Model Semantics

Read

- Foundations of Semantic Web Technologies: chapter 3, 2.

1 From the lecture

a) What does RDFS add to RDF?

b) What is formal semantics?

c) Why do we need a model semantics for RDF/RDFS?

d) What does a DL-interpretation consist of?

Solution

a) RDFS adds the concept of classes and a predefined vocabulary to add statements about classes

b) The study of how to model the meaning of a calculus

c) Can not afford ambiguity in interpreting RDF, it would be application dependent.

d) See Section 2.2.

2 Definitions

First, some notation and definitions collected from the lecture slides.

2.1 Syntax: Triple abbreviations

Triple pattern	Triple instance	Abbreviation
`indi prop indi .`	$i_1\ r\ i_2$	$r(i_1, i_2)$
`indi rdf:type class .`	i_1 `rdf:type` C	$C(i_1)$
`class rdfs:subClassOf class .`	C `rdfs:subClassOf` D	$C \sqsubseteq D$
`prop rdfs:subPropertyOf prop .`	r `rdfs:subPropertyOf` s	$r \sqsubseteq s$
`prop rdfs:domain class .`	r `rdfs:domain` C	$\mathsf{dom}(r, C)$
`prop rdfs:range class .`	r `rdfs:range` C	$\mathsf{rg}(r, C)$

2.2 Interpretation

An *interpretation* $\mathcal{I}$ consists of:

- A set $\Delta^{\mathcal{I}}$, called the *domain* $\mathcal{I}$
- For each individual URI i, an element $i^{\mathcal{I}} \in \Delta^{\mathcal{I}}$
- For each class URI C, a subset $C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
- For each property URI r, a relation $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$

2.3 Validity in Interpretations (RDF)

Given an interpretation $\mathcal{I}$, define $\models$ as follows:

- $\mathcal{I} \models r(i_1, i_2)$ iff $\langle i_1^{\mathcal{I}}, i_2^{\mathcal{I}} \rangle \in r^{\mathcal{I}}$
- $\mathcal{I} \models C(i)$ iff $i^{\mathcal{I}} \in C^{\mathcal{I}}$

2.4 Validity in Interpretations, cont. (RDFS)

Given an interpretation $\mathcal{I}$, define $\models$ as follows:

- $\mathcal{I} \models C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
- $\mathcal{I} \models r \sqsubseteq s$ iff $r^{\mathcal{I}} \subseteq s^{\mathcal{I}}$
- $\mathcal{I} \models \mathsf{dom}(r, C)$ iff $\mathsf{dom}\ r^{\mathcal{I}} \subseteq C^{\mathcal{I}}$
- $\mathcal{I} \models \mathsf{rg}(r, C)$ iff $\mathsf{rg}\ r^{\mathcal{I}} \subseteq C^{\mathcal{I}}$

3 Exercises

In these exercises use the notation and definitions above in your answers.

3.1 Exercise

Let Γ be the RDF graph below.

1. Create an interpretation $\mathcal{I}_1$ such that $\mathcal{I}_1 \models \Gamma$.
2. Create an interpretation $\mathcal{I}_2$ such that $\mathcal{I}_2 \not\models \Gamma$.
3. Create an interpretation $\mathcal{I}_3$ such that $\mathcal{I}_3 \models \Gamma$ and $|\Delta^{\mathcal{I}_3}| = 1$, i.e., the domain of the interpretation contains only one element.

```
@prefix : <http://www.example.org#> .
@prefix rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#> .
:Tweety rdf:type   :Bird .
:Nixon  rdf:type   :Republican .
:Nixon  rdf:type   :Quacker .
:Nixon  :listensTo :Tweety .
:Tweety :likes     :Tux .
```

Solution

1. There are many interpretations that satisfy the statements in the RDF graph. This is one:

   - $\Delta^{\mathcal{I}} = \{1, 2, 3, 4, 5\}$
   - `:Tweety` $^{\mathcal{I}} = 1$, `:Nixon` $^{\mathcal{I}} = 3$ `:Tux` $^{I} = 4$
   - `:Bird` $^{\mathcal{I}} = \{1, 2, 4, 5\}$, `:Republican` $^{\mathcal{I}} = \{3, 5\}$, `:Quacker` $^{\mathcal{I}} = \{3, 5, 4\}$
   - `:listensTo` $^{\mathcal{I}} = \{\langle 3, 1\rangle\}$, `:likes` $^{\mathcal{I}} = \{\langle 1, 4\rangle, \langle 3, 4\rangle\}$

2. There are also many ways to construct an interpretation that does not satisfy the RDF graph. Here are some examples of how:

   - `:Tweety` $^{\mathcal{I}} \notin$ `:Bird` $^{\mathcal{I}}$
   - `:Nixon` $^{\mathcal{I}} \notin$ `:Republican` $^{\mathcal{I}}$
   - `:Nixon` $^{\mathcal{I}} \notin$ `:Quacker` $^{\mathcal{I}}$
   - $\langle$ `:Nixon` $^{\mathcal{I}}$, `:Tweety` $^{\mathcal{I}}\rangle \notin$ `:listensTo` $^{\mathcal{I}}$
   - $\langle$ `:Tweety` $^{\mathcal{I}}$, `:Tux` $^{\mathcal{I}}\rangle \notin$ `:likes` $^{\mathcal{I}}$

3. Let

   - $\Delta^{\mathcal{I}} = \{b\}$ (some set with one element.)
   - `:Tweety` $^{\mathcal{I}} =$ `:Nixon` $^{\mathcal{I}} =$ `:Tux` $^{\mathcal{I}} = b$
   - `:Bird` $^{\mathcal{I}} =$ `:Republican` $^{\mathcal{I}} =$ `:Quacker` $^{\mathcal{I}} = \Delta^{\mathcal{I}}$
   - `:listensTo` $^{\mathcal{I}} =$ `:likes` $^{\mathcal{I}} = \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$

3.2 Exercise

Let Γ be the RDFS graph listed below.

1. Create an interpretation $\mathcal{I}_1$ such that $\mathcal{I}_1 \models \Gamma$.

2. Create an interpretation $\mathcal{I}_2$ such that $\mathcal{I}_2 \not\models \Gamma$.

```
@prefix rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#> .
@prefix rdfs: <http://www.w3.org/2000/01/rdf-schema#> .
@prefix owl: <http://www.w3.org/2002/07/owl#> .
@prefix : <http://example.org#> .
:Person    a                   rdfs:Class .
:Man       a                   rdfs:Class ;
           rdfs:subClassOf     :Person .
:Parent    a                   rdfs:Class ;
           rdfs:subClassOf     :Person .
:Father    a                   rdfs:Class ;
           rdfs:subClassOf     :Parent ;
           rdfs:subClassOf     :Man .
:Child     a                   rdfs:Class ;
           rdfs:subClassOf     :Person .
:hasParent a                   rdf:Property ;
           rdfs:domain         :Person ;
           rdfs:range          :Parent .
:hasFather a                   rdf:Property ;
           rdfs:subPropertyOf  :hasParent ;
           rdfs:range          :Father .
:isChildOf a                   rdf:Property ;
           rdfs:domain         :Child ;
           rdfs:range          :Parent .
:Ann       a                   :Person ;
           :hasFather          :Carl .
:Carl      a                   :Man .
```

Solution

1. *One* possible solution (i.e., there are many):

   - $\Delta^{\mathcal{I}} = \{A, B, C\}$
   - $\texttt{:Ann}^{\mathcal{I}} = A$, $\texttt{:Carl}^{\mathcal{I}} = C$
   - $\texttt{:Father}^{\mathcal{I}} = \texttt{:Man}^{\mathcal{I}} = \texttt{:Parent}^{\mathcal{I}} = \{C\}$, $\texttt{:Person}^{\mathcal{I}} = \{A, B, C\}$, $\texttt{:Child}^{\mathcal{I}} = \{B\}$,
   - $\texttt{:hasParent}^{\mathcal{I}} = \texttt{:hasFather}^{\mathcal{I}} = \{\langle A, C\rangle\}$ $\texttt{:isChildOf}^{\mathcal{I}} = \emptyset$

2. Let, e.g.,:

   - $\texttt{:Person}^{\mathcal{I}} \subset \texttt{:Child}^{\mathcal{I}}$
   - $\texttt{:Ann}^{\mathcal{I}} \not\in \texttt{:Person}^{\mathcal{I}}$
   - $\texttt{:hasFather}^{\mathcal{I}} \not\subseteq \texttt{:hasParent}^{\mathcal{I}}$

3.3 Exercise

Let Γ be the RDFS graph `entailments.n3`. Show by way of model semantics the following claims:

1. $\Gamma \models$ `:Father rdfs:subClassOf :Person .`
2. $\Gamma \not\models$ `:Ann a :Child .`
3. $\Gamma \models$ `:Ann :hasParent :Carl .`
4. $\Gamma \models$ `:Carl a :Person .`
5. $\Gamma \not\models$ `:Carl :hasChild :Ann .`

Solution

1. $\Gamma \models$ `:Father rdfs:subClassOf :Person .`
   - $\Gamma \vDash$:Father $\sqsubseteq$:Man
   - $\Gamma \vDash$:Man $\sqsubseteq$:Person
   - By definition: $\mathcal{I} \vDash C \sqsubseteq D$ iff $C^{\mathcal{I}} \subseteq D^{\mathcal{I}}$
   - Thus, $\mathcal{I} \vDash$:Father $\sqsubseteq$:Man $\sqsubseteq$:Person iff $\text{Father}^{\mathcal{I}} \subseteq \text{Man}^{\mathcal{I}} \subseteq \text{Person}^{\mathcal{I}}$
   - By set theory, we know that for any interpretation $\mathcal{I} \vDash \Gamma$, any $x \in \text{Father}^{\mathcal{I}}$ must also be in $\text{Person}^{\mathcal{I}}$, thus Father $\sqsubseteq$ Person

2. $\Gamma \not\models$ `:Ann a :Child .`
   - $\Gamma \vDash$:Ann a :Person
   - $\Gamma \vDash$:Ann :hasFather :Carl
   - $\Gamma \vDash$:hasParent rdfs:domain :Person
   - $\mathcal{I} \vDash Person(Ann)$: iff $\text{Ann}^{\mathcal{I}} \in \text{Person}^{\mathcal{I}}$
   - $\mathcal{I} \vDash hasFather(Ann, Carl)$ iff $\langle \text{Ann}^{\mathcal{I}}, \text{Carl}^{\mathcal{I}} \rangle \in \text{hasFather}^{\mathcal{I}}$
   - $\mathcal{I} \vDash dom(hasFather, Person)$ iff for all $\langle x, y \rangle \in hasFather^{\mathcal{I}}$, we have $x \in \text{Person}^{\mathcal{I}}$
   - By set theory there is no way to find that if x is in $\text{Ann}^{\mathcal{I}}$ x must also be in $\text{Child}^{\mathcal{I}}$, only that it is in $\text{Person}^{\mathcal{I}}$.

3. $\Gamma \models$ `:Ann :hasParent :Carl .`
   - $\Gamma \vDash hasFather(Ann, Carl)$
   - $\Gamma \vDash hasFather \sqsubseteq hasParent$
   - $\mathcal{I} \vDash$ hasFather $\sqsubseteq$ hasParent iff $\text{hasFather}^{\mathcal{I}} \subseteq \text{hasParent}^{\mathcal{I}}$

- We know that for any interpreation $\mathcal{I} \vDash \Gamma$, by using set theory, if $\langle x, y\rangle \in \text{hasFather}^{\mathcal{I}}$, $\langle x, y\rangle$ must also be in $\text{hasParent}^{\mathcal{I}}$. Thus $\Gamma \vDash hasParent(Ann, Carl)$

4. $\Gamma \models$ `:Carl a :Person .`

   - $\Gamma \vDash Man(Carl)$
   - $\Gamma \vDash \text{Man} \sqsubseteq \text{Person}$
   - $\mathcal{I} \vDash \text{Man} \sqsubseteq \text{Person}$ iff $\text{Man}^{\mathcal{I}} \subseteq \text{Person}^{\mathcal{I}}$
   - For any interpreation $\mathcal{I} \vDash \Gamma$ if $x \in \text{Man}^{\mathcal{I}}$ x must also be in $\text{Person}^{\mathcal{I}}$ because it is a subset. Thus, By set theory we find that $\Gamma \vDash Person(Carl)$.

5. $\Gamma \not\models$ `:Carl :hasChild :Ann .`

   - There is nothing in Γ that indicates that `:Carl :hasChild :Ann .`

3.4 Exercise

Let Γ be the RDFS graph `entailments.n3`. As we have seen in a previous week's exercises, using the standardised RDFS semantics the entailment $\Gamma \models$ `:hasFather rdfs:domain :Person.` does not hold. Does it hold in our simplified semantics?

Solution

- $\Gamma \vDash \text{hasFather} \sqsubseteq \text{hasParent}$
- $\Gamma \vDash dom(hasParent, Person)$
- $\mathcal{I} \vDash \text{hasFather} \sqsubseteq \text{hasParent}$ iff $\text{hasFather}^{\mathcal{I}} \subseteq \text{hasParent}^{\mathcal{I}}$
- $\mathcal{I} \vDash dom(hasParent, Person)$ iff for all $\langle x, y\rangle \in \text{hasParent}^{\mathcal{I}}$ we have that $x \in \text{Person}^{\mathcal{I}}$
- We have, by set theory that, for any interpreation $\mathcal{I} \vDash \Gamma$, any $\langle x, y\rangle \in \text{hasFather}^{\mathcal{I}}$ must also be in $\text{hasParent}^{\mathcal{I}}$
- And for any interpration, $\mathcal{I} \vDash \Gamma$, if $\langle x, y\rangle \in \text{hasParent}^{\mathcal{I}}$ $x \in Person^{\mathcal{I}}$, thus $\Gamma \vDash dom(hasFather, Person)$.