

IN3070/4070 – Logic – Autumn 2020

Lecture 9: Logic Programming

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15th October 2019



DEPARTMENT OF
INFORMATICS



UNIVERSITY OF
OSLO

Today's Plan

- ▶ Motivation
- ▶ SLD Resolution
- ▶ Prolog
- ▶ Syntax
- ▶ Semantics
- ▶ Lists & Arithmetic
- ▶ Negation/Cut/If-then-else
- ▶ Summary

Outline

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The First-Order Resolution Calculus

Definition 1.1 (First-Order Resolution Calculus).

$$\frac{}{C_1, \dots, \{\}, \dots, C_n} \text{ axiom}$$

$$\frac{C_1, \dots, C_i \cup \{L_1\}, \dots, C_j \cup \{L_2\}, \dots, C_n, C_i\sigma \cup C_j\sigma}{C_1, \dots, C_i \cup \{L_1\}, \dots, C_j \cup \{L_2\}, \dots, C_n} \text{ resolution}$$

with $\sigma(L_1) = \sigma(\bar{L}_2)$

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$$\frac{C_1, \dots, C_i \cup \{L_1, \dots, L_m\}, \dots, C_n, C_i\sigma \cup \{L_1\sigma\}}{C_1, \dots, C_i \cup \{L_1, \dots, L_m\}, \dots, C_n} \textit{ factorization}$$

with $\sigma(L_1) = \dots = \sigma(L_m)$

- ▶ a **resolution proof** for a set of clauses S is a derivation of S in the resolution calculus; the **substitution** σ is local for every rule application; variables in every clause C can be **renamed**

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- ▶ given a **query**, “**SLD resolution**” is used to prove that the query is a logical consequence of the program
- ▶ **unification** is used to calculate a substitution of the variables in the given query
- ▶ in **imperative** programming languages, computation is **explicitly** constructed by the programmer (using if-then-else, while, for, ...)
- ▶ in **logic programming**, the program is a **declarative** specification and the resolution inference engine provides an **implicit** control

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- ▶ **SLD Resolution**
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Horn Clauses and Logic Programs

Definition 2.1 (Horn Clause).

A *Horn clause* is a clause that contains at most one positive literal (a *positive literal* is a non-negated literal). A *definite clause* is a Horn clause that contains a (single) positive literal.

Horn Clauses and Logic Programs

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Definition 2.2 (Logic Program).

A *logic program* consists of definite clauses of the form:

- ▶ *facts*: $\{A\}$ (A)
- ▶ *rules*: $\{A, \neg B_1, \dots, \neg B_n\}$ $(A \leftarrow B_1 \wedge \dots \wedge B_n)$

where A, B_1, \dots, B_n are atomic formulae.

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where A, B_1, \dots, B_n are atomic formulae.

Definition 2.3 (Goal or Query).

A *goal/query clause* has the form $\{\neg B_1, \dots, \neg B_n\}$ where B_1, \dots, B_n are atomic formulae.

SLD Resolution

SLD resolution (Selective Linear Definite clause resolution) is the inference rule used in logic programming

- ▶ it is a **refinement** of the general resolution rule
- ▶ it is sound and complete for **Horn clauses**

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- ▶ **first step**: **1st parent clause** $C_i \cup \{L_1\}$ is the **query clause**
- step $n \geq 2$: **1st parent clauses** $C_i \cup \{L_1\}$ is **resolvent** $C_i\sigma \cup C_j\sigma$ of step $n-1$
- ▶ **2nd parent clauses** $C_j \cup \{L_2\}$ is always a **clause of the logic program**

An SLD Resolution Derivation

Let $\{\neg Q_1, \neg Q_2, \dots\}$ be a **query clause** and
 $\{A_1, \neg B_1, \neg B'_1, \dots\}, \dots \{A_n, \neg B_n, \neg B'_n, \dots\}$ be a **logic program**.

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An **SLD resolution derivation** has the following form:

$$\{\neg Q_1, \dots, \neg Q_j, \dots\}$$

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$$\begin{array}{ccc}
 \{\neg Q_1, \dots, \neg Q_i, \dots\} & \{A_j, \neg B_j, \neg B'_j, \dots\} & \text{with } \sigma(Q_i) = \sigma(A_j) \\
 \downarrow & \swarrow & \\
 \{\neg D_1, \dots, \neg D_k, \dots\} & &
 \end{array}$$

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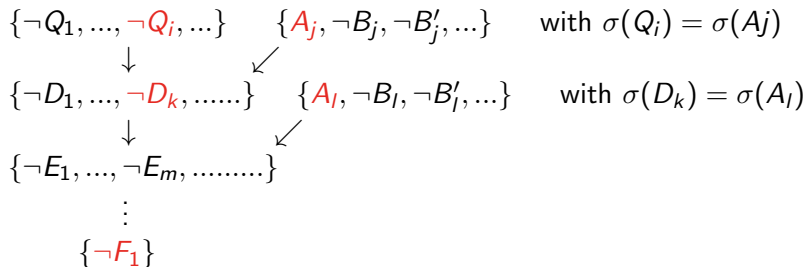
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 \{\neg D_1, \dots, \neg D_k, \dots\} \quad \{A_l, \neg B_l, \neg B'_l, \dots\} \quad \text{with } \sigma(D_k) = \sigma(A_l) \\
 \downarrow \quad \swarrow \\
 \{\neg E_1, \dots, \neg E_m, \dots\}
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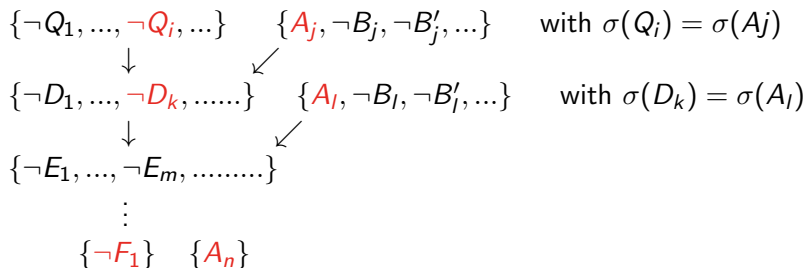
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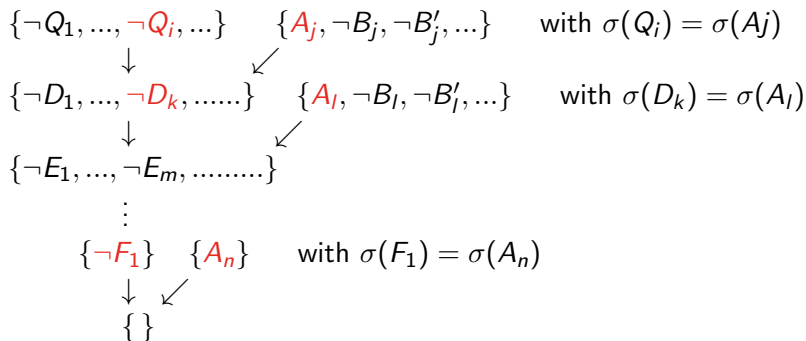
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The Programming Language Prolog

- ▶ Prolog (**P**rogramming in **L**ogic) is a **declarative** programming language invented in the early 1970s by **A. Colmerauer**, **R. Kowalski**, and **P. Roussel**

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- ▶ **declarative** programming: **specify** the problem and let the computer **solve** it
- ▶ **algorithm = logic + control** [Kowalski 1979]

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- ▶ A Prolog program is a **logic** program, i.e. a **set of definite clauses**
- ▶ the symbol **' :- '** is used to represent the implication **' \leftarrow '**
- ▶ A Prolog program is “executed” by the Prolog **interpreter** (**control**) that implements **SLD resolution**
- ▶ **search strategy**: choose **leftmost** literal in the first parent/goal clause (D_1) and choose second parent clause (D_2) from **top to bottom** among the program clauses

Prolog – An Example

- ▶ An example in Prolog (file `family.pl`)

```
male(thomas).  
male(rolf).
```

```
% these are facts
```

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```
male(thomas).
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```
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```
female(anna).
```

```
female(maria).
```

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```
female(anna).
```

```
female(maria).
```

```
parent(thomas,anna).
```

```
parent(maria,anna).
```

```
parent(rolf,maria).
```

```
% these are facts
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male(thomas).                                % these are facts
male(rolf).
female(anna).
female(maria).
parent(thomas,anna).
parent(maria,anna).
parent(rolf,maria).

father(X,Y) :- parent(X,Y), male(X).         % these are rules
mother(X,Y) :- parent(X,Y), female(X).
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- ▶ **start** Prolog and type `'[family].'` to load the program
- ▶ **Ctrl-C** **stops** Prolog; `'halt.'` **exits** Prolog

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- ▶ `?- grandfather(rolf,Y).`

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SLD Resolution Derivation – Example

program clauses:

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male(rolf).
```

```
parent(maria,anna).
```

```
parent(rolf,maria).
```

```
father(X,Y) :- parent(X,Y), male(X).
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```
grandfather(X,Z) :- father(X,Y), parent(Y,Z).
```

query: ?- grandfather(rolf,Y).

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grandfather(X,Z) :- father(X,Y), parent(Y,Z).
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query: ?- grandfather(rolf,Y).

```
{¬grandfather(rolf,Y)} {grandfather(X,Z),¬father(X,U),¬parent(U,Z)}
```

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$\{\neg\text{grandfather}(\text{rolf}, Y)\} \{\text{grandfather}(X, Z), \neg\text{father}(X, U), \neg\text{parent}(U, Z)\}$

↓

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with $\sigma(X)=\text{rolf}, \sigma(Y)=Z$

$\{\neg\text{father}(\text{rolf}, U), \neg\text{parent}(U, Z)\}$

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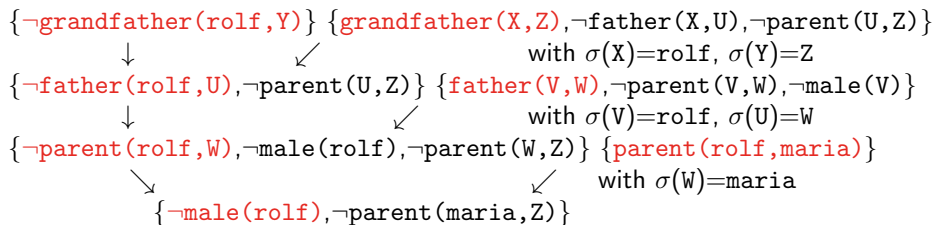
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SLD Resolution Derivation – Example

program clauses:

male(rolf).

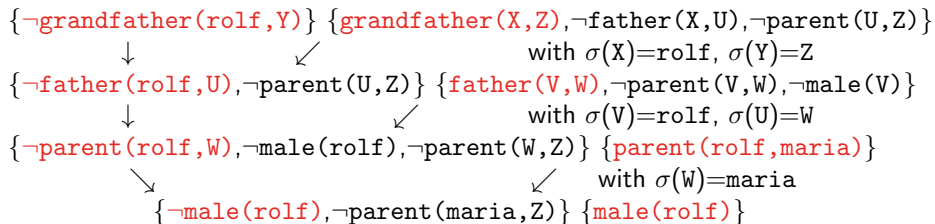
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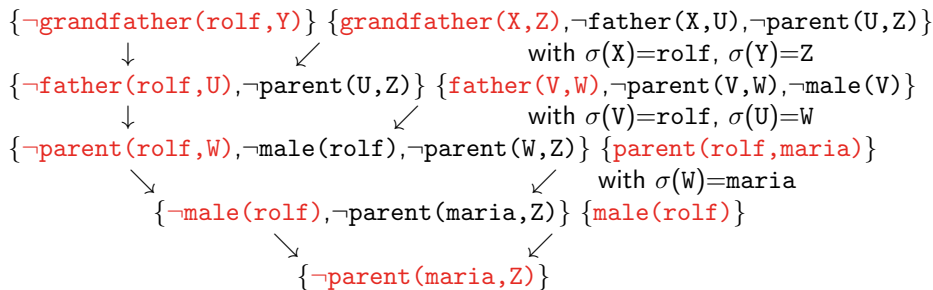
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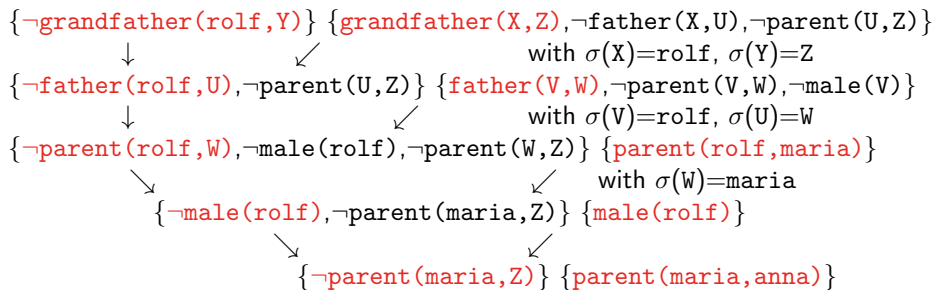
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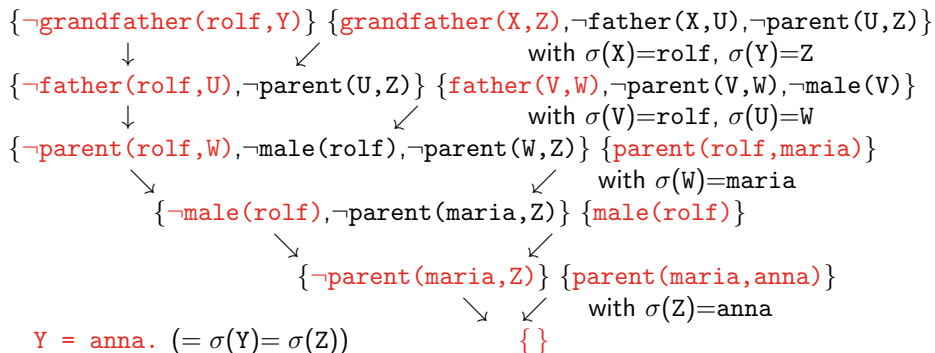
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Outline

- ▶ Motivation
- ▶ SLD Resolution
- ▶ Prolog
- ▶ **Syntax**
- ▶ Semantics
- ▶ Lists & Arithmetic
- ▶ Negation/Cut/If-then-else
- ▶ Summary

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Query:

- ▶ $\langle \text{predicate1} \rangle$, ... , $\langle \text{predicateN} \rangle$.
(e.g. `parent(maria,anna).` or `grandfather(rolf,Y).`)

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- ▶ if an appropriate **fact** matches, then the predicate/goal succeeds
- ▶ if the head of a **rule** matches, then Prolog continues by trying to fulfil the predicates of the rule's body
- ▶ the database is searched **top to bottom**
- ▶ if more than one fact or head of a rule matches, then **alternative options** are considered if the search fails (**via backtracking**)

Operational Semantics – Example

```
male(thomas). male(rolf). female(anna). female(maria).  
parent(thomas,anna). parent(maria,anna). parent(rolf,maria).  
father(X,Y) :- parent(X,Y), male(X).  
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-> father(X,Y)
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      male(thomas)
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?- grandfather(X,anna).
-> father(X,Y) -> parent(X,Y) -> parent(thomas, anna)
           male(thomas)
parent(anna,anna) -> fail

```

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```

```
                                -> parent(maria, anna)
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```
      male(maria) -> fail
```

```
                                -> parent(rolf, maria)
```

```
      male(rolf)
```

```
      parent(maria,anna)
```

```
grandfather(rolf,anna) succeeds
```

```
X = rolf.
```

- ▶ variables are **instantiated** (“bound”) during the unification of terms

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```
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```

```
...
```

```
fact_n.
```

```
head_1 :- body_1.
```

```
...
```

```
head_m :- body_m.
```

```
?- query.
```

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$$\forall X_1, \dots, X_n (\exists Y_1, \dots, Y_n \text{ body}_i \rightarrow \text{head}_i)$$

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- ▶ **inference engine** is a theorem prover based on **SLD resolution** (only **Horn clauses**, **depth-first** search (incomplete!), **no occurs-check** (unsound!))

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$[a | [b | [c | [d | [e]]]]]$

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$[a|[b|[c|[d|[e]]]]]$

$[a, b|[c, d, e]]$

$[a, b, c, d|[e]]$

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Predefined Predicates on Lists

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- ▶

```
?- member(a,[a,b,c]).  
true .  
?- member(X,[a,b]).
```

Predefined Predicates on Lists

- ▶ `member(Element,List)` succeeds iff `Element` occurs in `List`
- ▶ `append(List1,List2,List3)` succeeds iff appending `List1` and `List2` results in `List3`
- ▶ `length(List,Length)` succeeds iff `List` has length/size `Length`
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Lists – Examples

- ▶ **delete** all identical elements from list

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delete([],_, []).
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Example: Ordered Lists

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Comparison only works if variables are instantiated to numbers.

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Outline

- ▶ Motivation
- ▶ SLD Resolution
- ▶ Prolog
- ▶ Syntax
- ▶ Semantics
- ▶ Lists & Arithmetic
- ▶ **Negation/Cut/If-then-else**
- ▶ Summary

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- ▶ Very different from our classical notion of logical consequence

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```
parent(thomas,anna) :- !.
parent(maria,anna). parent(rolf,maria).
```

```
?- parent(X,anna).
```

```
X = thomas.
```

```
? grandfather(X,anna).
```


The Cut

- ▶ the **cut** `!` is used to restrict Prolog's backtracking mechanism
- ▶ the cut is a predefined predicate that **succeeds** when it is encountered for the first time; any attempt to re-fulfil it results in the **failure** of the calling (head) predicate
- ▶ **"green cut"**: does **not** change solutions, only affects efficiency

```
factorial(0,1) :- !.
```

```
factorial(I,N) :- I>0,I1 is I-1,factorial(I1,N1),N is I*N1.
```

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? grandfather(X,anna).
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```
false.
```

Example: Siblings

Disjunction and If-then-else

- ▶ `predicate :- predicate1 ; predicate2.`
succeeds if `predicate1` succeeds or `predicate2` succeeds; backtracking over `predicate1` and `predicate2` when re-fulfilling `predicate`

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grandparent(X,Y) :- grandfather(X,Y) ; grandmother(X,Y).
```

(backtracking over `grandfather(X,Y)` `grandmother(X,Y)`)

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 (information given by `male(X)` needs to be complete)

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 `male(X) -> grandfather(X,Y) ; grandmother(X,Y).`
 (information given by `male(X)` needs to be complete)
`grandparent(X,Y) :-`
 `grandfather(X,Y) -> true ; grandmother(X,Y).`
 (no backtracking over `grandfather`)

Problems with Prolog

- ▶ No type system
- ▶ No standardized module system
- ▶ Non-declarative arithmetic
- ▶ Cut needed for efficiency
 - ▶ Cut has non-declarative semantics
 - ▶ Cut can simulate negation as failure (non-monotonic)
 - ▶ Cut can be tricky to use
 - ▶ Cut makes automated optimization hard
- ▶ IO does not play nice with backtracking

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Outline

- ▶ Motivation
- ▶ SLD Resolution
- ▶ Prolog
- ▶ Syntax
- ▶ Semantics
- ▶ Lists & Arithmetic
- ▶ Negation/Cut/If-then-else
- ▶ **Summary**

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- ▶ **next week**: DPLL (efficient SAT solving)