Exam
IN3070/IN4070/INF4171
Autumn 2019

November 24, 2020

The maximum number of marks for the whole exam was 100. The minimum number of marks required for each grade was as follows: For E, min. 40. For D, min. 50. For C, min. 57. For B, min. 70. For C, min. 85.

These boundaries are based on an alignment of the delivered work with the grade definitions given here:

`https://www.uio.no/studier/eksamen/karakterer/`

The final grade for each candidate is based on an evaluation of the delivered work as a whole, and may therefore in some cases deviate from the bounraries given above.

Question 1 – Notions: Soundness and Completeness

A) Briefly explain what it means that a calculus is **sound** for a given logic. [2 marks]

B) Briefly explain what it means that a calculus is complete for a given logic [2 marks]

C) It is very easy to give a calculus (e.g. for first order logic) that is sound, but not complete. It is also very easy to give a calculus that is complete but not sound. Explain! [4 marks]

D) When we add a rule to sound and complete calculus (e.g. for efficiency), do we have to reconsider soundness, or completeness, or both? [2 marks]

E) When we remove a rule from a sound and complete calculus, or restrict its application in some way (e.g. regularity), do we have to reconsider soundness, or completeness, or both?

Question 2 – Sequent Calculus

Prove the validity of the following formulae using the given calculus. Use the hand drawing sheets

A) $(p \to (q \to r)) \to (q \to (p \to r))$ using **propositional LK** [6 marks]

B) $(p(a) \wedge \forall x\, (p(x) \to p(f(x)))) \to p(f(f(a)))$ using **ground** first-order LK [7 marks]

C) $(\forall x\, p(x)) \to \forall y\, (p(y) \vee q(y))$ using **free variable** first-order LK [6 marks]

D) $\exists x\, (p(x) \to \forall y\, p(y))$ using ground first-order LK [7 marks]

E) $\exists x\, (p(x) \to \forall y\, p(y))$ using **free variable** first-order LK [6 marks]

Question 3 – Resolution

We want to formalise arithmetic of natural numbers in first order logic.

We use a constant symbol a to denote zero, and a unary function symbol f to denote the "successor" of a number, i.e.the number that is larger by one. For instance $f(a)$ denotes 1, $f(f(a))$ denotes 2, etc.

We also use a predicate sum so that $sum(x, y, z)$ denotes that $x + y = z$. For instance $sum(f(f(a), f(a), f(f(f(a))))$ means that $2 + 1 = 3$.

We now formalise the properties of addition using the following two formulae:

$$A = \forall x\, sum(a, x, x)$$
$$B = \forall x \forall y \forall z\, (sum(x, y, z) \to sum(f(x), y, f(z))$$

A) Write these two formulae as Prolog clauses. [4 marks]

B) Write the formulae A and B in clause form, i.e. as a set of sets of literals. You can use the minus sign for negation. [4 marks]

C) Use the resolution calculus to prove that

$$C = \; sum(f(f(a), f(a), f(f(f(a))))$$

is a logical consequence of the formulae A and B. In other words, show that $(A \wedge B) \to C$ is valid. Please number the clauses you derive and make it clear in each step which clauses you resolve and with which substitutions. Remember that you might have to rename variables in clauses before resolving. [8 marks]

Question 4 – Classical vs. Intuitionistic Logic

In the following, there is a number of statements, that may be true in some logics, but not in others.

Reminder: validity in intuitionistic logic means that a formula is true in *every* world of *every* intuitinistic Kripke structure.

A) If A is a valid formula and B is a valid formula, then $A \wedge B$ is a valid formula

B) If $A \wedge B$ is a valid formula, then A is a valid formula and B is a valid formula.

B) Using the model semantics, show that it is valid in modal logic **T**, i.e. A holds in every world of any Kripke structure with a reflexive frame, i.e. $(x, x) \in R$ for all $x \in W$. [5 marks]

Question 6 – An Inductive Proof

Let A and B be two logically equivalent propositional formulae.

Let C be a propositional formula, and C' the result of replacing all occurrences of A in C by B.

E.g. $A = p \to q$ and $B = \neg q \to \neg p$ are logically equivalent.
If $C = (p \to q) \vee (q \to r)$ then $C' = (\neg q \to \neg p) \vee (q \to r)$.

Prove by structural induction on C that C and C' are logically equivalent.

You may use a hand drawing sheet, or the text field below. You can use ASCII renditions like `/\`, `/\`, `->`, `-`. Most of the cases are very similar, you do not need to spell them all out.

Question 7 – Multiple Choice Mix

A) Which of these is a correct formulation of the substitution lemma for for terms

[2 marks]

B) Are these terms unifiable? $f(g(x), x)$ and $f(y, h(y))$

C) The first order formula $\forall x\,(p(x) \wedge \neg p(x))$ is satisfiable, because it is true in an interpretation $\mathcal{I} = (\emptyset, \iota)$ with an empty domain.

E) If the clauses in a set S of propositional clauses contains one atomic formula only positively (e.g., only but not), then... [2 marks]