Grading Guidelines
IN3070/IN4070
Autumn 2023

The maximum number of marks for the whole exam was 100.

The minimum number of marks required for each grade will be published after the exam was graded.

Question 1 – Sequent Calculi LK and LJ

Prove the validity of the following formulae using the given calculus. Note that the first two formulas are to be proven in the classical logic LK, while the last one is to be proven in the intuitionistic logic LJ.

In this task you can submit a hand-written answer. Use the sketching paper handed to you in the exam room for this. See instructions in the link below the task bar.

A) $\neg(p \land q) \to (\neg p \lor \neg q)$ using propositional LK [5 marks]

C) $(p \lor \neg p) \to (\neg(p \land q) \to (\neg p \lor \neg q))$, using propositional LJ [10 marks]

Question 2 – Classical first-order semantics

A) Show that the formula $\forall x(p(x) \lor r(x)) \to \forall x p(x) \lor \forall x r(x)$ is not valid by constructing a falsifying interpretation. [8 marks]

B) Show that the formula $\forall x(p \lor r(x)) \to p \lor \forall x r(x)$ is valid by reasoning semantically. (That is, show that it is true in all interpretations by reasoning about interpretations, do *not* use the calculus and the soundness theorem.) [8 marks]

In this task you can submit a hand-written answer. Use the sketching paper handed to you in the exam room for this. See instructions in the link below the task bar.

Question 3 – Modal Sequent Calculus

In the sequent calculus for modal logics, the axiom that closes a branch requires the same label on the formulae in the antecedent and the succedent.

$$u : A, \Gamma \Rightarrow u : A, \Delta$$

If this were not required, i.e. if an axiom had the shape

$$u : A, \Gamma \Rightarrow v : A, \Delta$$

allowing different labels in the antecedent and succedent, the calculus would be unsound. Show this by giving a formula that is not valid in modal logic K but that has a closed derivation in the calculus with the wrong axiom. [10 marks]

Question 4 – An alternative beta rule

Consider replacing the $\vee$-left rule of the propositional sequent calculus LK by the following rule:

$$\frac{A, \Gamma \vdash B, \Delta \qquad B, \Gamma \vdash A, \Delta}{A \vee B, \Gamma \vdash \Delta}$$

A) Is the resulting calculus still sound? Explain why, or give a formula that can be proven although it is not valid. [6 marks]

B) Is the resulting calculus still complete? Explain why, or give a formula that cannot be proven all though it is valid. [6 marks]

C) If $A \vee B$ is true, then either A is true and B is not, or B is true and A is not, or A and B are both true. We could try to capture this using the following rule with three premisses:

$$\frac{A, \Gamma \vdash B, \Delta \qquad B, \Gamma \vdash A, \Delta \qquad A, B, \Gamma \vdash \Delta}{A \vee B, \Gamma \vdash \Delta}$$

Would this replacement for the usual $\vee$-left rule leave the calculus sound? Complete? [5 marks]

D) Given the discussions about branching in the lecture about DPLL, would it be a good idea to implement this for automated proof search? [3 marks]

In this task you can submit a hand-written answer. Use the sketching paper handed to you in the exam room for this. See instructions in the link below the task bar.

1 Question 5 – Hintikka Sets

In this question, we will work with **propositional** formulas in **negation normal form**, i.e. the set of formulas F is inductively defined as the smallest set such that

- $p \in F$ for any atomic formula p
- $\neg p \in F$ for any atomic formula p
- $A \vee B \in F$ if $A, B \in F$
- $A \wedge B \in F$ if $A, B \in F$

A set of formulae $H \subseteq F$ is called a **Hintikka set** if it satisfies the following conditions:

- There is no atomic formula p with both $p \in H$ and $\neg p \in H$
- For every $A \vee B \in H$, either $A \in H$ or $B \in H$ (or both)
- For every $A \wedge B \in H$, $A \in H$ and $B \in H$

Hintikka sets, named after the Finnish philosopher and logician Jaakko Hintikka, can be used in the completeness proof of one-sided sequent calculi: the formulae in the antecedents of a saturated open branch form a Hintikka set.

Show that every Hintikka set is satisfiable, i.e. that there is a propositional interpretation that makes all formulae in the set true.

Hints:

- you can define the interpretation from the literals in H, just like in the completeness proof shown in the course.
- to show that *all* formulae in H are satisfied, use structural induction on formulas.
- remember to properly explain what is the base of the induction, what are the induction steps, when you use the induction hypothesis, etc.

[16 marks]

2 Question 6 – Resolution

Prove that the following formula is valid, using the resolution calculus

$$(\forall x(p(x) \to p(f(x)))) \to (\forall x(p(x) \to p(f(f(x)))))$$

Remember that resolution is a refutation calculus, i.e. you can derive that a set of clauses is unsatisfiable. Also remember that variables should be made disjoint before applying resolution.

- Arriving at a correct set of clauses: 5 credits
- Correct resolution proof: 5 credits

Question 7 – Description Logics

A) The calculus presented for the description logic ALC has a "blocking condition." The application of ∃R-left and ∀R-right rules is restricted to "labels that are not blocked." You do not need to give the definition of these blocking conditions, but please write in one sentence why they are needed, i.e. what their effect on the calculus is. [4 marks]

B) One way of defining the semantics of description logics, as shown in the lecture, is by a translation to first order logic. Concepts are translated to formulas with one free variable, while ABox and TBox assertions are translated into closed formulas.

Can a similar translation be given from first-order logic to a description logic like ALC? Write why in one sentence. [4 marks]