IN3060/4060 – Semantic Technologies – Spring 2021

Lecture 8: Model Semantics & Reasoning

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Oblig 5

- Published today
- First delivery due 19th of March
- Final delivery 2 weeks after feedback
- Extra question for IN4060 students
- “Real” semantics of RDF and RDFS
- Foundations book: Section 3.2
- Still OK to ignore some complications, see oblig text
- We provide an excerpt of Sect. 3.2 with unimportant parts removed.
- Go to group sessions!

Today's Plan

1. Repetition: RDF semantics
2. Literal Semantics
3. Blank Node Semantics
4. Properties of Entailment by Model Semantics
5. Entailment and Derivability

Outline

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- Forget blank nodes and literals for a while!

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- Used much in particular for OWL

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- DL syntax, without namespaces:

$$\mathit{loves}(\mathit{romeo}, \mathit{juliet})$$
$$\mathit{Lady}(\mathit{juliet})$$

$$\mathit{Lady} \sqsubseteq \mathit{Person}$$
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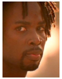
- Given these, it will be possible to say whether a triple holds or not.

An example “intended” interpretation

- $\Delta^{\mathcal{I}_1} = \Big\{$ , ,  $\Big\}$

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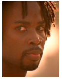
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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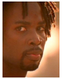
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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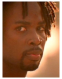
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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- In fact, there is *no way* of ensuring they denote only what we think!

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- $\mathcal{A}$ is valid in $\mathcal{I}$, written

$$\mathcal{I} \models \mathcal{A}$$

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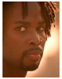
  $\textit{Lover}^{\mathcal{I}_1} = \Big\{$ ,  $\Big\} \subseteq \textit{Person}^{\mathcal{I}_1} = \Big\{$ , ,  $\Big\}$

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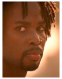
  $\mathit{Lover}^{\mathcal{I}_1} = \Big\{$ ,  $\Big\} \subseteq \mathit{Person}^{\mathcal{I}_1} = \Big\{$ , ,  $\Big\}$

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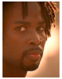
  $Lover^{\mathcal{I}_1} = \{$ ,  $\} \subseteq Person^{\mathcal{I}_1} = \{$ , ,  $\}$
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Finding out stuff about Romeo and Juliet

Statements

loves(*romeo*, *juliet*)
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Lady ⊑ *Person*
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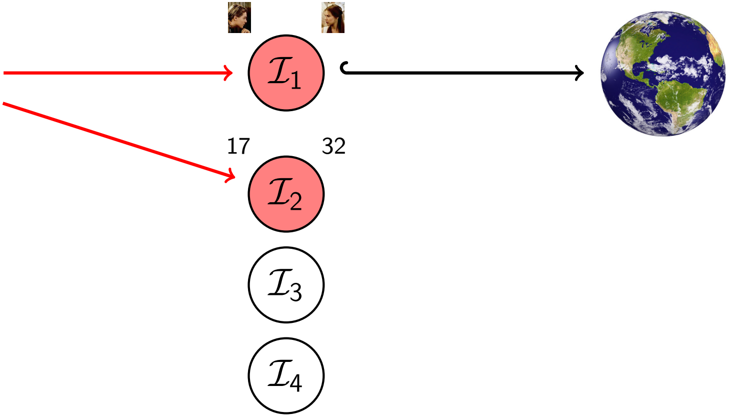

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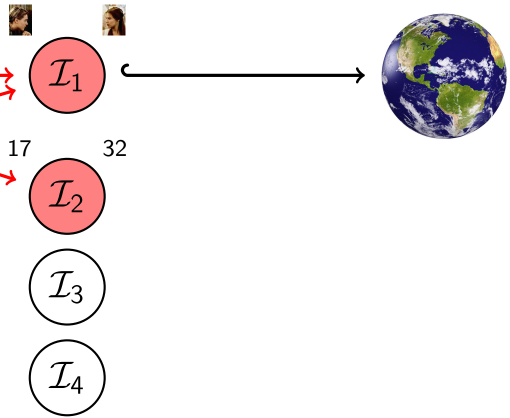

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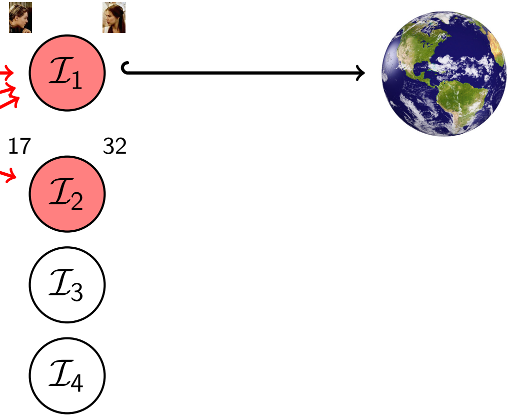

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  - *Built-ins*, a fixed set including `rdf:type`, `rdfs:domain`, etc.
  - *Individuals* (all the rest, "usual" resources)

Simplifying Literals

- Literals can only occur as *objects* of triples
- Have datatype, can be with or without language tag
- The same predicate can be used with literals and resources:

```
ex:me ex:likes dbpedia:Berlin .
ex:me ex:likes "some string" .
```

- We simplify things by:
  - considering only string literals without language tag, and
  - allowing either resource objects *or* literal objects for any predicate
- Five types of resources:
  - *Object Properties* like `foaf:knows`
  - *Datatype Properties* like `dc:title`, `foaf:name`
  - *Classes* like `foaf:Person`
  - *Built-ins*, a fixed set including `rdf:type`, `rdfs:domain`, etc.
  - *Individuals* (all the rest, “usual” resources)
- Why? – simpler, object/datatype split is in OWL

Allowed triples

Allow only triples using object properties and datatype properties as intended

Triples	Abbreviation
`indi o-prop indi .`	$r(i_1, i_2)$
`indi d-prop "lit" .`	$a(i, l)$
`indi rdf:type class .`	$C(i_1)$
`class rdfs:subClassOf class .`	$C \sqsubseteq D$
`o-prop rdfs:subPropertyOf o-prop .`	$r \sqsubseteq s$
`d-prop rdfs:subPropertyOf d-prop .`	$a \sqsubseteq b$
`o-prop rdfs:domain class .`	$\mathrm{dom}(r, C)$
`o-prop rdfs:range class .`	$\mathrm{rg}(r, C)$

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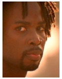
- Note: Literals l are in Λ, don't need to be interpreted.

Example: Interpretation with a Datatype Property

- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$

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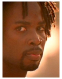
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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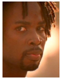
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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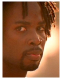
Outline

Motivating example

- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$

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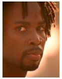
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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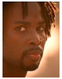
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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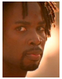
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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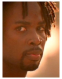
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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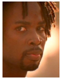
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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- Is $\mathcal{A}$ valid in $\mathcal{I}_1$? why?

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- Solution: pass in blank node interpretation, deal with sets later!

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  if there is a β such that $\mathcal{I}, \beta \models \mathcal{A}$

Sets of Triples with Blank Nodes

- Given a set $\mathcal{A}$ of triples with blank nodes...
- $\mathcal{I}, \beta \models \mathcal{A}$ iff $\mathcal{I}, \beta \models A$ for all $A \in \mathcal{A}$
- $\mathcal{A}$ is valid in $\mathcal{I}$
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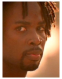
if there is a β such that $\mathcal{I}, \beta \models \mathcal{A}$
- I.e. if there exists some valuation for the blank nodes that makes all triples true.

Example: Blank Node Semantics

- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$

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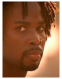
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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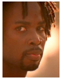
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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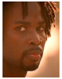
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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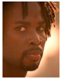
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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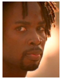
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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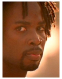
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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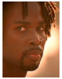
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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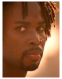
- $\Delta^{\mathcal{I}_1} = \{$ , ,  $\}$
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- So, yes, $\mathcal{I}_1 \models \mathcal{A}$.

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- Simple entailment: entailment with blank nodes, but no RDFS semantics

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Therefore,

- RDFS supports no reasoning services that require consistency-checking.
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Expressive limitations of RDFS

Note that,

- RDFS cannot express inconsistencies, so any RDFS graph is consistent.
- RDFS has no notion of negation at all
  - For instance, the two triples

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- More about that next week.

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- Answers remain valid with new information!

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  - $KB \cup \{isFatherOf(olav, harald)\} \models$ *harald does have a father*
  - In some models of KB, harald has a father, in others not.

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Outline

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  - All given SE/RDF/RDFS rules are sound w.r.t. the model semantics!

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- Won't bother to do that now. Will get completeness for OWL.

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