ consists of:

- A set $\Delta^{\mathcal{I}}$, called the *domain* $\mathcal{I}$
- For each individual URI $i$, an element $i^{\mathcal{I}} \in \Delta^{\mathcal{I}}$
- For each class URI $C$, a subset $C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
- For each property URI $r$, a relation $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$

### 2.3 Validity in Interpretations (RDF)

Given an interpretation $\mathcal{I}$, define $\models$ as follows:

- $\mathcal{I} \models r(i_1, i_2)$ iff $\langle i_1^{\mathcal{I}}, i_2^{\mathcal{I}} \rangle \in r^{\mathcal{I}}$
- $\mathcal{I} \models C(i)$ iff $i^{\mathcal{I}} \in C^{\mathcal{I}}$

### 2.4 Validity in Interpretations, cont. (RDFS)

Given an interpretation $\mathcal{I}$, define $\models$ as follows:

- $\mathcal{I} \models C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
- $\mathcal{I} \models r \sqsubseteq s$ iff $r^{\mathcal{I}} \subseteq s^{\mathcal{I}}$
- $\mathcal{I} \models \mathsf{dom}(r, C)$ iff $\mathsf{dom}\ r^{\mathcal{I}} \subseteq C^{\mathcal{I}}$
- $\mathcal{I} \models \mathsf{rg}(r, C)$ iff $\mathsf{rg}\ r^{\mathcal{I}} \subseteq C^{\mathcal{I}}$

## 3 Exercises

In these exercises use the notation and definitions above in your answers.

### 3.1 Exercise

Let $\Gamma$ be the RDF graph below.

1. Create an interpretation $\mathcal{I}_1$ such that $\mathcal{I}_1 \models \Gamma$.
2. Create an interpretation $\mathcal{I}_2$ such that $\mathcal{I}_2 \not\models \Gamma$.
3. Create an interpretation $\mathcal{I}_3$ such that $\mathcal{I}_3 \models \Gamma$ and $|\Delta^{\mathcal{I}_3}| = 1$, i.e., the domain of the interpretation contains only one element.

```
1  @prefix : <http://www.example.org#> .
2  @prefix rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#> .
3  :Tweety rdf:type   :Bird .
4  :Nixon  rdf:type   :Republican .
5  :Nixon  rdf:type   :Quacker .
6  :Nixon  :listensTo :Tweety .
7  :Tweety :likes     :Tux .
```

### 3.2 Exercise

Let $\Gamma$ be the RDFS graph listed below.

1. Create an interpretation $\mathcal{I}_1$ such that $\mathcal{I}_1 \models \Gamma$.
2. Create an interpretation $\mathcal{I}_2$ such that $\mathcal{I}_2 \not\models \Gamma$.

```
1  @prefix rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#> .
2  @prefix rdfs: <http://www.w3.org/2000/01/rdf-schema#> .
3  @prefix owl: <http://www.w3.org/2002/07/owl#> .
4  @prefix : <http://example.org#> .
5  :Person    a                  rdfs:Class .
6  :Man       a                  rdfs:Class ;
7             rdfs:subClassOf    :Person .
8  :Parent    a                  rdfs:Class ;
9             rdfs:subClassOf    :Person .
10  :Father    a                  rdfs:Class ;
11             rdfs:subClassOf    :Parent ;
12             rdfs:subClassOf    :Man .
13  :Child     a                  rdfs:Class ;
14             rdfs:subClassOf    :Person .
15  :hasParent a                  rdf:Property ;
16             rdfs:domain        :Person ;
17             rdfs:range         :Parent .
18  :hasFather a                  rdf:Property ;
19             rdfs:subPropertyOf :hasParent ;
20             rdfs:range         :Father .
21  :isChildOf a                  rdf:Property ;
22             rdfs:domain        :Child ;
23             rdfs:range         :Parent .
24  :Ann       a                  :Person ;
25             :hasFather         :Carl .
26  :Carl      a                  :Man .
```

### 3.3 Exercise

Let $\Gamma$ be the RDFS graph `entailments.n3`. Show by way of model semantics the following claims:

1. $\Gamma \models$ `:Father rdfs:subClassOf :Person .`
2. $\Gamma \not\models$ `:Ann a :Child .`
3. $\Gamma \models$ `:Ann :hasParent :Carl .`
4. $\Gamma \models$ `:Carl a :Person .`
5. $\Gamma \not\models$ `:Carl :hasChild :Ann .`

### 3.4 Exercise

Let $\Gamma$ be the RDFS graph `entailments.n3`. As we have seen in a previous week's exercises, using the standardised RDFS semantics the entailment $\Gamma \models$ `:hasFather rdfs:domain :Person.` does not hold. Does it hold in our simplified semantics?