IN3070/4070 – Logic – Autumn 2020

Lecture 1: Introduction

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Today's Plan

- What is Logic?
- Logic in Computer Science
- Three Ingredients
- Applications
- Course Information

Outline

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- Logic in Computer Science
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- Abstraction!

Computation with Knowledge

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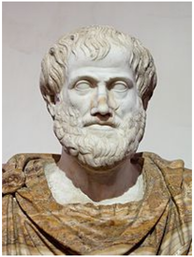

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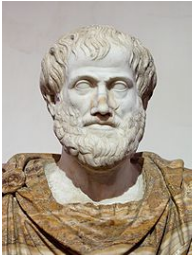

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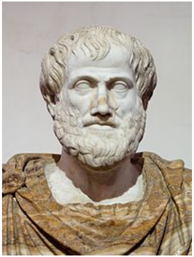

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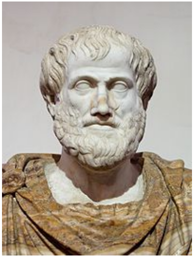

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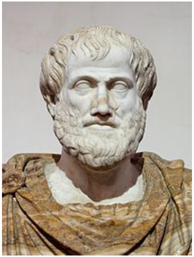

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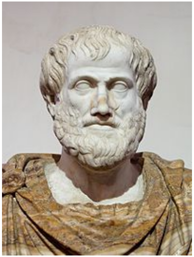

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- Also an abstraction!

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Sure, cool, but why bother?

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  - Numerical Models (Newtonian mechanics, Quantum mechanics)

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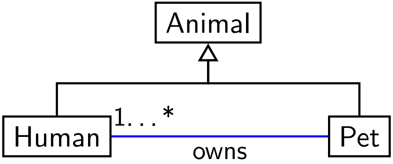
What is a Good Modelling Language

For a model that can be used by a computer…

- there has to be a 'language'
  - language says what is a model and what not
  - programs (and humans…) need to know what to expect
  - often defined by some grammar
  - UML, ER, ORM, OWL, SQL, Java, etc. are all languages
- the meaning of models should be very clear
  - otherwise, different implementations do different things
  - sometimes, 100s of pages of technical text (e.g. JLS)
  - sometimes meaning given by mathematical definitions

Models and Statements

Observation: much of the content of many models can be given as statements:



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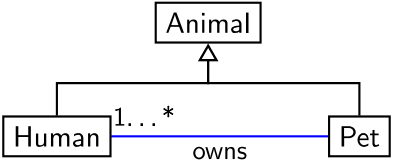

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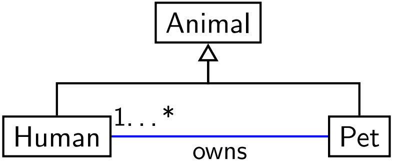

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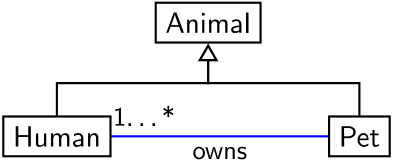

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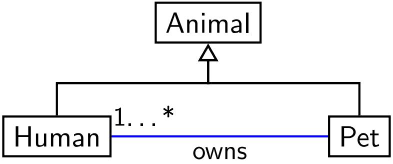

- Every Human is an Animal.
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- Every Pet is owned by at least one Human.
- Everybody owning a Pet is a Human (?)

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- $\forall x, y. ((Pet(x) \land owns(y, x)) \rightarrow Human(y))$

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- $\forall x, y.((Pet(x) \wedge owns(y, x)) \rightarrow Human(y))$

Logics are a very expressive and precise family of modelling languages

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Three Central Ingredients

- Syntax (i.e. the language)
- Semantics (i.e. the meaning)
- Calculus (i.e. method, algorithm, usually rules)

Syntax

Most logics have some kind of *formulas*.
The syntax says which strings of characters are formulas.

Syntax of Propositional Formulae

Propositional formulas are defined inductively as follows

- Every lower case letter (p, q, r,...) is a formula
- If A and B are formulae, then $\neg A$, $(A \wedge B)$, $(A \vee B)$ and $(A \rightarrow B)$ are formulae.

Inductively defined (remember IN1150): only what can be constructed using these rules is a formula.

$$p, \quad (p \wedge \neg p), \quad (p \rightarrow q) \vee (q \rightarrow p), \ldots$$

But not: $((p, \quad \neg \rightarrow q, \ldots$

Model Semantics

We usually define some kind of *interpretation* or *model* or *structure*...
Always the same idea:

- We don't know what we talk about (p, q, r, x, y, z)
- We don't know what is true (p or $\neg q$?)
- So we use a mathematical object that tells us what they mean and what is true or not

Interpretation

An interpretation is a function $\mathcal{I} : \mathit{Letters} \to \{T, F\}$ that assigns one of the truth values T or F to every lower case letter

Model Semantics (cont.)

Truth Value

The truth value of formulas $v_{\mathcal{I}}(A)$ is defined inductively by

- $v_{\mathcal{I}}(A) = \mathcal{I}(A)$ for letters A
- $v_{\mathcal{I}}(\neg A) = T$ if $v_{\mathcal{I}}(A) = F$ and
  $v_{\mathcal{I}}(\neg A) = F$ if $v_{\mathcal{I}}(A) = T$
- $v_{\mathcal{I}}(A \wedge B) = T$ if $v_{\mathcal{I}}(A) = T$ and $v_{\mathcal{I}}(B) = T$
  $v_{\mathcal{I}}(A \wedge B) = F$ otherwise
- ...

Entailment

Formula A *entails* formula B ($A \models B$) if for every $\mathcal{I}$ with $v_{\mathcal{I}}(A) = T$ it also holds that $v_{\mathcal{I}}(B) = T$.

Model Semantics: Take Aways

Model Semantics defines the meaning of logical formulas...

- i.e. truth/falsity in some interpretation/model/...
- relations between formulas like entailment, equivalence...

...by *mathematical definitions*.

- We assume that maths, set theory, etc. "work"
- We assume that people can read formulas, understand words like "and" or "not" or "otherwise," look up truth values in tables, etc.
- The definitions can often not be implemented directly
  - E.g. loop over infinitely many interpretations in 1st order logic

Calculi

- A calculus works on formulas, i.e. *syntax*
- Usually by *inference rules* saying how to *derive* new formulas

$$\frac{A \to B \qquad A}{B}$$

- Always with some machinery that says how to use the rules
- Can be used to check entailment etc. between formulas
- Can be implemented on a computer

Natural Deduction for Propositional Logic

- rules for $\land$ (conjunction)

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- rules for $\wedge$ (conjunction)

$$A \qquad B$$

Natural Deduction for Propositional Logic

- rules for ∧ (conjunction)

$$\frac{A \qquad B}{A \land B} \land\text{-I}$$

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- rules for ∧ (conjunction)

$$\frac{A \qquad B}{A \wedge B}\ \wedge\text{-I} \qquad\qquad A \wedge B$$

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- rules for ∧ (conjunction)

$$\frac{A \qquad B}{A \wedge B}\,\wedge\text{-I} \qquad\qquad \frac{A \wedge B}{A}\,\wedge\text{-E}$$

Natural Deduction for Propositional Logic

- rules for $\wedge$ (conjunction)

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Natural Deduction for Propositional Logic

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Calculi: Take Aways

Calculi allow to determine

- *semantic properties* like equivalence, satisfiability etc.
- by *syntactic means*

. . . i.e. in ways that can be implemented

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Learn techniques to handle languages and their semantics

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- Foundations are not (usually) our problem :-)

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SAT

- SAT-solving: given a propositional formula A, is there an interpretation $\mathcal{I}$ such that $v_{\mathcal{I}}(A) = T$?

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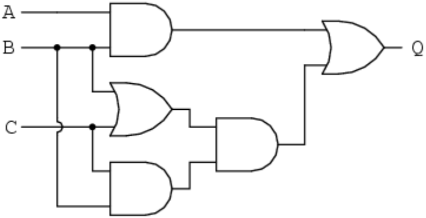
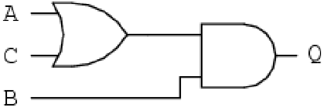
- See here for a talk about applications `http://www.carstensinz.de/talks/RISC-2005.pdf`

SAT applications: circuit verification

Are these two circuits the same?





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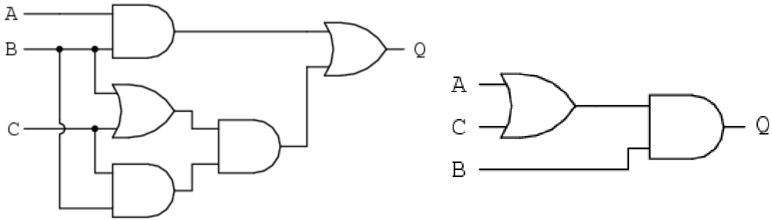

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Logically equivalent?

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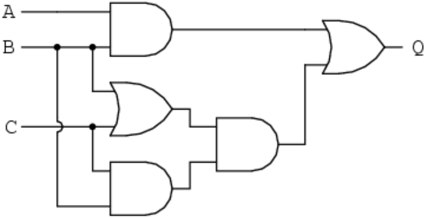

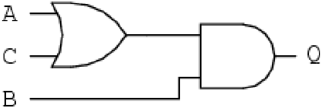

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Today, theorem provers are routinely used to check Boolean circuits

SAT applications: program verification

- A 32 bit int can be encoded as 32 boolean variables
- If SAT can handle 1M boolean variables, it can handle thousands of 32 bit words.
- Properties of programs (without loops) can be handled by SAT solvers
- Experimental, but works in many cases

SAT applications: puzzle solving

5	3			7				
6			1	9	5			
	9	8					6	
8				6				3
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SAT: problems that require finding one of a large, fixed number of combinations, but checking is easy

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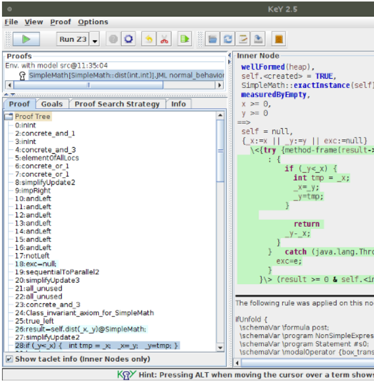
- First-order theorem provers have been used to prove difficult open problems in (unintuitive parts of) mathematics
- In combination with other techniques, first-order logic can used to reason about programs

The KeY tool

- `https://www.key-project.org/`
- Verify behaviour of Java programs
- Based on 1st-order logic
- Extended with program operators
  - $\langle Prog \rangle\ p$
    Prog terminates and p holds afterwards
- Based on a Sequent Calclus



The TimSort bug

Proving that Android's, Java's and Python's sorting algorithm is broken (and showing how to fix it)

February 24, 2015 Envisage Written by Stijn de Gouw. $s

Tim Peters developed the Timsort hybrid sorting algorithm in 2002. It is a clever combination of ideas from merge sort and insertion sort, and designed to perform well on real world data. TimSort was first developed for Python, but later ported to Java (where it appears as java.util.Collections.sort and java.util.Arrays.sort) by Joshua Bloch (the designer of Java Collections who also pointed out that most binary search algorithms were broken). TimSort is today used as the default sorting algorithm for Android SDK, Sun's JDK and OpenJDK. Given the popularity of these platforms this means that the number of computers, cloud services and mobile phones that use TimSort for sorting is well into the billions.

http://www.envisage-project.eu/proving-android-java-and-python-sorting-algorithm-is-broken-and-how-to-fix-it/

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- Knowledge logics, probabilistic logics, alternating logics, belief logics, ...

Outline

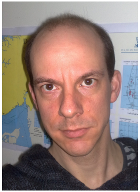
- What is Logic?
- Logic in Computer Science
- Three Ingredients
- Applications
- Course Information

When, Where, and Who

When and Where

- Lectures
  - Tuesdays 10:15–12:00 in OJD 2458, Postscript and/or
  - Thursdays 10:15–12:00 in KN Store Aud
  - When fewer students, move to OJD 3438, Caml
- Homepage: `https://www.uio.no/studier/emner/matnat/ifi/IN3070/index-eng.html`

Lecturer



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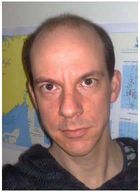

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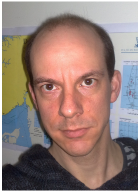

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Exercises

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- Practical exercises every week,
- Thursdays 12:15–14:00 in OJD 3468 Fortress, from **27 August** and
- Tuesdays 10:15–12:00 in OJD 2458, Postscript, from 1 Sept.
- Exercises available on website well in advance. Come prepared!
- In general: part repetition of lectures, part exercises

Teacher

Ida Sandberg Motzfeldt
(`idasmot@ifi.uio.no`)

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Mandatory Assignments

Assignments

- Two mandatory assignments (obliger)
- Will be in October/November
- Corrected by teacher.
- Pass/Fail
- Must have passed all assignments in order to attend exam
- For IN4070 (MSc version): one extra question in each oblig

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- May organise a little prover competition at the end of the semester.

Prover Hacking – Competition?

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- Doesn't need to be very powerful to get 'pass.'
- But hacking is fun, right?
- May organise a little prover competition at the end of the semester.
- Strictly for fun, no influence on grade.

Padlet

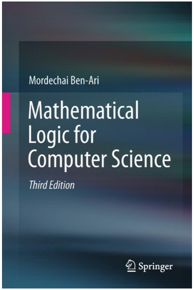
https://uio.padlet.org/martingi/8swc2uezt4sy2nsk

Exam

- Four hours written home exam
  - In case of few students, might be oral exam instead
- Same exam for IN3070 and IN4070
- Grades A–F
- Probably 4 December – Check semester page!
- Unsure, due to Corona

Textbook

- Mordechai Ben-Ari
  Mathematical Logic for Computer Science
  3rd edition, Springer, 2012.



- only chapters 1–4 and 6–12;
  not part of the curriculum:
  chapter 5 (binary decision diagrams) and
  chapters 13–16 (temporal logic, verification of programs)
- download for free (within the UiO network) from Springer's website at
  `http://www.springer.com/gp/book/9781447141280`

Next weeks. . .

- Propositional Logic
- Tableaux/Sequent calculi for propositional Logic
- Soundness and Completeness
- Resolution calculus for propositional Logic
- Soundness and Completeness
- First-order logic
- Tableaux/Sequent calculi and resolution for 1st order logic
- Soundness and Completeness for those calculi